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A New Integral Variable Structure Regulation Controller for Robot Manipulators with Accurately Predetermined Output Performance

로봇 매니플레이터를 위한 정확한 사전 결정 출력 성능을 갖는 새로운 적분 가변구조 레귤레이션 제어기

이 정 훈
Jung-Hoon Lee

Abstract

In this paper, a new integral variable structure regulation controller(IVSRC) is designed by using a special integral sliding surface and a disturbance observer for the improved regulation control of highly nonlinear robot manipulators with prescribed output performance. The sliding surface having the integral state with a special initial condition is employed in this paper to exactly predetermine the ideal sliding trajectory from a given initial condition to origin without any reaching phase. And a continuous sliding mode input using the disturbance observer is also introduced in oder to effectively follow the predetermined sliding trajectory within the prescribed accuracy without large computation burden. The performance of the prescribed tracking accuracy to the predetermined sliding trajectory is clearly investigated in detail through the two theorems together with the closed loop stability. The design of the proposed IVSRC is separated into the performance design and robustness design in each independent link. The usefulness of the algorithm has been demonstrated through simulation studies on the regulation control of a two link manipulator under parameter uncertainties and payload variations, in view of no reaching phase, no overshoot, predetermined response with prescribed accuracy, easy change of output performance, separation of design phase, and so on.

Keywords: variable structure system, sliding mode control, robot control, regulation control

요 약

본 연구에서는 고도의 비선형 로봇 매니플레이터를 위한 새로운 적분 가변 구조 제어기를 설계하였다. 특수 적분 슬라이딩 면과 외란 관찰기를 이용한 사전 성능을 갖는 개선된 레귤레이션 제어이다. 어떠한 리칭 구간도 없이 주어진 초기조건부터 원점까지 슬라이딩 궤적을 정확히 사전 결정하기 위하여 특수한 초기 조건을 갖는 적분 변수를 갖는 슬라이딩 면이 채택되었다. 그리고 외란 관찰기를 사용한 연속 입력은 큰 계산 부하 없이 사전 추적오차 범위내의 사전에 결정된 슬라이딩 궤적을 추적하게 한다. 사전에 결정된 슬라이딩 궤적을 사전에 결정된 추적 오차의 성능은 슬라이딩 면의 값과 슬라이딩 출력의 오차와 관계와 페루프 안정성과 함께 두 개의 정리를 통하여 명확히 검진되었다. 제안된 레귤레이션 제어기의 설계는 성능 설계와 강인성 설계로 각 독립 링크 상에 분리된다. 제안된 알고리즘의 유용성은 매개변수 불확실성과 페이로드 변동하의 이 축 로봇의 레귤레이션 제어에 대한 시뮬레이션 연구를 통하여 무 리칭 구간, 무 오버슈트, 사전 추적 오차를 갖는 사전 결정 출력, 용이한 출력 가변성, 설계 단계의 분리 등의 관점에서 입증되었다.

경상대학교 제어계측공학과
(Department of Control and Instrumentation . Eng. Gyeongsang National University)

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I. Introductions

In servo control, three fundamental problems are the point-to-point control(regulation) problem, tracking problem(trajectory following), and mixed problem. The point-to-point problem is concerned with moving control objects from a point to another. While the controllers for the point-to-point problem are required to provide a small positioning error and superior regulation. In the tracking control, control objects must be moved along the desired trajectory with the same initial position as that of plants. Particularly, the mixed problem is the tracking problem with the severely different initial position of plants from that of planned trajectory in which the features of both regulation and tracking problems exist. The regulation, tracking, and mixed controls are very important in many mechanical system such as robot manipulators, machining systems, tracking antennas etc. These three control problems may be combined in practical fields. Among them, the point-to-point control problem of robot manipulators is the theme of this paper.

A great deal of the researches on the control of highly nonlinear robot manipulators has been reported in order to improve the performance of controllers and to extend the application fields of robot manipulators. There are several approaches to attempt to obtain the desired performances such as decentralized linear PID, optimal control, state feedback control(linear techniques until now), computed torque method[1]-[5], adaptive control[6][7], sliding mode control[8]-[18], and others[22]-[24](nonlinear techniques). Each method has its merits and shortcomings. In the model based methods[17] among them, specially, all of highly nonlinear dynamics models are taken into account to calculate the control input which is a hard task in view of the computation time for controllers. Moreover the robot controllers needs the robustness property from all the modeling errors. In order to obtain the robustness against modeling uncertainties and parameter variations, the variable structure system(VSS) with the sliding mode control(SMC) for robot manipulators has been studied by many researchers[8]-[19]. The strong robustness with simple control structure can be obtained in spite of the existence for an acceptable modeling error and unknown payload using the sliding mode. The other advantages of a SMC are that the almost output performance can be predetermined by choosing the sliding surface. The first application of SMC to robot manipulator seems to be in the work of Young dealing with a set point regulation problem[8]. A modification of the Young's controller was presented by Morgan[10]. Other SMCs of robot manipulators may be found[15]-[17]. However, the existing SMCs for robot manipulators unfortunately have the problem of the reaching phase in the regulation controls during the transient period. Hence the whole output is not completely robust and it is difficult to obtain accurate pre-information on the control performance. Because of this reaching phase, the works about the reacheability and convergence to the sliding surface with finite time are reported. To increase the steady state performance of controllers, an integral action is simply introduced to the variable structure system, but which causes the inevitable overshoot problems in transient state as a side effect as pointed out in [29]. To alleviate computation burden due to the nonlinear dynamics of manipulators, the multi sampling technique is employed to the inner and outer two loop control scheme[21] which results in the complexity of the analysis and design. Currently the neural network is considered[30][31], it is good for static nonlinear dynamics but not effective for the unknown payload and external disturbances.

