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Exact Solutions of Fuzzy Goal Programming Problems using α-cut Representations

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Abstract

Ramik[7] introduced a fuzzy goal programming (FGP)problem that generalizes a standard goal programming (GP) problem with fuzzy alternatives, fuzzy objective functions and fuzzy deviation functions for measuring the deviation between attained and desired goals being fuzzy. However, it is known that this FGP tends to produce an approximate solution since it uses an approximate fuzzy multiplication operation to solve the resultant fuzzy model. In this paper, we show that this FGP sometimes leads to the wrong decision. We also propose a procedure that gets the exact solution to overcome these problems. The method is based on T_M(min norm)-based fuzzy operations using α-cut representations. We consider the same example as used in Ramik and investigate how our procedures are compared to Ramik's.

Keywords : Fuzzy alternatives, Fuzzy goals, Fuzzy operations, Goal programming

1. Introduction

In order to specify imprecise aspiration levels of the goals in a fuzzy environment, Narasimhan[3] had initially proposed fuzzy goal programming (FGP) problem by using membership functions. This work and some related studies [4,5,8] were actually inspired by a fuzzy programming approach introduced by Zimmermann[10]. The FGP formulation has widespread applications to various fields which need decision making.

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Applying fuzzy set theory into goal programming (GP) has the advantage of allowing for the vague aspirations of a decision maker to find out what attainments are desired for each objective function, see [3,6]. In [7], Ramik generalized a standard GP problem in [1] with fuzzy alternatives and fuzzy objective functions and fuzzy deviation functions for measuring the deviation between attained and desired goals being fuzzy. In fact, the resulting FGP is a unifying approach covering several approaches known from the literatures.

However, it is known that this FGP tends to produce an approximate solution since it uses an approximate fuzzy multiplication operation to solve the resultant fuzzy model. In this paper we show that this FGP sometimes leads to wrong decision. We also propose a procedure that get the exact solution to overcome these problems. The idea is based on T_M(min norm)-based fuzzy operations using α-cut representations. Through examples, we will show how the proposed procedures work.

To this end, we need some preliminaries on fuzzy arithmetic operations based on triangular norm. First, we state two types of fuzzy numbers. The first is a triangular fuzzy number $\tilde{a}$ denoted by (a, α, β), which is defined as

$$\tilde{a}(t)=\begin{cases} 1-\dfrac{|a-t|}{\alpha} & \text{if } a-\alpha \leq t \leq a, \\ 1-\dfrac{|a-t|}{\beta} & \text{if } a \leq t \leq a+\beta, \\ 0 & \text{otherwise}, \end{cases}$$

where $a \in R$ is the center and $\alpha > 0$ is the left spread, $\beta > 0$ is the right spread of $\tilde{a}$. If $\alpha = \beta$, then the triangular fuzzy number is called a symmetric triangular fuzzy number and denoted by (a, α).

The second is an $L-R$ fuzzy number $\tilde{a} = (a, \alpha, \beta)_{LR}$, which is a function from the reals into the interval [0, 1] satisfying

$$\tilde{a}(t)=\begin{cases} R\left(\dfrac{t-a}{\beta}\right) & \text{if } a \leq t \leq a+\beta, \\ L\left(\dfrac{a-t}{\alpha}\right) & \text{if } a-\alpha \leq t \leq a, \\ 0 & \text{else}, \end{cases}$$

where L and R are non-increasing and continuous functions from [0, 1] to [0, 1] satisfying $L(0) = R(0) = 1$ and $L(1) = R(1) = 0$. If $L = R$ and $\alpha = \beta$, then the symmetric $L-L$ fuzzy number is denoted $(a, \alpha)_L$.

Now, we describe T_M-based fuzzy operations using α-cut representations. Let $T = T_M$ and let $\widetilde{A}$ and $\widetilde{B}$ be two fuzzy numbers with $\{t \mid \widetilde{A}(t) \geq \alpha\} = [a_L(\alpha), a_R(\alpha)]$ and $\{t \mid \widetilde{B}(t) \geq \alpha\} = [b_L(\alpha), b_R(\alpha)]$ representations. For simplicity, we use $\{\widetilde{A} \geq \alpha\}$, $\{\widetilde{B} \geq \alpha\}$ instead of

