

# Effect of Residual Frequency Offsets on the Performance of Adaptive Equalizers

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## Abstract

This paper has interest in the effect of a fine frequency offset, defined in ITU-T G.225, to the training performance of an adaptive equalizer. This paper uses Hilbert filter in configuring a transmission system model in order to let it get a frequency offset. Also additive white Gaussian noise and band-limited filter are considered. The signal received from the above transmission system applies to an adaptive equalizer with LMS algorithm, and its training procedures are investigated. As a result, we could find that even small fine frequency offset can severely deteriorate training performance of adaptive algorithm.

**Keywords:** Frequency offset, Adaptive equalizer

## I. Introduction

The frequency offset is defined as the difference between the frequency applied to one end of channel and the frequency received at the other end. It is one of several impairments that degrade the service quality of data transmission. In particular, it cannot be tolerated in high-speed data transmission systems that use synchronous phase modulation[1]. ITU-T G.225 recommends that the frequency offset should not exceed 2 Hz for the public switched telephone network (PSTN)[2,3].

Although the carrier recovery circuit compensates the most of the frequency offset quite successfully at the front end of the receiver, any small amount of residual frequency offset may still remain due to the imperfection of the carrier recovery circuits. Surprisingly, not much work has been done on the effect of the residual frequency offset so far.

In this letter, the effect of the residual frequency offsets on the performance of the adaptive equalizer is investigated. A high-speed data communication system is considered, where there exists the undesirable residual frequency offset in the front end of the receiver and also the adaptive equalizer is necessary. We first describe the experimental model used in our discussion.

## II. Description of Experimental Model

The block diagram of our experimental model is illustrated in Figure 1, where the adaptive equalizer is employed to combat with the inter symbol interference (ISI) as well as the frequency magnitude distortion over an additive white Gaussian noise (AWGN) channel[3-5].

The band-limited transmission channel is modeled in such a way that its magnitude response is chosen to be that of the general PSTN as depicted in Figure 2. Note that the channel characteristic has the non-flat magnitude

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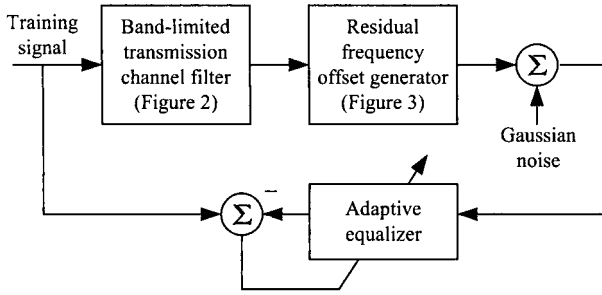


Figure 1. The block diagram of the experimental model

response for the purpose of taking the magnitude distortion of the transmission channel into account.

Figure 3 shows the block diagram for generating an arbitrary residual frequency offset  $\Delta f$ . The upper single sideband (SSB) modulator by making use of the Hilbert filter is employed. Some important parameters used in this experimental model are tabulated in Table 1.

Since a general PSTN (Public Switching Telephone Networks) have a passband of about 300 to 3400 Hz, a bandpass filter is adopted with the same passband of the PSTN channel [6]. In order to make the general transmission characteristics of the PSTN channel, the passband ripple of the band-limited filter is designed to be less than 10 dB and the attenuation of the stopband is designed to be above 40 dB attenuation from the passband gain. This band-limited filter is implemented by the FIR structure that has the filter order of 119 so that the filter takes the above PSTN channel characteristics. The magnitude response of the designed filter adopted in this paper is shown in Fig. 2. Also additive white Gaussian noise exists in the channel and it appears typical 27 dB signal to noise ratio in general long distance PSTN channel [6].