In this paper, a new improved integral variable structure controller with the prescribed tracking accuracy to the predetermined output is designed for regulation problem of highly nonlinear robot manipulators without the problems mentioned above. With the proposed technique, the reaching phase is completely removed by means of the sliding surface augmented by the integral state with special initial value. The ideal sliding dynamics of the new integral sliding surface is analytically obtained from a given initial point to origin. In consequence, it is possible to predetermine the desired output from a given initial point to origin by using the ideal sliding mode dynamics, with no overshoot as designed according to the choice of the coefficient of the integral sliding surface, which implies the design of the output performance. The relationship between the value of the sliding surface and the error to the sliding trajectory is analyzed in Theorem 1. A continuous sliding mode input based on the disturbance observer for efficient compensation of the nonlinear dynamics of robot manipulators can derive robot manipulators to follow the predetermined sliding trajectory within the prescribed accuracy. The calculation burden in control input is also avoided by using the disturbance observer effectively compensating the nonlinear dynamics of robots. The stability of the closed loop system is investigated in detail in Theorem 2. The results of Theorem 2 provide the stable condition for control gains and the stable region on the axis of the sliding surface. Combing the results of Theorem 1 and Theorem 2 gives rise to possibility of designing the integral variable structure regulation controller to guarantee the tracking error to the predetermined ideal sliding trajectory within the prescribed value. The usefulness of the algorithm has been demonstrated through the simulations of the point-to-point regulation control of a two-link robot under parameter uncertainties and payload variations.

II. A New Integral Variable Structure Regulation Controller (VSRC)

2.1 The State Equation of Robot Manipulators

The motion equations of an n degree-of-freedom manipulator can be derived using the Lagrange-Euler formulation as

$$J(q(t),\phi)\cdot\ddot{q}(t)+D(q(t),\dot{q}(t),\phi)=\tau(t) \tag{1}$$

where $J(q(t),\phi)\in R^{nxn}$ is a symmetric positive definite inertia matrix, $D(q(t),\dot{q}(t),\phi)\in R^n$ is called a smooth generalized disturbance vector as follows

$$D(q(t),\dot{q}(t),\phi)=H(q(t),\dot{q}(t),\phi)+F(q(t),\dot{q}(t),\phi)+G(q(t),\phi) \tag{2}$$

including centrifugal and Coriolis terms $H(q(t),\dot{q}(t),\phi)\in R^n$, Coulomb and viscous or any other frictions $F(q(t),\dot{q}(t),\phi)\in R^n$, gravity terms $G(q(t),\phi)\in R^n$, unknown payload and etc. where τ is an input vector, and $q(t)$, $\dot{q}(t)$, and $\ddot{q}(t)\in R^n$ are the generalized position, velocity, and acceleration vector, respectively. The ϕ is the vector composed of the parameters of robot manipulators(i.e. the masses, lengths, offset angles, and inertia of links). An exact modeling of physical robot dynamics is difficult because of the existence of parameter uncertainties, unknown frictions, and payload variations.

In this study for the point-to-point regulation problem, a desired position reference $q_d(t)\in R^n$ is given and $\dot{q}_d=\ddot{q}_d=0$ is satisfied. Let us define a state vector $X(t)\in R^{2n}$ in the error coordinate system for the SMC as

$$X(t)=[X_1(t)^T \quad X_2(t)^T]^T \tag{3}$$

where $X_1(\cdot)$ and $X_2(\cdot)$ are the trajectory errors and its derivative as

$$X_1(t)\equiv e(t)=q_d-q(t)$$
$$X_2(t)\equiv\dot{e}(t)=-\dot{q}(t) \tag{4}$$

Then the state equation of robot system for the regulation control becomes

$$\dot{X}(t)=\begin{bmatrix}0 & I\\ 0 & 0\end{bmatrix}\cdot X(t)-\begin{bmatrix}0\\ J(q(t),\phi)^{-1}\end{bmatrix}\cdot\tau(t)+\begin{bmatrix}0\\ J(q(t),\phi)^{-1}\cdot D(q(t),\dot{q}(t),\phi)\end{bmatrix},\quad X(0) \tag{5}$$

where $X(0)=[(q_d-q(0))^T \quad 0^T]^T$ is a given initial condition. For (5), a new improved integral variable structure regulation controller will be designed through the two steps, design of the integral sliding surface and choice of the continuous control input. And some analysis about the relationship between the error to the sliding trajectory and the value of the sliding surface together with the closed loop stability will be given in each step.

