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Direct Slicing with Optimum Number of Contour Points

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Abstract – In this work, a rational procedure has been formulated for the selection of points approximating slice contours cut in LOM (Laminated Object manufacturing) with first order approximation. It is suggested that the number of points representing a slice contour can be 'minimised' or 'optmised' by equating the horizontal chordal deviation (HCD) to the user-defined surface form tolerance. It has been shown that such optimization leads to substantial reduction in slice height calculations and NC codes file size for cutting out the slices. Due to optimization, the number of contour points varies from layer to layer, so that points on successive layer contours have to be matched by four sided ruled surface patches and triangular patches. The technological problems associated with the cutting out of triangular patches have been addressed. A robust algorithm has been developed for the determination of slice height for optimum and arbitrary numbers of contour points with different strategies for error calculations. It has been shown that optimisation may even lead to detection and appropriate representation of elusive surface features. An index of optimisation has been defined and calculations of the same have been tabulated.

***Keywords*: Direct slicing, Adaptive slicing, Laminated object manufacturing, LOM, RP, Optimisation**

1. Introduction

LOM (Laminated Object Manufacturing) is one method of RP (Rapid Prototyping) in which profiles or contours are cut out of sheet material and assembled together to build up a part. If the side walls of the profiles are slanted, it is known as first order approximation (Fig. 1). In first order approximation, the complexity of the manufacturing facility and postprocessing requirements is increased. However, the advantage is that in comparison with zero order approximation (straight or vertical side walls), first order approximation yields higher slice height for the same value of cusp height error [1-3]. Moreover, positional continuity exists between layers and hence staircase effect is absent. In this article, the discussion would be restricted to slicing with first order approximation only.

In order to cut out a profile, the cutter moves in steps from point to point, whereby errors are incurred in the layer plane in the form of horizontal chordal deviation (HCD, Fig. 2). This deviation can be kept within user-defined limits by selecting sufficient number of points on the layer contour. These points will be referred to as 'contour points' in this article.

Now, in most of the previously reported literature, the number of contour points is generally far in excess of that required to keep the HCD within limits. For example, Hope *et al.* [1] reported 1400 points per slice, which is not related to the limiting of HCD but to a machine imposed constraint.

However, excess number of points increases the total volume of slice height calculations and NC codes file size for cutting out the slices. In view of this, it is proposed in this work to optimize the number of contour points in every slice by equating the HCD to a user-defined value.

The main resultant problem is that the number of contour points would be varying from slice to slice such that for every point on one contour - there might not be a matching point on the next contour. The matching points, when joined, define the direction of cutting vector, i.e., the orientation of cutter trajectory (Figs. 2 & 3).

This problem did not arise in earlier reported work as optimisation in the number of contour points had not been attempted in such investigations [1-5]. In such cases, slices can be cut with point-to-point matching so that the side-walls would be consisting of a series of four sided ruled surface patches (Fig. 2).

If the number of contour points be optimised, one-to-one matching of points between two contours is not possible and hence two points on one contour might have to be joined with one point on the next contour resulting in a triangular patch. This means that the sloping walls of the slices would contain some triangular patches (Fig. 3) in addition to the four-sided ruled surfaces that are present in point-matched contours. The advantages would be : reduced slice height calculations, NC codes file size and an improved feature detection capability during cutter trajectory determination. In many cases,

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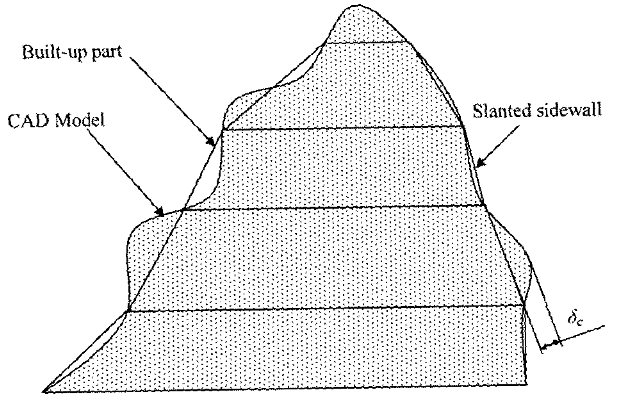


Fig. 1. Built-up part with slanted side walls - first order approximation.

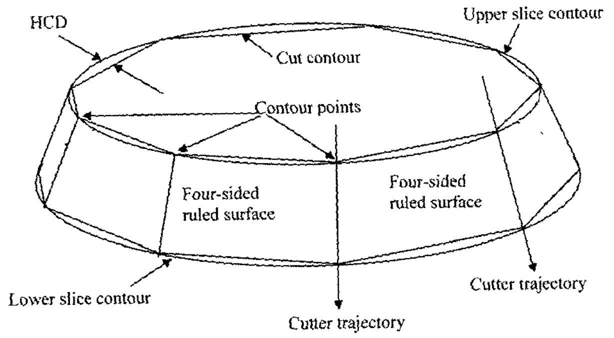


Fig. 2. Side-walls of slice consisting of four-sided ruled surface patches.

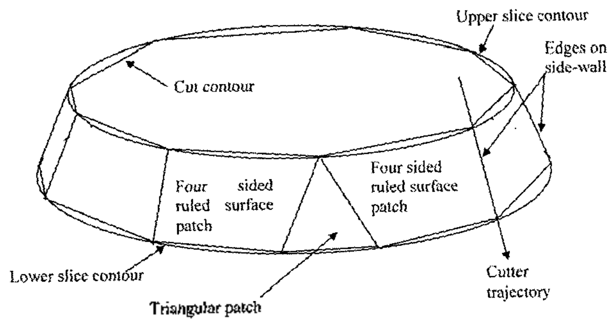


Fig. 3. Formation of triangular patch.

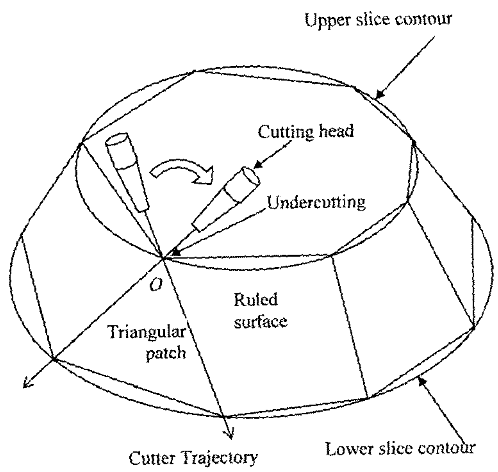


Fig. 4. Undercutting at apex of triangular patch.