In this paper, Hilbert filter is used for generating frequency offset. The ideal Hilbert filter response is follow:

$$H(e^{j\omega T}) = \begin{cases} -j & 0 < \omega \leq \frac{\pi}{T} \\ j & -\frac{\pi}{T} \leq \omega < 0 \end{cases} \quad (1)$$

where  $T$  is the sampling rate of the processed signal through this filter. This filter has continuous and periodic characteristics in frequency domain with  $2\pi/T$ . This can be transferred into a time series with  $n$  of sampling space

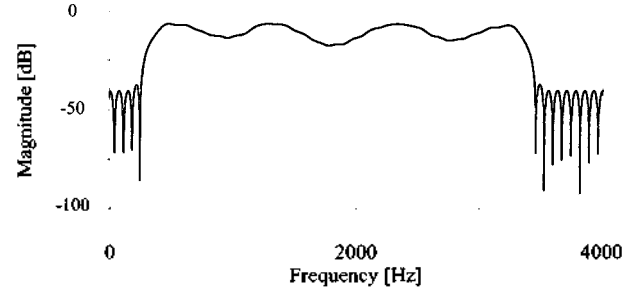


Figure 2. The characteristic of the band-limited channel

by using inverse Fourier transform as follows:

$$\begin{aligned} a(n) &= \frac{T}{2\pi} \int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} H(e^{j\omega T}) e^{j\omega n T} d\omega \\ &= \frac{T}{2\pi} \int_{-\frac{\pi}{T}}^0 j e^{j\omega n T} d\omega - \frac{T}{2\pi} \int_0^{\frac{\pi}{T}} j e^{j\omega n T} d\omega \\ &= \frac{Tj}{2\pi} \cdot \frac{1}{njT} \cdot [e^{j\omega n T}]_{-\frac{\pi}{T}}^0 - \frac{Tj}{2\pi} \cdot \frac{1}{njT} \cdot [e^{j\omega n T}]_0^{\frac{\pi}{T}} \\ &= \frac{1}{2\pi n} (1 - e^{-j\pi n}) - \frac{1}{2\pi n} (e^{j\pi n} - 1) \\ &= \frac{1}{\pi n} - \frac{1}{2\pi n} (e^{-j\pi n} + e^{j\pi n}) \\ &= \frac{1}{\pi n} (1 - \cos \pi n) \\ &= \begin{cases} \frac{2}{\pi n} & n: \text{odd} \\ 0 & n: \text{even or zero} \end{cases} \quad (2) \end{aligned}$$

This filter cannot be realized because it represents a non-causal system so that we have to transfer its response into a causal and finite response system by using time window and shift. In our simulation, the time window size is 103 sample spaces and its shape is rectangular that makes delay of 51 sample space in delay buffer in Figure 3.

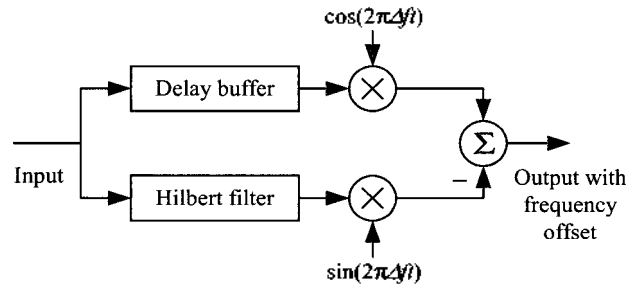


Figure 3. Residual frequency offset generator in the model

Table 1. Parameters of channel model

Band-limited filter order	119 (symmetric)
3 dB passband	300 - 3400 Hz
Ripple on the passband	10 dB
Attenuation on the stopband	Greater than 40 dB
Hilbert filter order	103 (symmetric)
Signal-to-channel noise	27 dB

### III. Simulations of the Adaptive Equalizer

The 128-tap normalized least mean square (NLMS) algorithm is used for the adaptive equalizer. A band-limited Gaussian random signal, which is uncorrelated with AWGN, is used as a training signal with signal power 0.06. The adaptive step-size is selected to be 0.0651.

Figure 4 shows the relative steady-state mean squared errors (MSE) (in dB) of the adaptive equalizer with different values of the residual frequency offsets when the adaptive equalizer is used for the ISI compensation problem. The MSE values are obtained by taking the average of the last 30,000 samples after the convergence takes place for each residual frequency offset. The relative MSE values displayed in the figure are normalized in such a way that the MSE value with zero residual frequency offset is 0 dB.

Note that the steady-state MSE increases more rapidly when  $\Delta f$  is smaller. This means that the performance of the adaptive equalizer becomes more sensitive to the small values of the residual frequency offset. It is observed that the adaptive equalizer experiences additional 11.5 dB loss in spite of  $\Delta f$  being only 0.1 Hz in our particular experiment.