2.2 An Integral Sliding Surface, Its Sliding Trajectories, and Error Analysis

First of all, let's define a new integral-augmented sliding surface vector

$$s(t)\equiv X_2(t)+K_p\cdot X_1(t)+K_I\cdot X_0(t) \tag{6}$$

$$X_0(t)=\int_0^t X_1(\tau)d\tau+X_0(0),$$
$$X_0(0)=-K_I^{-1}(X_2(0)+K_p\cdot X_1(0)) \tag{7}$$

where K_P and K_I are diagonal coefficient matrices and $X_0(t)$ is an integral of the error with the special initial condition $X_0(0)$ for removing the reaching phase by means of making the sliding surface be zero at $t=0$, i.e., $s(0)=0$. Thus this integral augmented sliding surface determines the ideal sliding mode dynamics to have the ideal second order dynamics exactly from a given initial state to the origin in each link independently, not straight line to origin like the conventional sliding surfaces. If $X_0(0)=0$ in (7) such as previous works on the integral variable structure systems[32], there is the reaching phase problems because $s(t)\neq 0$ at $t=0$ and an inevitable overshoot problems as the side effect[29] because the integral state accumulated from zero must re-converge to zero. The ideal sliding dynamics of the integral sliding surface (6) from a given initial state to origin is obtained as follows:

$$\dot{X}^*{}_2(t)+K_P\cdot X^*{}_2(t)+K_I\cdot X^*{}_1(t)=0 \tag{8}$$

Then re-write equation (8) into the state equation form as follows:

$$\dot{X}^*(t)=\Lambda\cdot X^*(t) \qquad X^*(0)=X(0) \tag{9}$$

which is the ideal sliding dynamics of the proposed integral sliding surface where $X^*(t)=[(q_d(t)-q_s^*(t))^T \quad -\dot{q}_s^*(t)^T]^T\in R^{2n}$ and $\Lambda\in R^{2n\times 2n}$

$$\Lambda=\begin{bmatrix} 0 & I \\ -K_P & -K_I \end{bmatrix} \tag{10}$$

The solution of the state equation (9), q_s^*, $\dot{q}_s^*$, and $\ddot{q}_s^*\in R^n$ theoretically predetermines the ideal sliding trajectories from $q(0)$ to the origin without any reaching phase, which point is not considered in [32]. Since $\det[\lambda I-\Lambda]=[\lambda^2 I+\lambda K_P+K_I]$, K_P and $K_I\times R^{n\times n}$ can be chosen so that all the eigenvalue of Λ have the negative real parts, which guarantees the exponential stability of the system (9). Then there exist the positive scalar constants K and κ such that

$$\left\|e^{\Lambda t}\right\|\le K\cdot e^{-\kappa t} \tag{11}$$

where $\|\cdot\|$ is the induced Euclidean norm.

Now, define $\overline{X}_1(t)$ and $\overline{X}_2(t)$ are the error to the sliding trajectory and its derivative, respectively as

$$\begin{aligned}\overline{X}(t)^T &= [\overline{X}_1(t)^T \quad \overline{X}_2(t)^T] \\ &= [(q_s^*(t)-q(t))^T \quad (\dot{q}_s^*(t)-\dot{q}(t))^T]\end{aligned} \tag{12}$$

If the input in the VSS is discontinuous, the value of the sliding surface can be zero for all time. However If the input of the VSS is continuous, the sliding surface may not be exactly zero. The effect of the non zero value of the sliding surface to the error to the sliding trajectory is analyzed in the following Theorem 1 as a prerequisite to the main theorem.

Theorem 1: *If the integral sliding surface defined by (6) satisfies* $\|s(t)\|\le\gamma$ *for any* $t\ge t_0$ *and* $\|\overline{X}(t_0)\|\le\gamma/\varkappa$ *is satisfied at the initial time, then*

$$\left\|\overline{X}_1(t)\right\|\le\varepsilon_1 \tag{13a}$$

$$\left\|\overline{X}_2(t)\right\|\le\varepsilon_2 \tag{13b}$$

is satisfied for all $t\ge t_0$ *where* ε_1 *and* ε_2 *are the positive constants defined as follows:*

$$\varepsilon_1=\frac{K}{\varkappa}\cdot\gamma, \qquad \varepsilon_2=\gamma\cdot\left[1+Z\cdot\frac{K}{\varkappa}\right], \qquad Z=\|[K_\rho\ K_I]\| \tag{14}$$

Proof: Let us define new error vector as

$$\widehat{X}^T=\left[\int_0^t q_s^*(\tau)-q(\tau)d\tau\right]\ \overline{q}-q(t) \tag{15}$$

The sliding surface can be re-written as

$$\begin{aligned}s(t)\equiv\ & X_2(t)+K_P\cdot X_1(t)+K_I\cdot X_0(t) \\ & -\left\{X_2^*(t)+K_P\cdot X_1^*(t)+K_I\cdot X_0^*(t)\right\}\end{aligned} \tag{16}$$

and can be re-expressed in a differential matrix form as

$$\dot{\widehat{X}}=\Lambda\cdot\widehat{X}+\begin{bmatrix}0\\1\end{bmatrix}\cdot s(t) \tag{17}$$

In (17), the sliding surface may be considered as the bounded disturbance, $|s(t)|\le\gamma$. The solution of (17) is expressed as

$$\widehat{X}(t)=e^{\Lambda t}\cdot\widehat{X}(0)+\int_0^t\left\{e^{\Lambda\tau}\cdot\begin{bmatrix}0\\1\end{bmatrix}\cdot s(\tau)\right\}d\tau \tag{18}$$