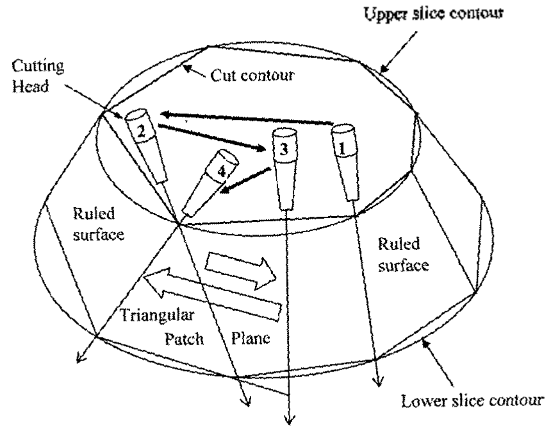


Fig. 5. Effective cutting of triangular patch.

machining time can also be expected to get reduced.

However, the presence of triangles in the contour would present a technological problem. Cutting a triangular patch or face would require the cutter (Abrasive water jet cutting / Laser beam cutting / Plasma Arc Cutting) to pause in its translation and swivel the cutting element (beam / jet / arc) about an axis (normal to patch) through the apex of the triangle (Fig. 4).

This would mean that the cutting element would be lingering at one point (the apex of the triangle) on the top surface of the sheet for a considerable amount of time and heavy undercutting / burning would take place. However, a possible solution is proposed to this problem here (Fig. 5).

The cutter, starting from location 1, moves till location 2, thus cutting out the ruled surface adjacent to the triangular patch. Next, instead of starting on the cut of the triangular face, the cutter moves away from the contour along the plane of the triangle to reach location 3 (movement 2→3). Thereafter, it retraces its path in the opposite direction to reach location 4 thereby cutting out the triangular face (3→4). The cutter does not linger anywhere and hence no undercutting / burning takes place.

2. Related Work

In most of the reported work on direct slicing with first order approximation (and also for slicing of tessellated models with first order approximation), optimisation in the number of contour points has not been attempted. On the other hand, equality in number of contour points for every slice has been considered by [1,3,4] and a host of other researchers.

Optimisation, however, has been attempted from other concepts: Jager *et al.* [3] suggested the optimum choice of cutter trajectory by geometric matching of the contour points of successive layers. The criteria of choice of the cutter trajectory was to find a combination of contour points that gave the best geometrical match to the original surface.

Matching of points between successive contours, which defines cutter trajectory orientation, happens to be an important issue in the work presented here. The effect of optimisation has been tested in various cases of cutter trajectory orientations. In this respect, a survey of investigations in the field of cutter trajectory orientation is extremely relevant and hence presented here.

In earlier reported work, various formulations for cutter trajectory orientation have been implemented. Hope *et al.* [1,2] considered the cutter trajectory at a contour point to be contained in a plane defined by the surface normal at mid-height of the slice and the vertical. This plane is called Normal vertical section or simply NVS. In the same paper, Hope *et al.* also proposed the joining of points having same s-parametric value on successive intersection curves to obtain the cutter trajectories.

Jager [3] had also suggested topological matching of contour points for determining cutter trajectory in case of contours originating by slicing of a BREP model. Topological information was used to find correspondence between points on adjacent contours.

Im and Walczyk [4] proposed two algorithms for cutter trajectory orientation: IEPS (Identical Equidistant Profile Segmentation) and AVPP (Adaptively Vectored Profiles Projection). In the former, they proposed that the contours be divided into equal number of segments of equal lengths and the corresponding contour points be joined. AVPP, on the other hand, entails the projecting of data points from one profile to the other by means of an adaptive tool vector.

Banerjee *et al.* [6] proposed two cutter trajectory algorithms in which the cutter could be oriented in the directions of maximum normal curvature or maximum flatness. It was shown that while cutter trajectory along maximum flatness direction would yield fewer deviations and hence higher slice thickness values, orienting the cutter along maximum normal curvature would result in more slices but a more accurate prototype.

Kumar *et al.* [7] proposed inclined cutter trajectory lines which would yield uniform cusp height at all points of the CAD model and hence uniformity in surface deviations throughout the built-up part.

The concept of 'Visibility pyramid' was proposed and implemented to determine the adaptive slice height and cutter trajectory by Chen and Song [8].

Last of all, Kumar and Roy Choudhury [5] proposed a patch-based error estimation procedure where the built-up part was represented by a series of taut cubic spline patches. It was shown that patch-based error estimation tends to yield more number of slices but more accurate prototypes.

In view of these investigations, the present work attempts to propose the selection of optimum number of contour points for each slice. Work in this exact direction has not been reported previously. Further, instead of meticulously applying the optimising technique in case of all the reported categories of cutter trajectory orientation, the present investigation has generalised the cutter orientations into broad categories and then applied the technique of optimisation.

3. Slicing of CAD Model

3.1. Rational B-spline surfaces

The free form surfaces considered for the present work (Figs. 6 and 7) can be modelled by employing rational B-Spline surfaces. A rational B-Spline surface [10] is expressed in the parametric form as,

$$R(u, w) = \frac{\sum_{i=1}^{n+1}\sum_{j=1}^{m+1} h_{i,j} B_{i,j} N_{i,k}(u) M_{j,l}(w)}{\sum_{i=1}^{n+1}\sum_{j=1}^{m+1} h_{i,j} N_{i,k}(u) M_{j,l}(w)} \tag{1}$$

Parameters u and w can assume values ranging from zero to any positive value. The $B_{i,j}$'s define the control polygon net which control the shape of the CAD model. The $h_{i,j}$'s are homogeneous coordinate transformation coefficients, and $N_{i,k}(u)$ and $M_{j,l}(w)$ are the basis functions in the parametric u and w directions respectively. k and l are the orders (degree+1) of the surface in u and w directions respectively. The basis functions are defined as : –

$$N_{i,1}(u) = \begin{cases} 1 & \text{if } x_i \le u < x_{i+1} \\ 0 & \text{otherwise} \end{cases} \quad \text{that is, for } k = 1 \tag{2}$$

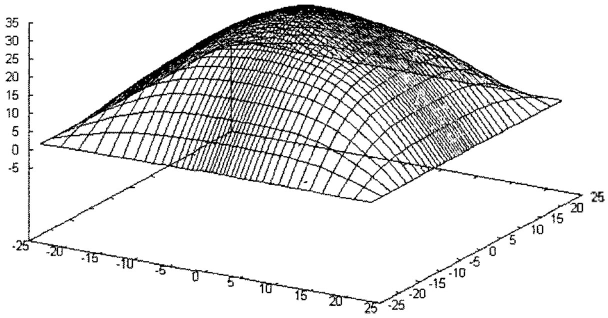


Fig. 6. CAD model of First free form surface.

