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A Numerical Method for a High-Speed Ship with a Transom Stern

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Abstract

A numerical method is developed for computing the free surface flows around a transom stern of a ship at a high Froude number. At high speed, the flow may be detached from the flat transom stern. In the limit of the high Froude number, the problem becomes a planning problem. In the present study, we make the finite-element computations for a transom stern flows around a wedge-shaped floating ship. The numerical method is based on the Hamilton's principle. The problem is formulated as an initial value problem with nonlinear free surface conditions. In the numerical procedures, the domain was discretized into a set of finite elements and the numerical quadrature was used for the functional equation. The time integrations of the nonlinear free surface condition are made iteratively at each time step. A set of large algebraic equations is solved by GMRES(Generalized Minimal RESidual, Saad and Schultz 1986) method which is proven very efficient. The computed results are compared with previous numerical results obtained by others.

Keywords: transom stern flow, nonlinear free surface condition, finite element method

1 Introduction

High-speed displacement ships are characterized by flat terminating in a transom. This type of ship has been familiar to naval architects for over 60 years. While there has been a remarkable development in the field of wave resistance, the treatment of transom stern flows has been received relatively little concern. At a high speed, the flow behind transom stern ships is characterized by smooth separation of the streamlines at the transom. This flow pattern has been credited with the reduced wave resistance for high-speed transom stern ships, as compared to their corresponding cruiser stern. In the extreme condition, i.e. at an even higher Froude number, this problem becomes a planing problem. Therefore, this phenomenon requires physical insights and suitable modeling obtaining a correct solution.

There have been largely two approaches in modeling the transom stern flows. One is to assume the detachment of flow from the transom stern. Most part of research to this problem belongs to this category. Others are to obtain the wetted surface on the transom stern as a part of the solution. Generally, this approach is based on solving the nonlinear free surface condition.

Investigations on steady nonlinear flows generated by sterns of two-dimensional semi-infinite hulls are Dagan and Tulin(1972), Vanden-Broeck and Tuck(1977), Vanden-Broeck et al(1978). Tulin and Hsu(1986) developed a model for high-speed slender ships with transom sterns Making the assumption that both the beam and draft are small relative to the length, Cheng(1989) introduced a practical method based on Dawson's method (1977) for 3D transom stern flows. As a dry transom boundary condition, he imposed the static pressure to be atmospheric and restricted the flow to leave tangentially at transom. Vanden-Broeck(1980) used an integro-differential formulation for steady two-dimensional potential flow past a flat-bottomed body with a transom stern. Assuming that the flow detaches from the corner of the body, he found that a steady-state nonlinear solution exists only for draft-based Froude numbers greater than 2.23. Haussling(1980) and Coleman and Haussling(1981) considered the unsteady nonlinear flows caused by transom sterns of 2-D, semi-infinite hulls. Especially, Coleman and Haussling(1981) provided the information in what ranges of Froud number the stable solutions could be obtained asymptotically. Van Eseltine and Haussling(1981) studied the stern flow generated by a semi-infinite, 3-D transom stern hull moving at a constant speed. The location and shape of the hull were fixed, and the wetted area was determined as part of the linearized solution. Another 3-D approaches are Reed et al(1981), Telste and Reed(1993), Doctors and Day(2001). Recently attempts have been made to approach this problem in full scale Reynolds number (Eca and Hoekstra 1997).

In this study, the model is assumed as vertically wall-sided for simplicity. Specifically the model has a wedge-shaped bow, a parallel body cut off at the stern and flat bottom, thus a transom stern ship model. Computations for the model are made to investigate the generation of a dry transom behind the transom stern. Computations for the model at moderate high Froude number show the separation of streamlines behind the transom stern resulting in a dry transom. Dry transom area is obtained as a part of the solution. A simple condition for dry transom is suggested.

2 Mathematical formulation

We used the Cartesian coordinate in this paper. Oxyz is the coordinate system with Oz opposing the direction of gravity and z = 0 coincides the undisturbed free surface. The body moves to the negative x-direction with velocity U. Figure 1 shows the coordinate system used in this study.