From the boundeness of the sliding surface and (11), the Euclidean norm of vector $\overline{X}$, i.e.

$$\| \widehat{X}(t) \| = \left\{ \left(\int_0^t q_s(\tau) - q(\tau) d\tau \right)^2 + (q_s(t) - q(t))^2 \right\}^{1/2} \quad (19)$$

and becomes

$$\| \widehat{X}(t) \| = \| e^{\Lambda t} \| \cdot \| \widehat{X}(0) \| + \int_0^t \left\| \left\{ e^{\Lambda\tau} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot s(\tau) \right\} \right\| d\tau$$

From (11) and simple manipulation, one obtain

$$\| \widehat{X}(t) \| \le K e^{\kappa t} \cdot \| \widehat{X}(0) \| + \int_0^t \left\{ \| e^{\Lambda\tau} \| \cdot \left\| \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\| \cdot \| s(\tau) \| \right\} d\tau$$

And from (11) and some manipulation, the following equation is obtained

$$\| \hat{X}(t) \| \le \frac{K}{\kappa} \cdot \gamma + \left(\| \hat{X}(0) \| - \frac{\gamma}{\kappa} \right) \cdot K \cdot e^{-\kappa \cdot t}.$$

Finally, one can obtain

$$\| \hat{X}(t) \| \le \frac{K}{\kappa} \cdot \gamma \quad (20)$$

for all time $t \ge 0$, which satisfies the following inequality

$$\| \hat{X}_1 \| \le \frac{K}{\kappa} \gamma \quad (21)$$

From the sliding surface, $\overline{X}_2$ can be simply obtained as

$$\| \hat{X}_2 \| = s(t) - \begin{bmatrix} K_P & K_I \end{bmatrix} \cdot \hat{X}(t) \quad (22)$$

If the norm operation is taken on both sides, (22) becomes

$$\| \hat{X}_2 \| \le \gamma \cdot (1 + Z \cdot K / \kappa) \quad (23)$$

which completes the proof of Theorem 1.

The above Theorem 1 implies that the error to the ideal sliding trajectory and its derivative are uniformly bounded provided the sliding surface is bounded, i.e., $\| s(t) \| \le \gamma$ for all time $t \ge t_0$. Using this result of Theorem 1, we can give the specifications on the error to the ideal sliding trajectory defined by the sliding surface, (6). In the next section, we will designed a variable structure regulation control input with the efficient compensation which can guarantee the boundedness of s(t), i.e., $\| s(t) \| \le \gamma$.

2.3 A Continuous VSS Input and its Analysis

Robot manipulators activated by several servo motor amplifiers are subject to a variety of disturbances. The robust control of highly nonlinear robot manipulators is essential for developing robotics. It is often noted that the generalized nonlinear disturbances $D(q(t), q(t), \phi)$, must be compensated for improving the performance. As an ideal control input in the sliding mode control, the equivalent control of the augmented sliding surface (6) for the robot system (5) is obtained from equation (8)

$$\tau_{eq}(t) = D(q(t), q(t), \phi) + J(q(t), \phi) \cdot (\ddot{q}_d(t) + K_P X_2 + K_I X_1) \quad (24)$$

The smooth generalized disturbance $D(q(t), q(t), \phi)$ is included in the equivalent control $\tau_{eq}(t)$. Since generally this smooth generalized disturbance is very complex, a direct calculation of the smooth generalized disturbance from the robot model results in a long sampling time, limitations of the control performance, difficulties of controller design, and so on.

In this paper, using the efficient compensation method, so called disturbance observer[19], we consider the following continuous control input $\tau(t)$.

$$\tau(t) = \tau_c(t) + \tau_s(t) \quad (25)$$

where $\tau_c(t)$ is the compensation term for the smooth generalized disturbance as well as the error of nominal inertia matrix, not direct calculation from $\hat{D}(q(t), q(t), \phi)$ in the robot model but the efficient estimation of the generalized disturbance $D(q(t), \dot{q}(t), \phi)$, only using the nominal inertia matrix, J_N of the robot model (1) and an available acceleration information which can be calculated from the speed information by means of the Euler method[19].

$$\begin{aligned} \tau_c(t) &= \tau(t-h) - J_N \cdot \tilde{\ddot{q}}(t) \\ &= D(q(t), \dot{q}(t), \phi) + \Delta J(q(t), \phi) \cdot \tilde{\ddot{q}}(t) \\ &\quad - J(q(t), \phi) \cdot \Delta\ddot{q}(t) - \Delta\tau(t) \end{aligned} \quad (26)$$

where $\tilde{\ddot{q}}$, $\Delta J(q(t), \phi)$, $\Delta\ddot{q}(t)$, and $\Delta\tau(t)$ are defined by