$\{t \mid \widetilde{A}(t) \geq \alpha\}$, $\{t \mid \widetilde{B}(t) \geq \alpha\}$, respectively. Then, we have the following facts:

$$\{\widetilde{A} \oplus \widetilde{B} \geq \alpha\} = [a_L(\alpha) + b_L(\alpha), a_R(\alpha) + b_R(\alpha)] \tag{1}$$

and

$$\begin{aligned}\{\widetilde{A} \otimes \widetilde{B} \geq \alpha\} = [\, &\min\{a_L(\alpha)b_L(\alpha), a_L(\alpha)b_R(\alpha), a_R(\alpha)b_L(\alpha), a_R(\alpha)b_R(\alpha)\},\\ &\max\{a_L(\alpha)b_L(\alpha), a_L(\alpha)b_R(\alpha), a_R(\alpha)b_L(\alpha), a_R(\alpha)b_R(\alpha)\}]\end{aligned} \tag{2}$$

If $\widetilde{A}$ and $\widetilde{B}$ are both non-negative, i.e., $\widetilde{A}(x) = \widetilde{B}(x) = 0$ for $x \leq 0$, then

$$\{\widetilde{A} \otimes \widetilde{B}(t) \geq \alpha\} = [a_L(\alpha)b_L(\alpha), a_R(\alpha)b_R(\alpha)]. \tag{3}$$

2. Fuzzy goal programming elements

Since we here deal with the FGP in [7], we need some basic concepts such as fuzzy alternatives, fuzzy goal functions and fuzzy deviation functions regarding this FGP from there.

As the components of alternatives we consider the symmetric non-negative triangular fuzzy numbers with the spread ratio ξ

$$\widetilde{R}_{\xi} = \{\tilde{x}; \tilde{x} = (x, \xi x), 0 \leq x, 0 \leq \xi \leq 1\}.$$

Then, as alternatives we consider non-negative fuzzy vectors $\widetilde{\mathbf{x}}$ with the spread ratio ξ, i.e., $\widetilde{X} \subseteq \widetilde{R}^n_{\xi}$, where

$$\widetilde{R}^n_{\xi} = \left\{\widetilde{\mathbf{x}}; \widetilde{\mathbf{x}} = (\tilde{x}_1, \cdots, \tilde{x}_n), \tilde{x}_j \in \widetilde{R}_{\xi}, j = 1, 2, \cdots, n\right\}.$$

A set of goals G is a set of triangular fuzzy numbers (T-numbers for short), i.e. $G = \widetilde{T}$, where

$$\widetilde{T} = \{\tilde{g}; \tilde{g} = (g, \underline{g}, \overline{g}), \underline{g} \geq 0, \overline{g} \geq 0\}. \tag{4}$$

As usually done in [7], we here confine ourselves to linear fuzzy goal functions on $\widetilde{X} \subseteq \widetilde{R}^n_{\xi}$, i.e.,

$$\tilde{c}_i : \widetilde{R}^n_{\xi} \to \widetilde{R}, \quad i = 1, 2, \cdots, m,$$

$$\tilde{c}_i(\widetilde{\mathbf{x}}) = (\tilde{c}_{i1} \otimes \tilde{x}_1) \oplus \cdots \oplus (\tilde{c}_{in} \otimes \tilde{x}_n), \tag{5}$$

where $\widetilde{R}$ is the set of fuzzy numbers, $\tilde{c}_{ij} = (c_{ij}, \underline{c}_{ij}, \overline{c}_{ij})$, and $\tilde{x}_j = (x_j, \xi x_j)$, $i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, n$. In formula (5), Ramik[7] used the following binary operations on $\widetilde{T}$. Let $\tilde{a} = (a, \underline{\alpha}, \overline{\alpha})$, $\tilde{b} = (b, \underline{\beta}, \overline{\beta})$.

Addition :

$$\tilde{a} \oplus \tilde{b} = (a + b, \underline{\alpha} + \underline{\beta}, \overline{\alpha} + \overline{\beta}), \tag{6}$$

Multiplication:

$$\tilde{a} \otimes \tilde{b} = (ab, \underline{a}b + a\underline{\beta} - \underline{a}\,\underline{\beta}, \overline{a}b + a\overline{\beta} + \overline{a}\,\overline{\beta}). \tag{7}$$

Addition operation is a natural extension of the crisp addition when applying Zadeh[9]'s extension principle based on T_M. On the other hand, multiplication is an approximate operation [2]. Therefore, the FGP using this approximate multiplication tends to produce an approximate solution and sometimes lead to the wrong decision.