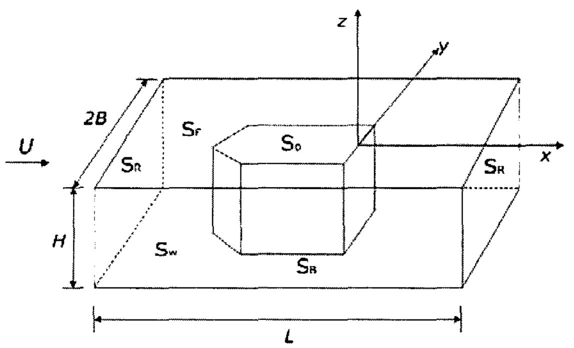

Figure 1: Coordinate System

The formulation is given in an inertial coordinate system. However, in the numerical procedures, the computing domain is moving with the body. We assume that the fluid is inviscid, incompressible and its motion is irrotational. So the velocity potential exists and is defined as

$$\vec{u}(\vec{x},t)=\nabla\phi(\vec{x},t) \tag{1}$$

where $\vec{x}=(x,y,z)$ and ϕ is the velocity potential. From the continuity condition we obtain the Laplace equation

$$\nabla^2\phi(\vec{x},t)=0 \qquad \text{in the fluid domain D} \tag{2}$$

The boundary condition on the body boundary surface, S_0, is

$$\phi_n=-Un_x \tag{3}$$

Where the vector, $\vec{n}=(n_x,n_y,n_z)$, denotes the outward unit normal vector on the boundaries. The conditions on the free surface, $z=\zeta(x,y,t)$ can be given by the kinematic and dynamic boundary conditions as follows,

$$\zeta_t=-U\zeta_x+\frac{1}{n_z}\phi_n, \tag{4}$$

$$\phi_t=-U\phi_x-\frac{1}{2}|\nabla\phi|^2-g\zeta-\frac{p}{\rho}, \tag{5}$$

where g and ρ denote the gravitational acceleration and the density of fluid, respectively. The pressure, $p=p(x,y,t)$ is taken as zero unless a non-zero pressure distribution is specified. The fluid motion is assumed to be at rest initially, therefore the initial condition may be given as

$$\phi=\phi_t=0 \qquad \text{at} \qquad t=0 \tag{6}$$

and the radiation condition ;

$$\phi\to 0 \qquad \text{as} \qquad x^2+y^2\to\infty \tag{7}$$

The depth of water is h, and the width of numerical wave tank, $y=\pm B$. The bottom and wall boundary conditions are given as follows.

$$\phi_n=0 \text{ on } z=-h \tag{8}$$

$$\phi_n=0 \text{ on } y=\pm B \tag{9}$$

To make our formulation scale-independent, we nondimensionalized all physical variables by $d,\rho d^3,\sqrt{d/g}$ for length, mass and time, respectively. The length of draft is d. After nondimensionalization, the governing equation and the boundary conditions can be written as

$$\nabla^2\phi(\vec{x},t) = 0 \qquad \text{in} \quad D \tag{10}$$

$$\phi_n = -F_d n_x \qquad \text{on} \quad S_o \tag{11}$$

$$\zeta_t = -F_d\zeta_x + \frac{1}{n_z}\phi_n \qquad \text{on} \quad S_F \tag{12}$$

$$\phi_t = -F_d\phi_x - \frac{1}{2}|\nabla\phi|^2 - \zeta - p \qquad \text{on} \quad S_F \tag{13}$$

$$\phi_n = 0 \qquad \text{on} \quad z = -1 \tag{14}$$

$$\phi_n = 0 \qquad \text{on} \quad y = \pm B \tag{15}$$

where $F_d = U/\sqrt{gd}$ is the draft Froude number, $n_z = 1/\sqrt{1+\zeta_x^{\,2}+\zeta_y^{\,2}}$ is defined on the free surface and all variables are redefined as nondimensionalized ones.

3 Variational formulation

We introduce a variational formulation equivalent to the above. First we define the variational functional, J and the Lagrangian L as

$$J = \int_0^{t^*} L dt \tag{16}$$

$$L = \iint_{\bar{S}_F} \phi\zeta_t dS - \frac{1}{2}\iint_{\bar{S}_F} \zeta^2 dS - \frac{1}{2}\iiint_D |\nabla\phi|^2 dV \tag{17}$$

where $\bar{S}_F$ is the projection of S_F on Oxy plane and t^* is the final time. Taking the variations on J first with respect to ζ, we can obtain δJ_ζ as

$$\begin{aligned}\delta J_\zeta &= \int_0^{t^*} dt\left[\iint_{\bar{S}_F}\left(\phi\delta\zeta_t - \zeta\delta\zeta - \frac{1}{2}|\nabla\phi|^2\delta\zeta\right)dS\right] \\ &= \iint_{\bar{S}_F} [\phi\delta\zeta]_{t=t^*} - [\phi\delta\zeta]_{t=0} dS - \int_0^{t^*} dt\left[\iint_{\bar{S}_F}\left(\phi_t + \frac{1}{2}|\nabla\phi|^2 + \zeta\right)\delta\zeta dS\right]\end{aligned} \tag{18}$$