$$\tilde{\ddot{q}} = \{\dot{q}(t) - \dot{q}(t-h)\} / h \quad (27)$$

$$\Delta J(q(t), \phi) = J(q(t), \phi) - J_N \quad (28)$$

$$\Delta\ddot{q}(t)=\tilde{\ddot{q}}(t)-\ddot{q}(t) \quad (29)$$

$$\Delta\tau(t)=\tau(t-h)-\tau(t) \quad (30)$$

respectively, where $\Delta J(q(t),\phi)$ is the deviation between the real inertia matrix and its nominal value, $\Delta\ddot{q}(t)$ is the acceleration information error from the real acceleration value, $\Delta\tau(t)$ is control input delay error resulted from the digital control, and h is sampling time for digital implementation. If the sampling time is sufficiently small and the control input continuously implemented, then, the acceleration information error $\Delta\ddot{q}(t)$ and the control input delay error $\Delta\tau(t)$ can be small. This disturbance observer fails at the initial time because $\tau(t-h)$ is unknown, hence $\tau_c(0)$ is once calculated by using the model of robots with off-line in advance. The detail features of disturbance observer is explained in [19]. The second term in the right hand side of the equation (25) is defined as

$$\tau_s(t)=(\tilde{\tau}_{eq}(t)+\tau_x(t)) \quad (31)$$

where $\tilde{\tau}_{eq}(t)$ is the modified equivalent control for the compensated dynamics of equation (1), and is so designed that the error dynamics of the controlled system has the sliding surface dynamics defined by equation (9), which is defined as

$$\tilde{\tau}_{eq}(t)=J_N\cdot(\ddot{q}_d(t)+K_P\cdot X_1+K_I\cdot X_2) \quad (32)$$

The $\tau_\chi(t)$ is the continuous feedback term of the sliding surface for correcting the small compensation error as follows:

$$\tau_\chi(t)=J_N\cdot\{\kappa_{\chi^1}\cdot s(t)+\kappa_{\chi^2}\cdot\sigma(t)\}$$

$$\sigma(t)=\frac{s(t)}{\|s(t)\|+\delta} \quad (33)$$

After effectively compensating an almost part of the highly nonlinear dynamics of robot manipulators based on the disturbance observer for avoiding a heavy computation burden, the sliding control input is totally continuously implemented. As the function of the disturbance observer, the effective compensation for highly nonlinear generalized disturbances and modeling errors of the inertia matrix will be studied. If we apply the input control torque given by equation (25)-(33) to the robot system (5), the following equation is obtained

$$\dot{X}_2(t)=-J^{-1}(q(t),\phi)\cdot(\Delta J(q(t),\phi)\cdot\ddot{q}(t)-\Delta\tau(t))+\Delta\ddot{q}(t)+\ddot{q}_d -J^{-1}(q(t),\phi)\cdot J_N\cdot\left[\ddot{q}_d+K_v\cdot X_2+K_\rho\cdot X_1+\kappa_{\chi^1}\cdot s(t)+\kappa_{\chi^2}\cdot\sigma(t)\right] \quad (34)$$

and the dynamics of s(t) is expressed in the following simple form

$$\dot{s}(t)=n_1(t)-\left[k_{\chi^1}\cdot s(t)+k_{\chi^2}\cdot\sigma(t)\right] \quad (35)$$

where $n_1\in R^n$ is the resultant disturbance vector given by

$$n_1(t)=n_1(\Delta\ddot{q}(t),\Delta\tau(t),\hat{\phi})=J_N^{-1}\cdot J(q(t),\phi)\cdot\Delta\ddot{q}+J_N^{-1}\cdot\Delta\tau(t). \quad (36)$$

From the equation (35), the 2n-th order original point-to-point regulation control problem is converted to the n-th stabilization problems with a three degree of freedom k_{x^1}, k_{x^2}, δ against the resultant disturbance n_1 by means the proposed algorithm, which means the robustness problems. For some positive constants ε_1 and ε_2 defined in (14), let the constant N be defined as follows:

$$N=\max\{\|n_1(\Delta\ddot{q}(t),\Delta\tau(t),\hat{\phi})\|\,|\,q(t)\in B(\varepsilon_1;q_s^*(t))\,and\,\dot{q}(t)\in B(\varepsilon_2;\dot{q}_s^*(t))\}. \quad (37)$$

where the matrix norm is defined as the induced Euclidean norm, and for a positive number $\rho>0$ and a vector $\nu\in R^n$, the boundary set is defined by as

$$B(\rho;\nu)=\{w\in R^n;\|w-\nu\|\le\rho\}. \quad (38)$$

In equation (36), the resultant disturbances are mainly dependent on the acceleration information error and the control input computation delay error and not the system uncertainties or the modeling errors of robot manipulators. The disturbance observer can compensate for modeling errors of the inertia matrix besides the smooth generalized disturbance (2). Thus the design of the IVSRC is independent of the maximum bound of modeling errors in the parameter space, but dependent on only the resultant disturbance composed of the acceleration information error and the control time delay due to the digital implementation.