Now, we describe the deviation D of the two fuzzy numbers that play an important role in our solution procedures. Let $\tilde{c}$ and $\tilde{g}$ be fuzzy numbers with $\{\tilde{c} \geq \alpha\} = [c_L^{-1}(\alpha), c_R^{-1}(\alpha)]$ and $\{\tilde{g} \geq \alpha\} = [g_L^{-1}(\alpha), g_R^{-1}(\alpha)]$. Here, $c_L(t)$, $g_L(t)$ are strictly increasing on $(c_L^{-1}(0), c_L^{-1}(1)]$, $(g_L^{-1}(0), g_L^{-1}(1)]$, respectively. On the other hand, $c_R(t)$, $g_R(t)$ are strictly decreasing on $[c_R^{-1}(1), c_R^{-1}(0))$, $[g_R^{-1}(1), g_R^{-1}(0))$, respectively. Let $c_L^{-1}(1) = c_R^{-1}(1) = c$ and $g_L^{-1}(1) = g_R^{-1}(1) = g$. Then, to solve the FGP problem, we need to use one of three deviations defined below.

First, the deviation D of $\tilde{c}$, $\tilde{g}$ is defined as follows:

$$1 - D(\tilde{c}, \tilde{g}) = \begin{cases} \max\{\text{ solution of } \; c_R^{-1}(\alpha) = g_L^{-1}(\alpha), 0\} & \text{if } c \leq g \\ \max\{\text{ solution of } \; c_L^{-1}(\alpha) = g_R^{-1}(\alpha), 0\} & \text{if } c > g. \end{cases} \tag{8}$$

We notice that the deviation $D(\tilde{c}, \tilde{g})$ is equal to one minus the membership grade of intersection point of the corresponding membership functions of the fuzzy numbers.

Second, if $\tilde{c} = (c, \underline{c}, \overline{c})$ and $\tilde{g} = (g, \underline{g}, \overline{g})$, then the deviation D of $\tilde{c}, \tilde{g} \in \tilde{T}$ is defined as follows:

$$D(\tilde{c}, \tilde{g}) = \min\left\{1, \ \max\{\frac{c - g}{\underline{c} + \overline{g}}, \ \frac{g - c}{\overline{c} + \underline{g}}\}\right\}. \tag{9}$$

Note that denominators in (9) should not be zero in case of utilizing this definition for deviation.

Third, if $\tilde{c} = (c, \underline{c}, \overline{c})_{LR}$ and $\tilde{g} = (g, \underline{g}, \overline{g})_{L'R'}$ where L, R, L' and R' are strictly decreasing, then the deviation D of $\tilde{c}$, $\tilde{g}$ is defined as follows:

$$1 - D(\tilde{c}, \tilde{g}) = \begin{cases} \max\{\text{ solution of } \; c + \overline{c}R^{-1}(\alpha) = g - \underline{g}(L')^{-1}(\alpha), 0\} & \text{if } c \leq g \\ \max\{\text{ solution of } \; c - \underline{c}L^{-1}(\alpha) = g + \overline{g}(R')^{-1}(\alpha), 0\} & \text{if } c > g. \end{cases} \tag{10}$$

3. Fuzzy goal programming problem

Now, we describe our FGP problem in what follows. Let ξ, $0 \leq \xi \leq 1$, be a spread ratio of fuzzy alternatives from $\tilde{X} \subset \tilde{R}^n_\xi$ and let $\tilde{g}_1, \tilde{g}_2, \cdots, \tilde{g}_m$ be the given fuzzy goals defined as (4). Set

$$\tilde{\mathbf{g}} = (\tilde{g}_1, \tilde{g}_2, \cdots, \tilde{g}_m),$$
$$\tilde{\mathbf{c}}(\tilde{\mathbf{x}}) = (\tilde{c}_1(\tilde{\mathbf{x}}), \tilde{c}_2(\tilde{\mathbf{x}}), \cdots, \tilde{c}_m(\tilde{\mathbf{x}})).$$