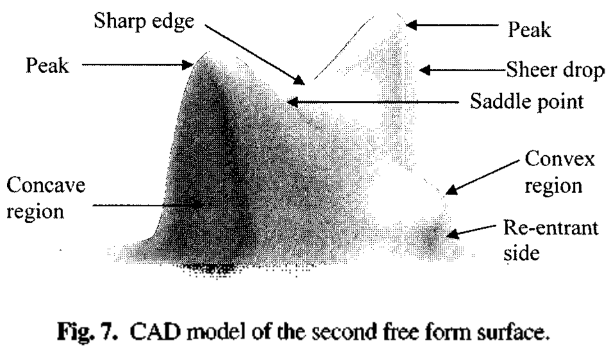


Fig. 7. CAD model of the second free form surface.

and

$$N_{i,k}(u) = \frac{(u-x_i)N_{i,k-1}(u)}{(x_{i+k-1}-x_i)} + \frac{(x_{i+k}-u)N_{i+1,k-1}(u)}{(x_{i+k}-x_{i+1})} \quad \text{for } k > 1 \qquad (3)$$

With similar expressions for $M_{j,l}$ as a function of w instead of u.

The x_i's are the elements of knot vectors in u direction and satisfy the relation $x_i \le x_{i+1}$. The elements of knot vectors can assume any value ranging from 0 to any positive value. The convention 0/0=0 is adopted in the rational B-Spline surfaces.

3.2. Salient features of the selected surfaces

The two surfaces selected for the present work have varying degree of complexity. The first free form surface is relatively simple so that the basic traits of optimisation can be easily identified. The second surface is of more complex nature which would rigorously test out such traits to finally identify the stable characteristic features of optimisation. The second surface is having the following features incorporated (Fig. 7) :

a. Multiple Peaks
b. Saddle point
c. Sharp edge
d. Sheer drop
e. Convex and concave regions
f. Re-entrant sides

Thus, it can be considered as a surface with a reasonable degree of complexity.

3.3. Location of optimised contour points

The location of optimised points on a contour can be determined by equating the HCD to the user-defined error value. HCD or Horizontal chordal deviation is the maximum deviation between the CAD-model and the cut contour in the horizontal plane (Fig. 2). The cut contour is represented by straight lines joining the contour points. The correct positions of the optimised points are arrived at iteratively.

At first, from a point P on the contour, another point Q is determined on the contour by the application of the Surface-Plane Intersection (SPI) algorithm [11]. Here, the surface is the CAD model surface and the plane the horizontal plane (Fig. 8). A guess value is used for the step length δ_S in the Eqns. (4) and (5) :

$$(R_u \cdot c)\Delta u + (R_w \cdot c)\Delta w = \delta_s \qquad (4)$$

$$(R_u \cdot p)\Delta u + (R_w \cdot p)\Delta w = 0 \qquad (5)$$

Here, R_u and R_w are the partials to the surface $R(u,w)$ w.r.t. u and w respectively; N is the unit outward normal to the surface = $\{R_u \times R_w / |R_u \times R_w|\}$; p is the normal to the slicing plane and c is the unit tangent to the intersection curve (slice contour) between the CAD Model and the slicing plane = $\{n \times p / |n \times p|\}$.

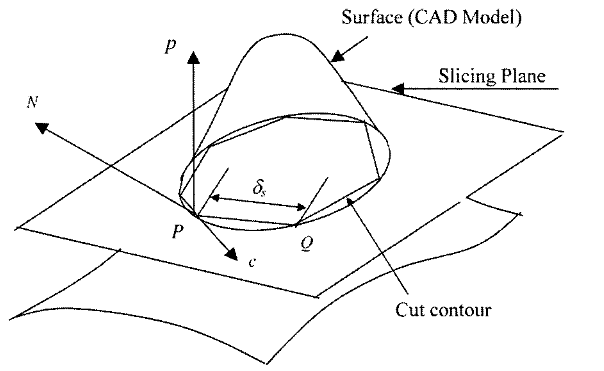


Fig. 8. Surface-plane intersection showing relevant nomenclature.

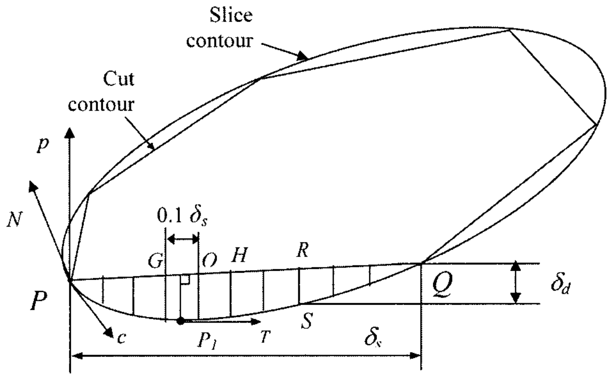


Fig. 9. Slice contour showing important surface-plane parameters.

Now, ten more points are selected on the chord PQ (e.g., R) such that these points lie evenly between P and Q. Distances between these points and the curve PQ (e.g., distance = RS = dd, as shown in Fig. 10) are computed and the maximum (OP_l in the figure) is compared with the user-defined error. If the absolute difference is just less than a preset value, the ten-point comparison is now applied between the two points on either side of O (G and H in the figure). This process is repeated till the difference stabilizes and is, of course, just less than the preset value. In such a case, Q is accepted as the optimal point next to P. If not, δ_s is changed by successive approximation method and the process iterated till the HCD equals the user-defined error. This strategy of optimisation of contour points ensures that no circular arc approximation is resorted to.

3.4. Error estimation strategies

Slice height calculations are carried out by equating errors in the prototype to a user-defined value. The errors, which are basically the deviation of the actual built-up part from the CAD model, may be estimated by section-based methods and patch-based methods (Fig. 10).

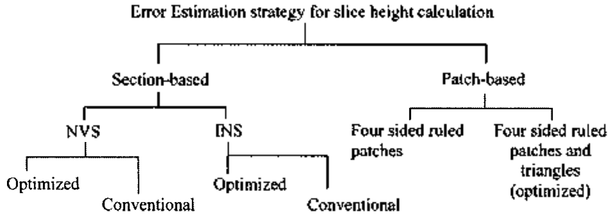


Fig. 10.