Next the variations on J with respect to ϕ, δJ_ϕ can be obtained as

$$\begin{aligned}\delta J_\phi &= \int_0^{t^*} dt\left[\iint_{\bar{S}_F} \zeta_t\delta\phi\, dS - \iiint_D \nabla\phi\cdot\nabla\delta\phi dV\right] \\ &= \int_0^{t^*} dt\left[\iint_{\bar{S}_F}\left(\zeta_t - \frac{1}{n_z}\phi_n\right)\delta\phi\, dS + \iiint_D \nabla^2\phi\ \delta\phi dV\right]\end{aligned} \tag{19}$$

Here $\delta J = \delta J_\zeta + \delta J_\phi$. Equation (18) means that the stationary condition on J for the variation with respect to ζ recovers the dynamic free surface condition in each time and that the wave elevation at t=0, t^* should be specified as the constraints. Equation (19) shows that the stationary condition on J for the variation of ϕ recovers the kinematic condition on S_F and the governing equation. The above variational form is previously given by Miles(1977) and slightly different from that by Luke(1967). In the present variational formulation the wave elevation ζ assumed to be known at t=0, t^*, whereas Luke assumed the potential ϕ to be known at both initial and final times additionally. The present functional has an advantage over the original Luke's variational functional in treating the nonlinear free surface boundary conditions. More details on the finite-element discretization, treatments for the numerical instability and time integration method can be found in the earlier paper by Bai et al(2002).

4 Dry transom condition

Here we will describe how to treat the dry transom condition in the numerical computations. The presence of a dry transom causes a difficulty in numerical computation, especially because wetted transom becomes dry as Froude number increases and in addition to this, it should be obtained as a part of the solution. Bai et al(2002) showed that a body with a transom stern mounted on the bottom would make a dry bottom at high depth Froude number. In this work, they found out that the critical Froude number for drying condition behind the flat body should be larger than $\sqrt{2}$ by a simple analysis. But in the transom stern problem where the body is not mounted on the bottom but floating, the analysis is not so simple. For the dry transom, we assumed that the flow is detached smoothly from the bottom of the ship. We imposed a condition for dry transom similar to Cheng(1989) based on the Bernoulli equation. Due to the Bernoulli equation, the flow on the free surface in two dimensions should satisfy the following steady relation. The velocity of the external flow is defined as U.

$$\frac{1}{2}|\nabla\phi|^2 + g\zeta = \frac{1}{2}U^2 \tag{20}$$

Since the flow is dominant in the x-direction, the above equation could be rearranged as follows.

$$\phi_x^{\,2} = U^2 - 2g\zeta \tag{21}$$

This should be satisfied as the dynamic free surface condition in the time domain. After simple manipulations, we can get a final dry transom condition as follows.

$$\frac{\partial\phi}{\partial t} + F_d\frac{\partial\phi}{\partial x} + \frac{1}{2}|\nabla\phi|^2 + \zeta + \frac{P^*}{\rho} = 0 \tag{22}$$

$$\frac{P^*}{\rho} = -\frac{1}{2}F_d\left(F_d + \sqrt{F_d^{\,2} + 2}\right) \quad \text{at transom stern} \tag{23}$$

The pressure term in Equation (23) is a fictitious pressure to make the wetted transom stern dry. As noted in Tulin(1986), there are vorticites generated behind the transom by the existence of the transom stern. Smooth separation condition that we proposed on the free surface condition acts like a pressure drop induced by vorticity generated at the transom stern. Figure 2 shows a simple sketch showing a dry transom and water front behind stern.

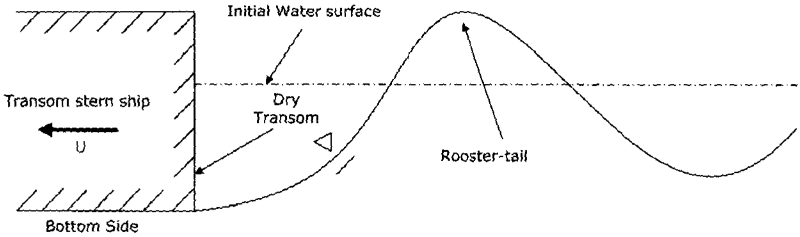

Figure 2: Sketch of dry transom.

Transom stern flows accompany with the rooster-tail behind the dry transom. As the Froude number approaches a critical value, the transom stern flow has a sharp crest behind the transom and finally breaks as in Coleman and Haussling(1981).