We here have the same deviation functions, i.e., $D_i = D$ for all pairs of $\tilde{c}_i(\tilde{\mathbf{x}})$ and $\tilde{g}_i$. Then, our FGP problem can be formulated as follows:

$$D(\tilde{\mathbf{c}}(\tilde{\mathbf{x}}), \tilde{\mathbf{g}}) \to \min$$
$$\text{subject to } \tilde{\mathbf{x}} \in \tilde{X}$$

i.e. to find $\tilde{\mathbf{x}}^* \in \tilde{\mathbf{X}}$ which minimizes $D(\tilde{\mathbf{c}}(\tilde{\mathbf{x}}), \tilde{\mathbf{g}})$, where

$$D(\tilde{\mathbf{c}}(\tilde{\mathbf{x}}), \tilde{\mathbf{g}}) = \max\{D(\tilde{c}_1(\tilde{\mathbf{x}}), \tilde{g}_1), \cdots, D(\tilde{c}_m(\tilde{\mathbf{x}}), \tilde{g}_m)\}.$$

This FGP problem can be transformed to the problem of nonlinear programming

$$\lambda \to \min \tag{11}$$
$$\text{subject to } D(\tilde{c}_i(\tilde{\mathbf{x}}), \tilde{g}_i) \le \lambda, \quad i = 1, 2, \cdots, m, \tag{12}$$
$$\tilde{\mathbf{x}} \in \tilde{X}. \tag{13}$$

We evaluate the deviation $D(\tilde{\mathbf{c}}(\tilde{\mathbf{x}}), \tilde{\mathbf{g}})$ directly instead of solving the above nonlinear programming problem and we can also obtain rather analytically the exact optimal solution of FGP problem as follows:

Algorithm.

Step 1 : Compute each deviation $D(\tilde{c}_i(\tilde{\mathbf{x}}), \tilde{g}_i)$ by using α-cut representations (1), (2), (3) and the definition of deviation (10).

Step 2 : Compute

$$D(\tilde{\mathbf{c}}(\tilde{\mathbf{x}}), \tilde{\mathbf{g}}) = \max\{D(\tilde{c}_1(\tilde{\mathbf{x}}), \tilde{g}_1), \cdots, D(\tilde{c}_m(\tilde{\mathbf{x}}), \tilde{g}_m)\}.$$

Step 3 : Find the solution that minimizes $D(\tilde{\mathbf{c}}(\tilde{\mathbf{x}}), \tilde{\mathbf{g}})$.

We think there will be no difficulty in solving the minimum and maximum problems associated with steps 2 and 3 if we utilize commercial softwares.

4. Illustrative Examples

Through numerical examples, we investigate how our procedures are comparable to Ramik's. We here use Algorithm to get the exact optimal solution of the FGP problem introduced by Ramik[7]. We basically use two examples. The first is the same example as used in Ramik[7]. The second is the example we made to demonstrate Ramik's solution leads to the wrong decision.

Example 1. Using the same example as in [7], we here investigate our procedure based on T_M-based fuzzy operations using α-cut representations. Given $\xi = 0.2 \in [0,1]$, we consider the FGP problem defined as follows:

$$\tilde{c}_1(\tilde{x}_1, \tilde{x}_2) = \tilde{6} \otimes \tilde{x}_1 \oplus \tilde{13} \otimes \tilde{x}_2 \quad ; \quad \tilde{g}_1 = \widetilde{1000},$$
$$\tilde{c}_2(\tilde{x}_1, \tilde{x}_2) = \tilde{10} \otimes \tilde{x}_1 \oplus \tilde{7} \otimes \tilde{x}_2 \quad ; \quad \tilde{g}_2 = \widetilde{1000},$$

subject to

$$(\tilde{x}_1, \tilde{x}_2) \in \tilde{X} = \{(\tilde{x}_1, \tilde{x}_2) \in \tilde{R}^2_\xi \; ; \; x_1 + x_2 = 100, \quad x_1, x_2 \geq 0\},$$

where $\tilde{x}_i = (x_i, \underline{x}_i, \overline{x}_i) = (x_i, \xi x_i, \xi x_i)$, $i = 1, 2$, being fuzzy alternatives and, particularly, $\tilde{6} = (6, 1, 1)$, $\tilde{13} = (13, 3, 1)$, $\tilde{10} = (10, 1, 2)$, $\tilde{7} = (7, 1, 1)$, and $\widetilde{1000} = (1000, 200, 0)$, being fuzzy parameter of the fuzzy objective functions.