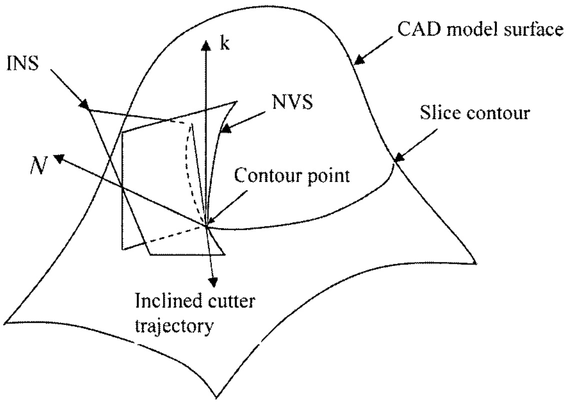


Fig. 11. Location of NVS and INS at a contour point.

3.4.1. Section-based methods

In these methods, the errors are calculated in sections containing the cutter trajectory and the surface normal. This error is the maximum deviation between the actual part and the CAD model and called cusp height. It is essentially a 2-D check. The section-based methods selected for the present study are the NVS and the INS (Inclined Normal Section) (Fig. 11). The reason for the choice is that the INS method is the most generalised cutter trajectory determination strategy with no preconditions while NVS is one of the most widely used. When the cutter trajectory is not contained in the NVS of the concerned point, INS has to be resorted to. INS is formally defined as the plane or section containing the cutter trajectory and the surface normal at the concerned point [5].

3.4.2. Patch-based methods

In first order approximation, slanted side walls of the layers consist of four sided ruled surface patches. For slice height determination, maximum deviation of the ruled patch from the CAD model surface is equated to a user-defined value. Optimisation introduces triangular patches in addition to the four sided ruled surface patches. In the present work, both section-based and patch-based strategies of slice height calculations have been considered for examining the effect of optimisation.

Error estimation has been carried out in all cases for two categories : taking excess / arbitrary number of contour points or taking optimum number of points. Cases with excess number of points have been termed as 'Conventional'.

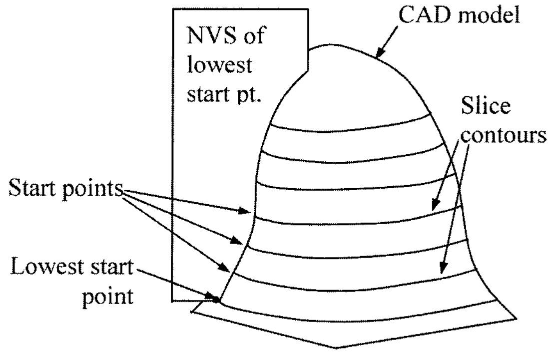


Fig. 12. Determination of start points.

Cutter trajectory orientations, which are obtained by contour point matching, are strongly dependent on the choice / selection of the starting contour point. In the present work, the start points on every contour have been selected to be on the same vertical plane. This plane is the NVS of the start point of the lowest contour (Fig. 12). The start point of the lowest contour is selected arbitrarily.

(a) Error calculation in NVS with **arbitrary / excess number of** contour points - *The NVS conventional method*

This case is applicable where the cutter trajectory is in the NVS and an arbitrary number of contour points are taken into consideration. On the lowermost layer, an arbitrary number of contour points are chosen with equal spacing, and an initial slice height, (the lowest possible) is considered as guess value. Sectional error is found in NVS (Fig. 13) as distance from points on CAD model to the cutter trajectory. If the error is within user-defined limits at the points of consideration, a next higher value of slice height is taken and error is checked similarly. If it is found to exceed the user defined limit, then the lower slice height is accepted. If the error exceeds the tolerance at the initial slice height, then in such case the lowest possible slice height is taken. Once the slice height value is decided, the intersection of the NVS planes with the upper slice contour (carried

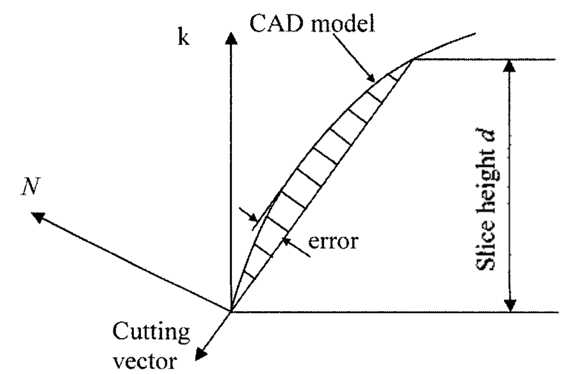


Fig. 13. Error incurred due to first order approximation in NVS.

out as per equations (4) and (5)) decides the location of the contour points in the upper slice.

(b) Error calculation in NVS with **optimum number of** contour points - *The NVS optimised method*

On the lowermost contour, optimum number of points (=N_l) are selected following the procedure described under the subsection 3.3 : location of optimum points; and NVS-based slice height calculation is carried out as in the case of conventional method described earlier. At the height thus decided upon, location of optimum number of points is again determined for the user-defined HCD with a starting contour point obtained as per discussion in subsection 3.4 in connection with Fig. 12. This way, N_2 number of optimum points is obtained on the second (upper) contour and NVS based calculation is applied to obtain the next slice height. This means that N_1 and N_2 numbers of contour points would co-exist at a slice plane. One obvious disadvantage of this procedure is the formation of ledges and overhangs that reduce the smoothness and continuity of the surface (Fig. 14). The G0 (positional continuity) continuity would be lost in that case.

However, an algorithm has been developed for reducing the severity of overhangs and ledges. The severity of the ledges and overhangs can be estimated by their total area - as shown in grey colour in Fig. 15. This area is calculated by polygonal clipping. Now, from the figure, it is clear that total magnitude of the area is a function of the contour point positions. Contour point positions can be changed by shifting the location of the first or start contour point. Hence, the start point is changed and area of the ledges and overhangs re-calculated. The process is carried out iteratively till the area is reduced below a user-defined limit. Hence, it is not exactly a global minimum but an acceptable solution to the present problem.

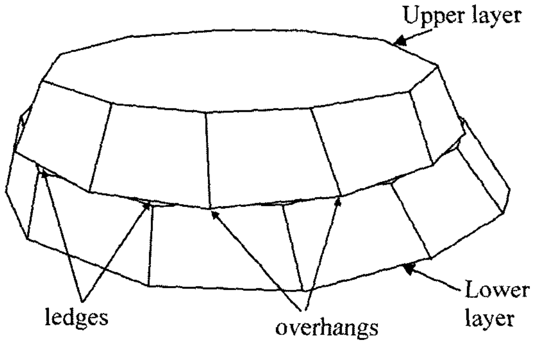


Fig. 14. Successive layers showing ledges and overhangs.