5 Results and discussions

Numerical validations of the present scheme have been reported previously in Bai, et al.(2002,2003) for various kinds of problem in a restricted finite-depth channel. In the previous researches for transom stern flows, the bow flow has not been considered due to its numerical difficulty. To avoid this difficulty in the previous work a rather simple semi-infinite body has been treated to investigate the transom stern flow.

Length of Wedge-shaped bow (Lw/d)	*20*
Length of Parallel middle body (Lm/d)	*10*
Water depth (h/d)	*11*
Beam width (W/d)	*10*
Time interval($\sqrt{d/g}$ *)*	*0.1*

In this study, both finite bow and transom stern are considered simultaneously. However, a long bow is chosen in our computation because of local breaking waves appearing in front of the blunt bow at high Froude numbers. Characteristic dimensions used in the computations are shown in Figure 3.

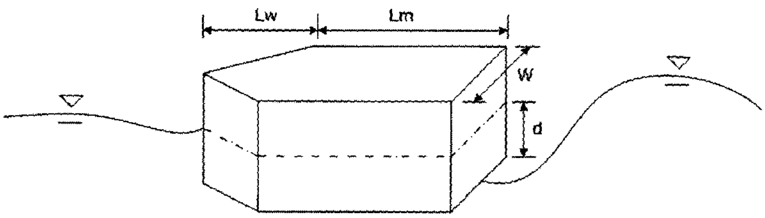

Figure 3: Definition of major dimensions.

For the case $F_d = 2.35$, we compared our results with previous two-dimensional results of Venden-Broeck(1980). The comparison in Figure 4 shows a good agreement with wave height behind the stern. Tangential detachment of the flow is more distinctive in the present three-dimensional result.

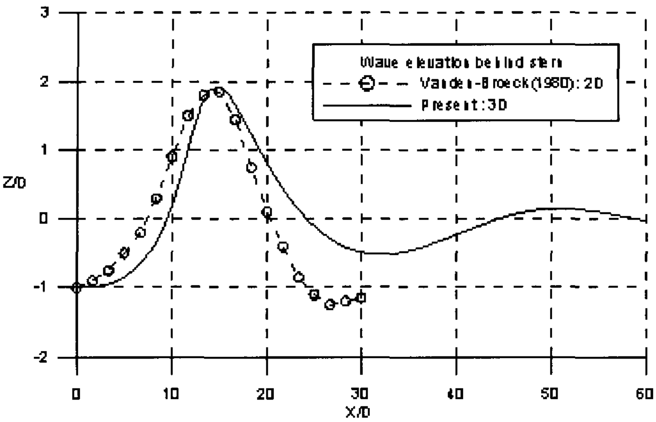

Figure 4: Free surface elevations behind the stern at $F_d = 2.35$

Figure 5 shows the wave profile at $F_d = 2.35$. Dry transom area in the center plane of wedge-shaped transom body can be well observed. The well-known rooster-tail can be also observed behind the transom stern.