The stability property or the system (5) with control laws (25)-(33) will be stated in the next theorem:

Theorem 2: *Consider the robot system with the control given by equation (25)-(33). Assume that for some positive* γ, $\|s(t_0)\|\le\gamma$ $\|x(t_0)\|\le\gamma/\kappa$ *are satisfied at the initial time* $t=t_0$, *and if the gain* k_{x^2} *satisfies*

$$k_{x^2}\ge N-k_{x^1}\cdot\delta \tag{39}$$

for a given k_{x^1} *and* δ, *then the global control system is uniformly bounded (i.e. the solution X is uniformly bounded at origin in error coordinate state space) for all* $t\ge t_0$ *until* $\|s(t)\|\le\eta$ *where* η *is defined by*

$$\eta=\sqrt{\alpha_1^2+\beta_1}-\alpha_1,\quad \alpha_1=\delta/2+(k_{x^2}-N)/(2k_{x^1}),\quad \beta_1=\frac{\delta\cdot N}{\kappa_{x_1}} \tag{40}$$

Proof: The proof is straightforward, first take Lyapunov candidate function as

$$V(t)=1/2s^T(t)\cdot s(t) \tag{41}$$

and differentiate with respect to time, it leads to

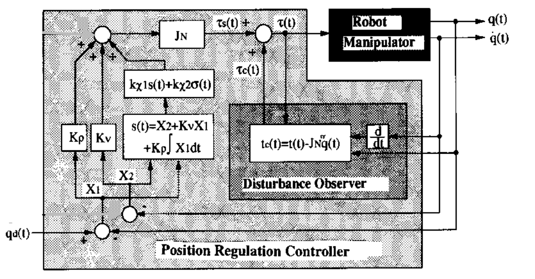

Fig. 1 The proposed integral variable structure regulation controller

그림 1 제안된 적분 가변구조 레귤레이션 제어기

$$\dot V(t)=s^T(t)\cdot\dot s(t)=s^T(t)n_1(t)-s^T(t)\lfloor k_{\chi^1}\cdot s(t)+k_{\chi^2}\cdot\sigma(t)\rfloor \tag{42}$$

By the matrix inequality, (42) becomes

$$\dot V(t)\le\|s(t)\|\cdot\|n_1(t)\|-\|s(t)\|\lfloor k_{\chi^1}\cdot\|s(t)\|+k_{\chi^2}\cdot\sigma(t)\rfloor \tag{43}$$

From the definition of N, the following equation can be obtained

$$\dot V(t)\le\|s(t)\|\cdot\{N-\lfloor k_{\chi^1}\cdot\|s(t)\|+k_{\chi^2}\cdot\sigma(t)\rfloor\}$$

From simple manipulation, one can obtain

$$\dot V(t)=-\frac{k_{\chi^1}\|s(t)\|}{\|s(t)\|+\delta}\cdot\{\|s(t)\|^2+2\alpha\|s(t)\|-\beta\}.$$

The gains, k_{x^1} and $k_{x^{21}}$ satisfy the inequality (39), then finally we can conclude

$$\dot V(t)<0 \tag{44}$$

at $t\ge t_0$ as long as $\|s(t)\|\le\eta$, which completes the proof of Theorem 2.

Theorem 2 guarantees the uniform bounded stability of the proposed continuous IVSRC for robot manipulators. The smaller δ in control algorithm (33), the lower bound of η. The η can be decreased by an increase of k_{x^1} for a given δ and N so that η is sufficiently smaller than γ the bound of the integral sliding surface in Theorem 1 ($\eta<\gamma$). If the initial value of the integral sliding surface($\|s(t)\|\le\gamma$) which is reasonable in the case of the known initial state of robot manipulators, the feedback control (25)-(33) designed by Theorem 1 and Theorem 2 maintains the bounded stability of the system with the prescribed performance:

$$q(t)\in B(\varepsilon_1;q_s^*(t))\ and\ \dot q(t)\in B(\varepsilon_2;\dot q_s^*(t))\quad for\ t\ge 0 \tag{45}$$

which implies guaranteeing the prescribed tracking error ε_1 to the ideal sliding trajectory $q_s^*(t)$ predetermined by the integral sliding surface from a given initial condition $q(0)$ to the origin without any reaching phase, in other words, guaranteeing the predetermined output response with the prescribed accuracy ε_1. Fig. 1 shows the structure of the proposed algorithm composed of the compensation term, modified equivalent term, and continuous feedback term of the sliding surface, which is relatively simple because of the nature of VSS and avoidance of large computation burden. Thus the sampling time can be as small as possible so that the acceleration information calculated by Euler method and the delayed control input are almostly exact to each real value, therefore, the maximum value N can be very small. Therefore, a new IVSRC can be realized effectively. The output is controlled to follow the predetermined sliding trajectory with the ε_1 accuracy. The sliding trajectory can be obtained from the solution of the sliding dynamics of (9). Hence the output is predictable. The design procedure of the proposed sliding mode controller to guarantee the predetermined output with prescribed accuracy is as follows: First, choose the desired sliding surface defining the desired sliding dynamics (9) which means the determination of the coefficients, K_P and K_I and calculate the ideal sliding trajectory off-line(performance design phase). Second, find the constants K and κ satisfying the equation (11). Third, determine the bound of the sliding surface γ using (14) in Theorem 1 for a given accuracy of the tracking error to the sliding trajectory ε_1. and finally design the gains k_{x^1} and k_{x^2} in equation (33) based on Theorem 2 so that η_1 is smaller than γ (robustness design phase). In the whole design procedure, it does not need the information of maximum bound of variations of the system parameter or uncertainties because of the efficient on-line compensation of the disturbance observer.