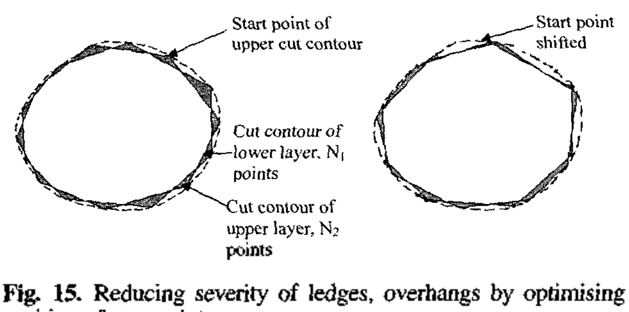


Fig. 15. Reducing severity of ledges, overhangs by optimising position of start point.

(c) Error calculation in INS with **arbitrary / excess number** of contour points - *The INS conventional method (section-based)*

In this method, an arbitrary number of points (but equal in number to those in the other conventional methods) are chosen on the lowest contour with equal spacing. On the next slice, after selecting the first point as per discussion in subsection 3.4 in connection with Fig. 12, an equal number of points are again selected with equal spacing. These points are now joined with one-to-one correspondence to yield the cutter trajectories and INS based slice height calculations are carried out at all contour points. The least value is selected as slice height.

For slice height calculations in INS, an SPI method similar to that elucidated through equations (4) and (5) is applied. In this case, the plane of intersection is the INS defined by the normal to the surface at the concerned point and the cutter trajectory.

One point to note is that no optimisation is attempted in the orientation of the trajectories. They are simply taken as obtained by the matching of arbitrarily placed contour points.

(d) Error calculation in INS with **optimum number** of contour points - *The INS optimised method (section-based)*

Optimum number of points is chosen on the lower contour and a guess value of slice height is assumed. Slicing is done at this height and optimum points are obtained. The selection of starting points is done as per discussion in subsection 3.4 in relation to Fig. 12. Points are matched such that triangles and four sided ruled surfaces result. Triangles are placed in the regions of lowest curvature (to be discussed in appropriate subsection 3.7 Determination of locations for placement of triangular patches); slice height calculations are carried out in INS. These INS are defined by the lateral / upright edges of the four sided ruled patches and triangular patches with their respective surface normals (N). This way, new slice height is obtained. This process is iterated till slice height converges.

(e) Error calculation in ruled patches for INS with **arbitrary / excess number** of contour points - *The INS conventional method (ruled patch-based)*

This process is similar to (c) with the difference that error is considered to be the maximum deviation between the CAD-model and ruled surface along normal to CAD-model (Fig. 16). For the four-sided ruled surface

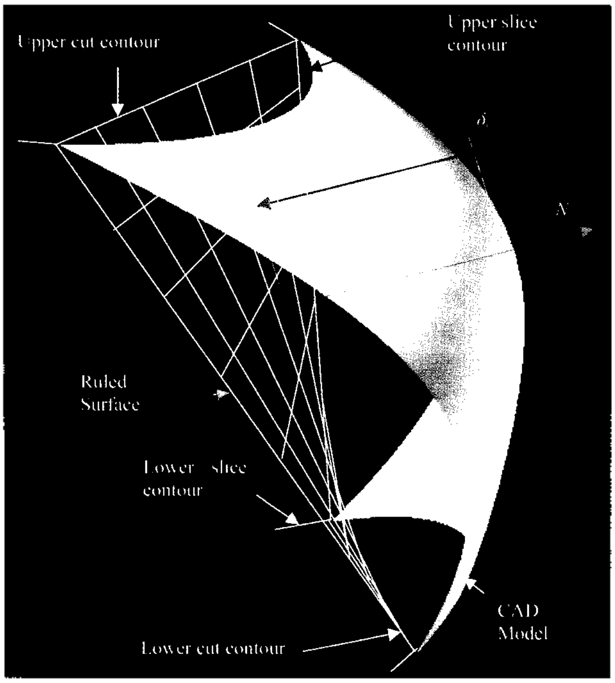


Fig. 16. Deviation between four-sided ruled surface patch and CAD Model.

patch, deviation is calculated at a number of points on the patch and the maximum of those is compared with the user-defined error. When the difference is less than a preset small value, the slice height is accepted. Expectedly, it yields thinner slices.

(f) Error calculation in four-sided ruled patches and triangular patches for INS with **optimum number** of contour points - *The INS optimised method (four-sided ruled patch and triangular patch-based)*

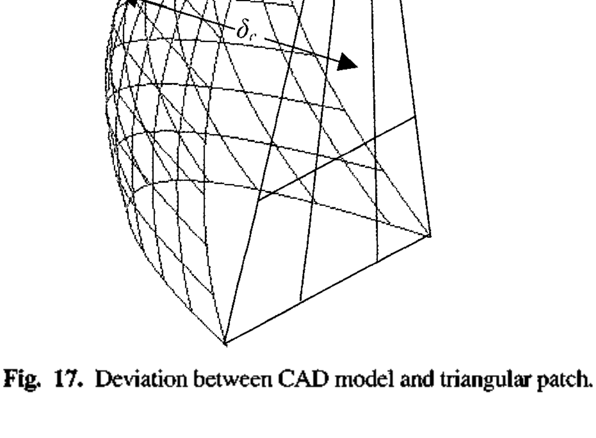


Fig. 17. Deviation between CAD model and triangular patch.

This process is similar to (d) with the difference that instead of INS-based error calculations, error is calculated between four sided ruled surface patches and CAD model and also between triangular patches and CAD-model (Fig. 17).

Among the different strategies of error calculations, methods (e) and (f) check the deviations between the surface patches from the CAD model. They might be four sided ruled surface patches and triangular patches. In either case, deviation between the patch in question and the CAD model has to be found out.

3.5. Deviation in case of ruled patch

The deviation between a ruled surface and the CAD model (Fig. 16) can be found out by determining the maximum deviation between CAD model and ruled surface along the normal to the CAD-Model. The reader may kindly refer to the work of Kumar and Roy Choudhury [5] where a very similar method has been covered in detail.