By using (1) and (3), we get the following α-cuts:

$$[c_{1L}^{-1}(\alpha), c_{1R}^{-1}(\alpha)] = \{\tilde{c}_1(\tilde{x}_1, \tilde{x}_2) \geq \alpha\}, = \{\tilde{6} \otimes \tilde{x}_1 \oplus \tilde{13} \otimes \tilde{x}_2 \geq \alpha\}$$
$$= [(6 + (\alpha - 1))(x_1 + 0.2x_1(\alpha - 1))$$
$$+ (13 + 3(\alpha - 1))(100 - x_1 + 0.2(100 - x_1)(\alpha - 1)),$$
$$(6 + (1 - \alpha))(x_1 + 0.2x_1(1 - \alpha))$$
$$+ (13 + (1 - \alpha))(100 - x_1 + 0.2(100 - x_1)(1 - \alpha))],$$
$$[c_{2L}^{-1}(\alpha), c_{2R}^{-1}(\alpha)] = \{\tilde{c}_2(\tilde{x}_1, \tilde{x}_2) \geq \alpha\} = [\tilde{10} \otimes \tilde{x}_1 \oplus \tilde{7} \otimes \tilde{x}_2 \geq \alpha\}$$
$$= [(10 + (\alpha - 1))(x_1 + 0.2x_1(\alpha - 1))$$
$$+ (7 + (\alpha - 1))(100 - x_1 + 0.2(100 - x_1)(\alpha - 1)),$$
$$(10 + 2(1 - \alpha))(x_1 + 0.2x_1(1 - \alpha))$$
$$+ (7 + (1 - \alpha))(100 - x_1 + 0.2(100 - x_1)(1 - \alpha))],$$
$$[g_L^{-1}(\alpha), g_R^{-1}(\alpha)] = \{\tilde{g}_1 \geq \alpha\} = [800 + 200\alpha, 1000].$$

We also get $c_1 = 1300 - 7x_1, c_2 = 3x_1 + 700$ by using the relation $x_1 + x_2 = 100$. Then, by using (10), we have

$$1 - D(\tilde{c}_1(\tilde{\mathbf{x}}), \tilde{g}_1) = \begin{cases} \max\{\text{solution of } c_{1L}^{-1}(\alpha) = 1000, 0\} & \text{if } 0 \leq x_1 \leq 42\frac{6}{7}, \\ \max\{\text{solution of } c_{1R}^{-1}(\alpha) = 800 + 200\alpha, 0\} & \text{if } 42\frac{6}{7} \leq x_1 \leq 100. \end{cases} \tag{14}$$

Since the equations in (14) are simple quadratic functions with regard to α, we can get analytic solutions as follows:

$$1-D(\tilde{c}_1(\widetilde{\mathbf{x}}),\tilde{g}_1)=\begin{cases}\dfrac{2.6x_1-440+\sqrt{241600-1648x_1+0.36x_1^2}}{120-0.8x_1} & \text{if } 0\le x_1\le 42\frac{6}{7},\\[2mm] \dfrac{600-1.4x_1-\sqrt{289600-1008x_1+1.96x_1^2}}{40} & \text{if } 42\frac{6}{7}\le x_1\le 100,\end{cases}$$

and similarly, we can get

$$\begin{aligned}1-D(\tilde{c}_2(\widetilde{\mathbf{x}}),\tilde{g}_2) &= \max\{\text{ solution of } c_{2R}^{-1}(\alpha)=800+200\alpha,\ 0\} \quad \text{if } 0\le x_1\le 100\\ &= \frac{(480+2x_1)-\sqrt{217600+1408x_1+0.16x_1^2}}{40+0.4x_1} \quad \text{if } 0\le x_1\le 100.\end{aligned}$$

Hence, after finding the x_1 points at which two individual deviation functions intersect and comparing them, we have the maximum deviation as follows:

$$\begin{aligned}D(\widetilde{\mathbf{c}}(\widetilde{\mathbf{x}}),\widetilde{\mathbf{g}}) &= \max\{D(\tilde{c}_1(\widetilde{\mathbf{x}}),\tilde{g}_1),D(\tilde{c}_2(\widetilde{\mathbf{x}}),\tilde{g}_2)\},\\ &=\begin{cases}1-\dfrac{(480+2x_1)-\sqrt{217600+1408x_1+0.16x_1^2}}{40+0.4x_1} & \text{if } 0\le x_1\le 58.66\\[2mm] 1-\dfrac{600-1.4x_1-\sqrt{289600-1008x_1+1.96x_1^2}}{40} & \text{if } 58.66\le x_1\le 100\end{cases}\end{aligned} \tag{15}$$