III. Numerical Simulations

3.1. Descriptions of a Two Link Manipulator

Numerical simulations are performed to show the accurate and robust control property of the proposed IVSRC. The dynamic model of a SCARA-type two degree-of-freedom manipulator shown in Fig.2 used in this simulation is as follows:

Fig. 2 A SCARA type two degree-of freedom manipulator
그림 2 SCARA 형 2 자유도 매니플레이터

$$\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix}=l\cdot\Lambda=\begin{bmatrix}\frac{1}{3}m_1+\frac{4}{3}m_2+m_2C_2 & \frac{1}{3}m_2+\frac{1}{2}m_2C_2\\ \frac{1}{3}m_2+\frac{1}{m_2}C_2 & \frac{1}{3}m_2\end{bmatrix}\cdot\begin{bmatrix}q_1\\ q_2\end{bmatrix}$$
$$+l^2\cdot\begin{bmatrix}\frac{1}{2}m_2S_2\dot{q}_1^2-m_2S_2\dot{q}_1\dot{q}_2\\ \frac{1}{3}m_2S_2\dot{q}_1^2\end{bmatrix}+l\cdot\begin{bmatrix}\frac{1}{2}m_1gC_1-\frac{1}{2}gC_{12}+m_2gC_1\\ \frac{1}{2}m_2gC_{12}\end{bmatrix} \tag{46}$$

where C_i, S_i and C_{ij} imply $\cos(q_i)$, $\sin(q_i)$ and $\cos(q_i+q_j)$, respectively. The parameters are m_1= m_2 = 0.782[kg], l = *0.23*[m] and g = *9.8*[*m/sec2*].

3.2 A Design Example Based on the IVSRC

The reference command $[90^\circ\ -60^\circ]$ is given for the two links as an example. Following the design procedure, the coefficients of the integral *sliding surface 1* is designed as $K_P=36$ and $K_I=12$ in (8) for locating double pole at -6 into the sliding dynamics (9). The corresponding constants in (11) K and κ become 4 and 2.715, respectively. By the results of Theorem 1, the error to the sliding trajectory and its derivative, $\overline{X}_1$ and $\overline{X}_2$, are bounded as $\varepsilon_1=1.473\gamma$ and $\varepsilon_2=57\gamma$ for a given γ of the sliding surface. For a $\varepsilon_1=0.2^\circ$ maximum error, γ is selected as 0.13. Now, the controller gain k_{x^1} and k_{x^2} are chosen as 20 and 10 for $\delta = 0.05$ and $N = 5$ by Theorem 2 which satisfy the condition (39) so that $\eta = 0.037$ is sufficiently small with respect to the chosen $\gamma = 0.13$ by Theorem 1 in order to guarantee the prescribed error $\varepsilon_1 = 0.2°$ to the sliding trajectory previously determined by the integral *sliding surface 1*.

3.3 Discussions on Simulations Results

For illustrating the robustness of the proposed algorithm, the simulations are carried out under the three different conditions, i.e., *case 1*:no modeling error, *case 2*: 10 [%] modeling error, and *case 3*: 10 [%] modeling error and 1[kg] unknown payload. The sampling time is selected as 2 [msec]. The position error responses of two links and corresponding phase trajectories for the three case conditions by the IVSRC are shown in Fig. 3 and Fig. 4, respectively. As can be seen, there is no reaching phase, and the three error outputs and phase trajectories are exactly identical and accurate to the predetermined sliding trajectories which means the high robustness of the suggested algorithm IVSRC for all parameter uncertainties and play load variations as theoretically expected. Therefore, using the response of the sliding trajectory from a given initial to the origin, the real output response can be predicted with the accuracy $\varepsilon_1 = 0.2°$. the The control inputs for link 1 and link 2 are depicted in Fig. 5 and Fig. 6, respectively. Fig. 7 shows the position errors of two links for three sliding surfaces, i.e., *sliding surface 1*:previously designed surface, *sliding surface 2*:the coefficients $K_P = 64$ and $K_I = 16$ for a double pole at -8 in the sliding dynamics, *sliding surface 3*:the coefficients $K_P = 144$ and $K_I = 24$ for a double pole at -12 in the sliding dynamics. As can be seen in Fig. 7, the convergence speed of the position error can be changed according to the design of the integral sliding dynamics, (9), which means that the real output can be changed according to the choice of the integral sliding surface.

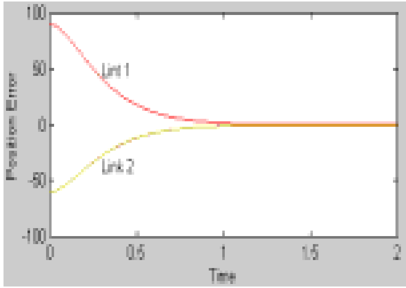

Fig. 3 Position error responses of two links for three cases by IVSRC
그림 3 제안된 알고리듬의 세 가지 경우의 두 링크의 위치 오차 응답

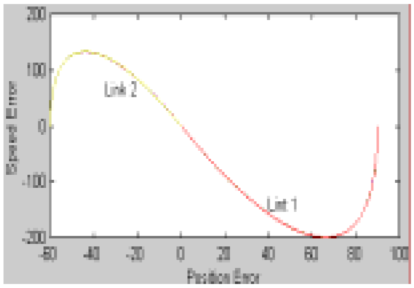

Fig. 4 Phase trajectories of two links for three cases by IVSRC
그림 4 세 가지 경우의 두 링크의 상 궤적