3.6. Deviation in case of triangular patch

A triangular patch is essentially a plane. Hence there is a unique maximum distance between this plane and the point of local maxima on the Rational B-Spline surface within the confines of the triangular patch. This means that the Rational B-Spline surface has a critical point with reference to the plane of the triangle. The distance of this critical point from the triangular plane (= δ_c as shown in Fig. 17) is described as the surface-to-patch deviation for triangular patch. The critical point can be determined by Newton's iteration scheme [11].

3.7. Determination of locations for placement of triangular patches

It has been proposed in this work that if there is a difference in the number of contour points in successive slice contours, four sided ruled surface patches and triangular patches would be making up the outer wall of the slices. The exact number of triangular patches required for the purpose can be found out from the difference in the number of contour points in the lower and upper slices. However, the particular locations at which the triangular patches would be introduced have not been decided till this point. For the first surface, a simple algorithm has been developed which positions the triangular patches at different locations all around the contours. In the logic supporting the algorithm, it is argued that the four-sided ruled patches can better approximate a curved surface than a triangular patch as the former itself is a curved surface while the latter is just a plane. Thus, it would be better to locate the triangular patches at those places on the CAD model with the least curvature. Hence, at all contour points, the maximum curvature was determined and triangular patches were placed at those places where the maximum curvature was least.

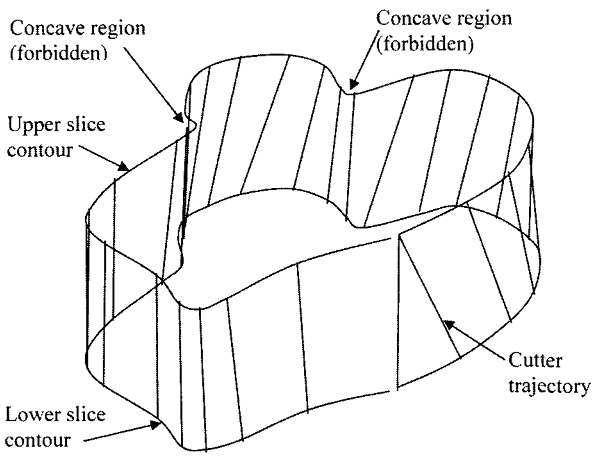


Fig. 18. Slice showing concave region forbidden for triangular segments.

In addition to the above criteria for the placement of the triangular patches, a constraint was applied. Triangles were to be avoided in regions where the (horizontal) curvature of the periphery of the slice was concave (Fig 18). This is because the triangle cannot be cut by the proposed method in a horizontally concave region.

In this procedure, several regions of the slice may qualify for the placement of triangles. So, initially, the triangles are placed in one such region and slice height calculations are carried out according to the strategy under consideration. If calculations support the slice height chosen, the positions of the triangles are accepted. If not, another region which had qualified for triangle placement is chosen and tested. It is thus carried out on a first satisfy - first select basis.

4. Results and Discussions

Optimisation in the number of contour points has been applied in case of two surfaces. While the first one is a relatively simple surface with no complex features, the second one incorporates a number of specific surface characteristics. It is expected that optimisation for the simple surface would identify and reveal fundamental characteristics of the process of optimisation. These would then be tested on the more complex surface with a rigorous check.

After slicing the first free form surface (Fig. 19, (a) to (d)) in accordance with the different strategies of error calculation, the results have been tabulated in Table 1.

The observations that can be made about the slices is that

a. The number of slices varies according to the strategy of error calculations
b. Optimisation in the number of contour points does not affect the number or thickness of slices. This is valid for all strategies of error calculations.

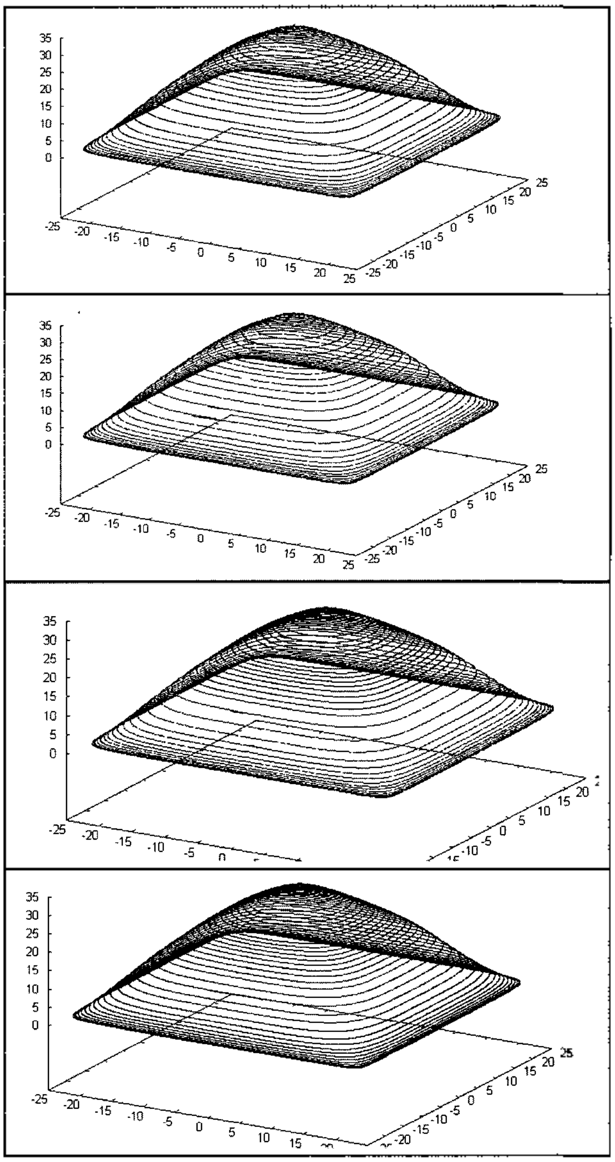


Fig. 19. Sliced figures of the first free form surface.

It would be meaningful to discuss the first observation from the point of view of the relation between cutter trajectory and error calculations. The slice height calculation for the case of patch-based methods is the most conservative of all the strategies of error calculations. As a result, the number of slices obtained for building up a CAD-model for this method is the maximum among all the methods. This fact can easily be verified from Table 1 and Fig. 19. This is generally applicable in cases of stringent acuracy conditions.

Here, the total number of slices for INS-based calculations (=37) being higher than that for the NVS-based calculations (≈34) should not be treated as a significant observation as this depends on the orientation of the INS and could even be less than 34 in some cases. In fact, it was felt that some form of optimisation could be introduced in the choice of the orientation of the cutter trajectory in the INS.