Figure 5 illustrates the output magnitude responses of

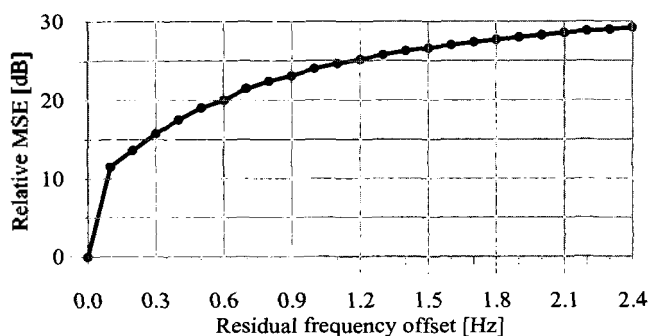
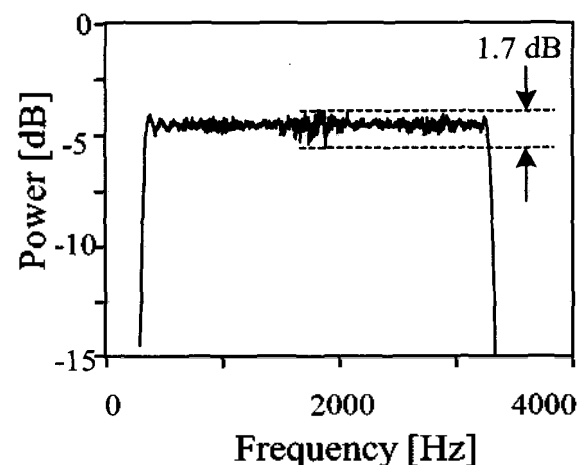


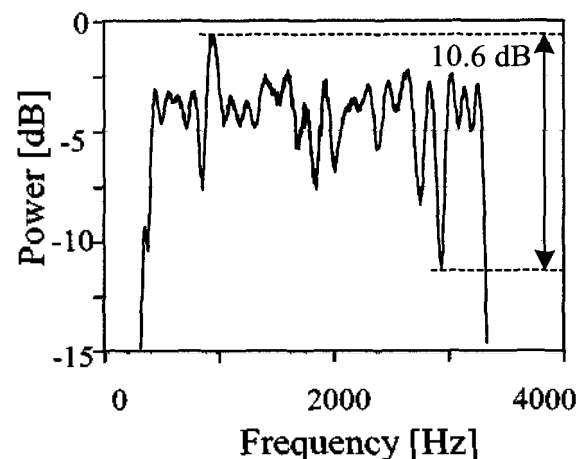
Figure 4. Variations of error power with respect to  $\Delta f$

the adaptive equalizer in the steady-state. Figures 5(a) and 5(b) are the two cases of  $\Delta f$  being 0 Hz and 0.9 Hz, respectively. Recall that the non-flat magnitude response of the transmission channel employed in this experiment is depicted in Figure 2, and that the frequency ripple of the channel is approximately 10 dB.

As can be seen in the figure, when  $\Delta f$  is equal to 0 Hz, the adaptive equalizer compensates the magnitude distortion of the transmission channel satisfactorily so that the output frequency ripple of the equalizer becomes less than 1.7 dB for all passband frequencies. When  $\Delta f$  is equal to 0.9 Hz, on the other hand, the output frequency ripple of the equalizer is measured as large as 10.6 dB. This result proves clearly that the adaptive equalizer cannot compensate the magnitude distortion of the transmission channel adequately, or it sometimes becomes another source of distortion, when there exists a certain amount of the residual frequency offset.



(a)  $\Delta f = 0.0$  Hz case



(b)  $\Delta f = 0.9$  Hz case

Figure 5. Overall magnitude response

It is usually not very difficult to maintain the residual frequency offset small enough to be ignored in most practical communication systems. It is, however, very important to take good care of the residual frequency offset, no matter how small it might be, when the adaptive equalizer is employed in the receiver of high-speed data transmission systems. We are currently working on development of the techniques for accurate detection and compensation of the residual frequency offset.

## IV. Conclusion

In this letter, the effect of the residual frequency offsets on the performance of the adaptive equalizer has been empirically investigated. The experimental model utilizing the adaptive equalizer has been described. We have observed that the steady-state MSE of the equalizer increases dramatically even for small values of the residual frequency offset. It has also been observed that the adaptive equalizer cannot compensate the magnitude distortion of the transmission channel adequately when there exists a certain amount of the residual frequency offset.

## [Profile]

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