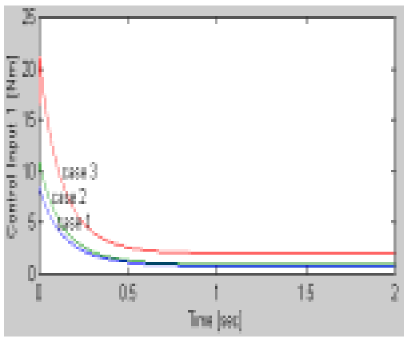

Fig. 5 Control inputs of link 1 for three cases
그림 5 세 가지 경우의 링크 1의 제어 입력

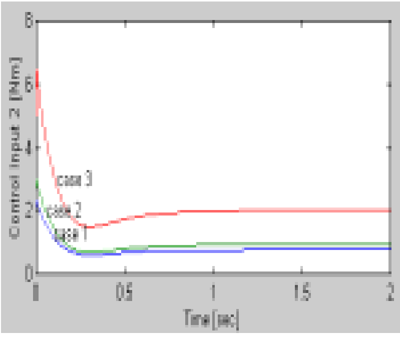

Fig. 6 Control inputs of link 2 for three cases
그림 6 세 가지 경우의 링크 2의 제어 입력

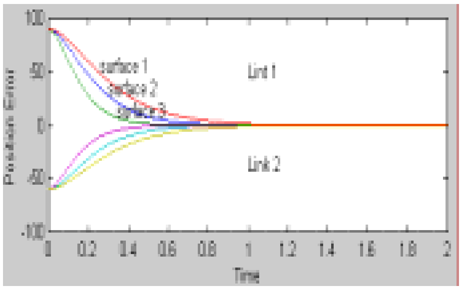

Fig. 7 Position error responses of two links for different three sliding surfaces
그림 7 세 가지 다른 슬라이딩 면에 대한 두 링크의 위치오차 응답

sliding surface 1: $K_P = 36$ and $K_I = 12$, *sliding surface 2*: $K_P = 64$ and $K_I = 16$, and *sliding surface 3*: $K_P = 144$ and $K_I = 24$

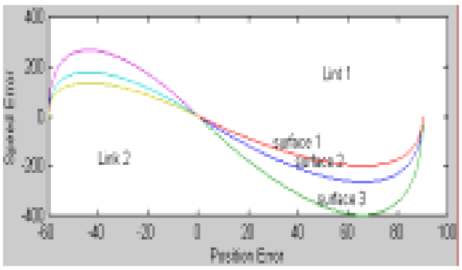

Fig. 8 Phase trajectories of two links for different three sliding surfaces
그림 8 세 가지 다른 슬라이딩 면에 대한 두 링크의 상 궤적

sliding surface 1: $K_P = 36$ and $K_I = 12$, *sliding surface 2*: $K_P = 64$ and $K_I = 16$, and *sliding surface 3*: $K_P = 144$ and $K_I = 24$

The corresponding phase trajectories are shown in Fig. 8 for the three different integral sliding surfaces. From the results of the simulation studies until now, the advantages of the proposed algorithm can be pointed out in view of no reaching phase, no overshoot, the strong robustness, the predetermined output with designed accuracy, design phase separation, and easy changeability of output performance which has been illustrated.

IV. Conclusions

In this paper, a new improved integral variable structure regulation controller for highly nonlinear robot manipulators with the prescribed tracking accuracy to the predetermined sliding trajectory is suggested based on the special integral sliding surface and the efficient disturbance observer. In the proposed algorithm, the special integral sliding surface is adapted for removing the reaching phase. The sliding dynamics of the special integral sliding surface is obtained as the differential equation in matrix form. Therefore, by using the solution of the sliding dynamics, the desired sliding trajectory is predetermined from a given initial state by the choice of the new special integral sliding surface without any reaching phase. The relationship between the maximum bound of the tracking error to the predetermined sliding trajectory and the value of the sliding surface is derived in Theorem 1. By the suggested continuous input based on the disturbance observer, robot manipulators can be controlled to follow the predetermined sliding trajectory within the prescribed accuracy for all the modeling errors and payload variations without computation burden. The bounded stability of the suggested algorithm is investigated in theorem 2. Through the two theorems, it is proved that the predetermination of the output response with prescribed accuracy is possible. The some usefulness of the algorithm has been demonstrated by the simulations about the point-to-point position control of a two-link robot under parameter uncertainties and payload variations with the example designs. The advantages of the proposed algorithm can be pointed out in view of no reaching phase, no overshoot, strong robustness with prescribed accuracy, the predetermined output with designed accuracy, design phase separation and easy changeability of output performance, etc.

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저 자 소 개

이 정 훈 (李 政 勳)

1966년 2월 1일생.
1988년 : 경북대학교 전자공학과 졸업(공학사), 1990년 : 국과학기술원 전기 및 전자공학과 졸업(석사).
1995년 : 한국과학기술원 전기 및 전자공학과 졸업(공박).
현재 2004년 : 현재 경상대학교 전기전자공학부 제어계측공학전공 부교수.
1997-1999 : 경상대학교 제어계측공학과 학과장.
마르퀘스사의 Who's Who in the world 2000년 판에 등재.
American Biograhpical Institute(ABI)의 500 Leaders of Influence에 선정.