Table 1. Slice data and index of optimisation for first surface

Analysis of Slicing data for first surface, HCD = 0.001 mm											
NVS conventional		NVS optimised		INS conventional		INS Optimised		INS with ruled patches		INS with ruled patch and triangles	
No. of Slices	Total no. of pts	No. of Slices	Total no. of Pts	No. of Slices	Total no. of points	No. of Slices	Total no. of points	No. of Slices	Total no. of points	No. of Slices	Total no. of points
34	47600	33	18402	37	51800	37	19687	39	54600	39	20935
η_{opt} = 0.6134				η_{opt} = 0.6199				η_{opt} = 0.6165			

The second observation is very important from the point of view of the efficacy of optimization of the number of contour points. It can be clearly seen from Table 1 and Fig. 19 that for a particular strategy of error calculation, the number of slices is invariant with respect to the number of contour points considered (whether excess / optimised number). (The fact that NVS-based calculations yield marginally different numbers of slices for excess and optimised number of contour points is not considered significant here).

However, it is expected that in case of patch-based errors, reduction in the number of contour points due to optimisation should reduce slice height. In this case, however, such a phenomenon is not observed as vertical curvature on the surface is largely absent and the HCD is low.

Optimization in the number of contour points has not affected the efficiency of the slicing process, i.e., the slices do not vary in thickness due to optimization. The same calculations would apply and hence, the same number of slices would result. On the other hand, the advantage is that due to optimization, less number of points would have to be considered (as compared to arbitrary or excess number of points) which would subsequently reduce the volume of slice height calculations and size of NC-codes file to be transferred to the slice cutting machine / facility. The size of the NC-codes file is a significant item in computer-machine interaction as there is sometimes a restriction in the number of points a machine can handle [1]. The number of points in each slice in each of the cases is shown in Table 1. An index of optimization may be defined for each of the strategies of slicing as :

$$\eta_{opt}=\frac{\sum t_a-\sum t_o}{\sum t_a} \qquad (6)$$

where t_a is the number of arbitrary contour points and t_o is the number of optimised contour points. This index has been determined for all three methods applied here. The closer the index is to unity, the higher is the efficiency of optimization. It is found from Table 1 that this efficiency is more or less independent of the slicing strategy employed. Further, the value is above 0.6, which suggests appreciable saving in slice height calculation and reduction in NC-code file size due to optimization.

Over and above the reduction in calculations and file size, it may be noted that the primary objective of optimization, namely : the maintenance of horizontal error within user defined limits, is a significant improvement in the slicing of free form surfaces.

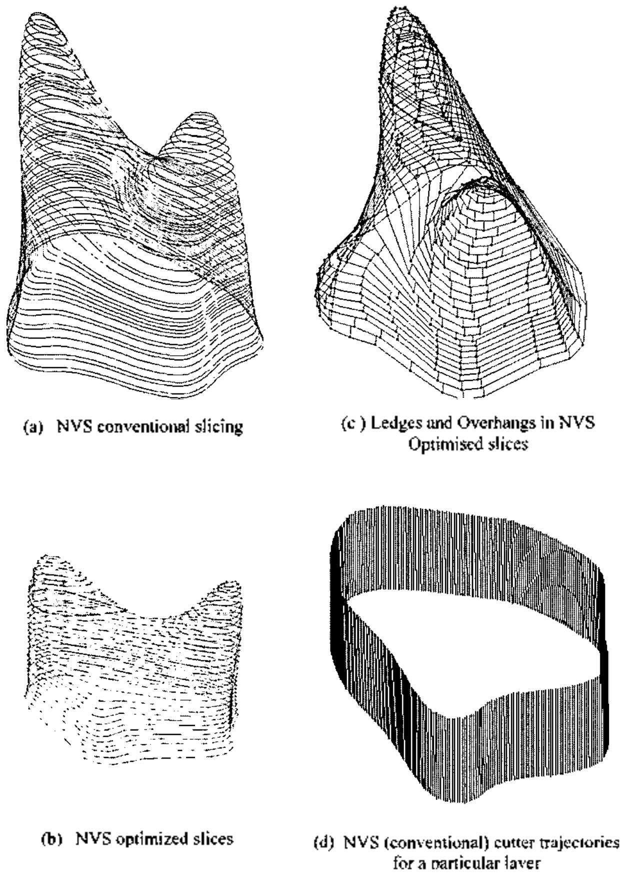
In view of the above observations on the slicing of the first free form surface, a second free form surface was taken into consideration. However, feedback from the slicing of the first surface initiated the authors to introduce the following improvements and modifications in the procedure for slicing and optimisation.

1. The value of the HCD and the cusp height (δ_c) was increased to 0.1 mm. This was done so that the effect of optimisation would be more pronounced and hence observations could be made with confidence.
2. In case of INS conventional method (i.e., with excess number of contour points), the cutter trajectories were arbitrarily chosen (by matching of arbitrarily placed points). For the case of the second surface, these trajectory lines were optimised so as to yield least cusp height. The optimum trajectory was chosen from a range of ±30° to the vertical in steps of 1° [7]. However, this optimisation here refers to that of the cutter trajectory orientation and not optimisation of the number of contour points.
3. The slice heights are now considered to be discrete and three slice thickness values : 0.5, 1.0 and 1.5 mm are considered.
4. Machining time for cutting the slices out has been found out for the second slice as per calculations in [6].
5. The entries under the item 'Computation time' in Table 2 include time for optimisation for contour points, slicing and optimisation of cutter trajectory orientation, whichever is applicable in a particular case.

Regarding the slicing of the second surface, Fig. 20(a) shows slicing as per NVS conventional method while Fig. 20(b) shows the slices for NVS optimised case. The ledges and overhangs together with the cutter trajectories are shown in Fig. 20(c) for the NVS optimised method. Fig. 20(d) shows cutter trajectory

Table 2. Slice data and index of optimisation for second surface

Analysis of Slicing data for second surface, HCD = 0.1 mm					
Error estimation strategy	No. of slices	No. of points	Computation time, secs	Machining time, secs	Index of Optimisation
NVS (conventional)	52	16779	40,099	2371.4	0.93
NVS (optimized)	42	1198	5,062	1856.7	
INS (conventional), Section-based errors	41	9996	115,764	1404.9	0.76
INS (optimised), Section-based errors	47	2341	27,091	1975.6	
INS (conventional with four-sided ruled patches)	47	14637	151,019	2183.3	0.86
INS (optimized with four-sided ruled patch and triangle-based errors)	48	1981	26,051	1790.1	