Therefore, from (15) we can get the optimal solution $\widetilde{\mathbf{x}}^*=(\tilde{x}_1^*,\ \tilde{x}_2^*)=(\widetilde{58.66},\widetilde{41.34})$ with the minimal deviation $\alpha^*=0.23$, which is slightly different from Ramik[7]'s solution $\widetilde{\mathbf{x}}^*=(\tilde{x}_1^*,\ \tilde{x}_2^*)=(\widetilde{56},\widetilde{44})$ with the minimal deviation $\alpha^*=0.05$.

Example 2. In this example, we show Ramik[7]'s FGP problem leads to the wrong decision since it uses an approximate fuzzy multiplication operation. To do this, we consider an FGP problem with two possible candidates for solution as follows:

$$\tilde{c}(\tilde{x})=\tilde{1}\otimes\tilde{x}\ ;\quad \tilde{g}=(g,0,0)$$

subject to

$$\tilde{x}\in\widetilde{X}=\{(1,0.5,0.5),(10,5,5)\},$$

where $\tilde{1}=(1,1,1)$. Note that two elements of $\widetilde{X}$ have the form of $(x,\ \xi x,\ \xi x)$ for $\xi=0.5$.

Let us put $\tilde{x}_1=(1,0.5,0.5)$ and $\tilde{x}_2=(10,5,5)$.

Then, when applying the approximation fuzzy multiplication operation (7), we obtain

$$\tilde{c}(\tilde{x}_1) = \tilde{1} \otimes \tilde{x}_1 = (1,1,1) \otimes (1,0.5,0.5) = (1,1,2),$$

$$\tilde{c}(\tilde{x}_2) = \tilde{1} \otimes \tilde{x}_2 = (1,1,1) \otimes (10,5,5) = (10,10,20).$$

When applying T_M-based fuzzy arithmetic operations using α-cut representations, we obtain

$$\tilde{c}(\tilde{x}_1)(t) = \begin{cases} \dfrac{-1+\sqrt{1+8t}}{2} & \text{if } 0 \le t \le 1 \\ \dfrac{5-\sqrt{1+8t}}{2} & \text{if } 1 \le t \le 3 \end{cases},$$

and

$$\tilde{c}(\tilde{x}_2)(t) = \begin{cases} \dfrac{-5+\sqrt{25+20t}}{10} & \text{if } 0 \le t \le 10 \\ \dfrac{25-\sqrt{25+20t}}{10} & \text{if } 10 \le t \le 30 \end{cases}.$$

Since the deviation can be computed as follows:

$$D(\tilde{c}(\tilde{x}), g) = \min\{1 - \tilde{c}(\tilde{x}_1)(g), 1 - \tilde{c}(\tilde{x}_2)(g)\},$$

in case of using the approximate fuzzy multiplication operation (7), we obtain the optimal solution,

$$\begin{cases} \tilde{x}_1 & \text{if} \quad g \le 2.5 \\ \tilde{x}_2 & \text{if} \quad g \ge 2.5 \end{cases}.$$

On the other hand, when applying T_M-based fuzzy operations using α-cut representations, we have the optimal solution,

$$\begin{cases} \tilde{x}_1 & \text{if} \quad g \le 2.22 \\ \tilde{x}_2 & \text{if} \quad g \ge 2.22 \end{cases}.$$

Therefore, when $g \in (2.22, 2.5)$, we make the wrong decision if we apply the approximate fuzzy multiplication operation (7).

5. Conclusion

In this paper, we have proposed two procedures of obtaining the exact solution of a generalized fuzzy goal programming (FGP) introduced by Ramik[7]. The method is based on T_M-based fuzzy operations using α-cut representations. We also have shown that the proposed procedures produce the exact solution, whereas Ramik's method produces an approximate solution and sometimes leads to the wrong decision since it uses an approximate fuzzy multiplication operation to solve the resultant fuzzy model. Thus, we realize that the proposed procedures provide the exact information for decision making. To conclude, we think the proposed procedures are promising methods for solving the FGP problem illustrated here.

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