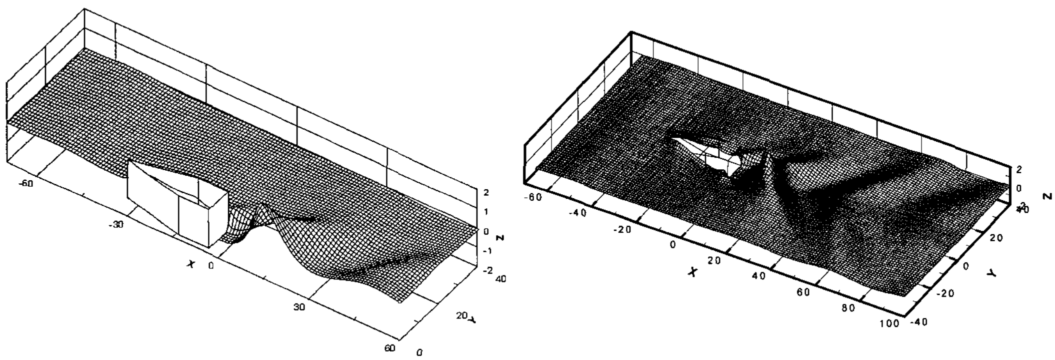

Figure 5: 3-D half and full domain views of surface elevations at $F_d = 2.35$

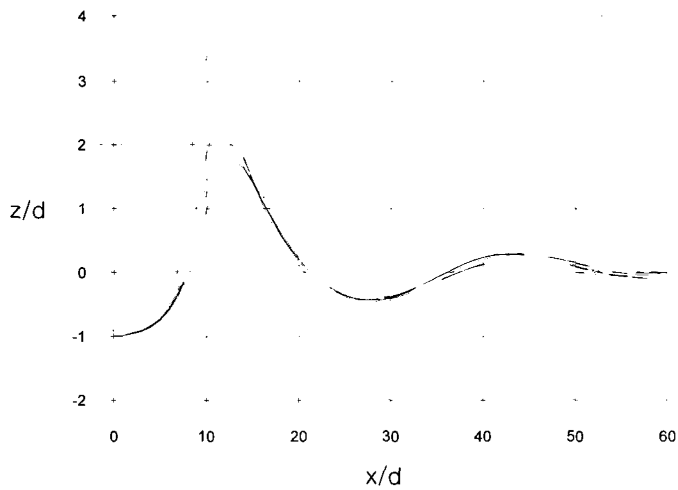

Figure 6: Time variation of the surface elevations behind the stern at $F_d = 2.2$ from t=40sec to 240sec with Δt=40sec until wave breaks.

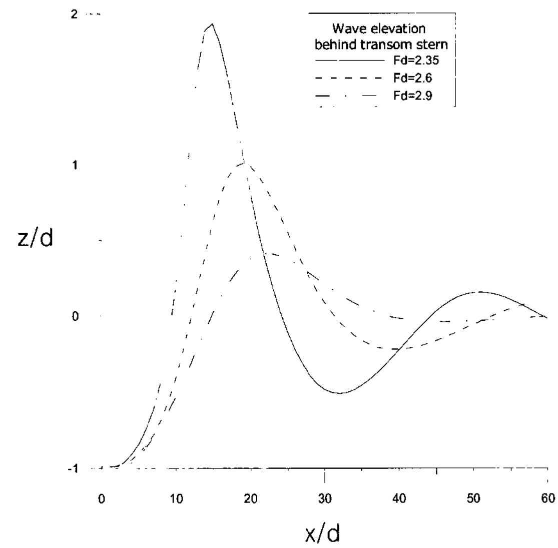

Figure 7: Comparison of surface elevations behind the transom stern in some draft Froude number.

Figure 6 shows the time variation of the surface elevations behind the transom stern when $F_d = 2.2$. As the computation time elapses, the wave elevation behind the transom stern becomes steeper, and eventually the wave breaks. These results agree with the previous observations of Coleman & Haussling(1981) and those of Vanden Broeck(1980). As the Froude number increases, the wave elevation behind transom stern shows a tendency of decreasing its slope as shown in Figure 7. Figure 8 show the contour plots of wave elevation at $F_d = 2.35$ and $F_d = 2.5$, respectively. In this figure it can be seen that the dry zone behind the transom stern at $F_d = 2.5$ becomes wider than that in case of $F_d = 2.35$ and the maximum height of wave elevation behind the transom stern becomes lower than in the case of the smaller Froude number.

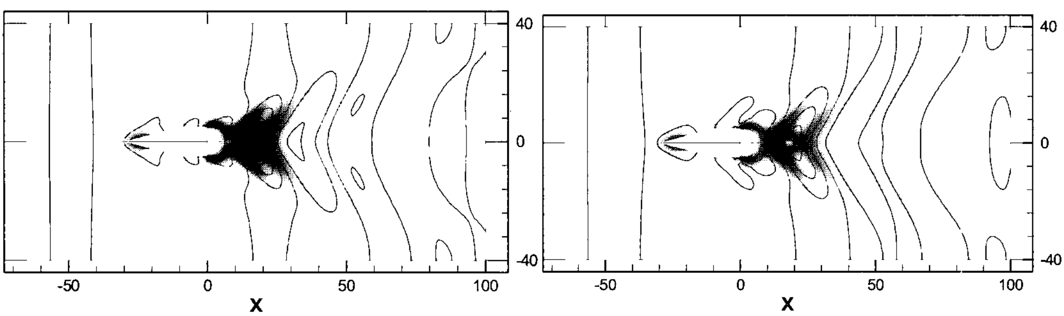

Figure 8: Contour plots of wave elevation. Left : $F_d = 2.35$, Right : $F_d = 2.5$

As concluding remarks we developed a numerical method for 3D transom stern flow using a finite element method based on variational principle. We also proposed a dry transom condition for an unsteady nonlinear free surface problem. The dry transom area can be obtained as part of the solution by this condition. From the comparison with previous works, it can be found that the present method is quite satisfactory for treating the transom stern flow.

Acknowledgments

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