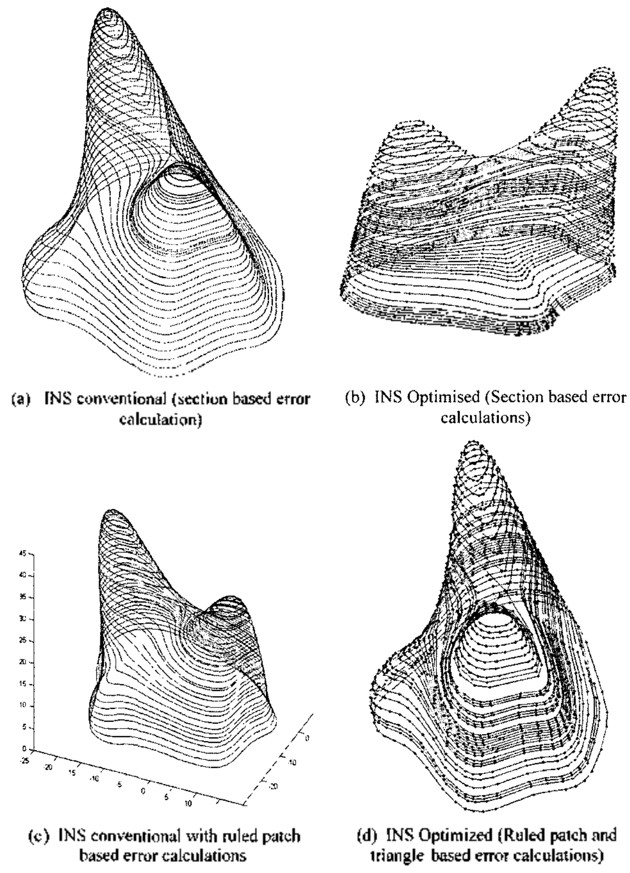
(a) NVS conventional slicing

(c) Ledges and Overhangs in NVS Optimised slices

(b) NVS optimized slices

(d) NVS (conventional) cutter trajectories for a particular layer

Fig. 20. Sliced figures for the second free form surface for NVS conventional and NVS optimised strategies.



(a) INS conventional (section based error calculation)

(b) INS Optimised (Section based error calculations)

(c) INS conventional with ruled patch based error calculations

(d) INS Optimized (Ruled patch and triangle based error calculations)

Fig. 21. Sliced figures for the second free form surface for INS conventional and INS optimized strategies.

for NVS conventional and they are expectedly dense-packed due to excess number of points. Fig. 21(a) shows slicing as per INS conventional method with section-based error estimation. Fig. 21(b) shows the slices with location of the contour points in case of INS optimised with section-based error calculations. It is not possible to show the contour points in case of conventional methods as they are densely packed. Figs. 21(c) and (d) show the slices for the patch-based methods for the conventional and optimised cases respectively. Fig. 22 shows the placement of triangles and four-sided ruled surface patches in the NVS optimised case (patch based). Fig. 23 shows the loss of sharp features while employing NVS conventional method of slicing.

A critical observation of the table of data for the second surface reveals that

(a) Computational time is consistently and considerably less due to optimisation for all strategies of error estimation. It is exceptionally less in case of NVS-based error calculations. This is because in

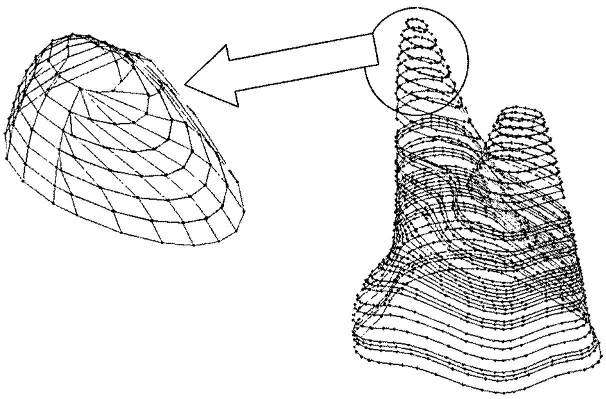


Fig 22. Inset showing the formation of ruled patches and triangles.

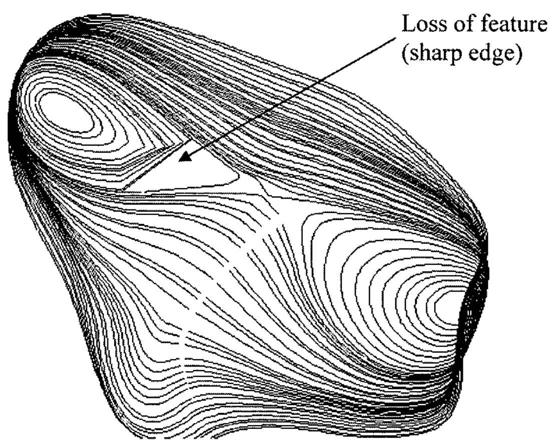


Fig. 23. Figure showing loss of feature in NVS conventional.

NVS, there is only one plane in which the error has to be checked and that is the NVS. In case of other strategies of error estimation, the procedure is not so straightforward, as will be apparent from the discussion that follows.

(b) In comparison to the results of the first surface, it may be observed that the number of slices varies according to the error calculation strategy.

(c) The number of slices is the maximum in case of the NVS method as no optimisation in the orientation of the NVS cutter trajectory is possible. However, the number of slices in NVS optimised case is not the same as in NVS conventional. It is here that the results of the second surface are in direct contradiction of those obtained in case of the first surface. This is due to the fact that conventional methods have more contour points and hence carry out a more rigorous check on the slicing error. As the first surface was relatively simple, more contour points (and therefore a more rigorous check) did not really matter.

(d) The computational time is consistently and considerably high for the INS based conventional methods in comparison to the conventional NVS method. This is a direct consequence of the optimisation process applied to cutter trajectory orientation for minimising cusp height. A considerable number of possible cutter trajectory orientations (within $\pm 30°$) are checked for every contour point for the INS based methods and the one yielding least cusp height error is selected. Hence the computation time is higher compared to NVS-based methods.

(e) The number of slices is the minimum for the INS (conventional with section based errors and conventional with four-sided patch based) methods. This is a direct consequence of optimisation in the orientation of the cutter trajectory that minimises the cusp height error and hence maximises the slice height

The significance of the above observation lies in the fact that there is always the possibility of the optimisation of cutter trajectory in INS while in NVS that possibility is not there.

(f) The number of slices has not changed appreciably due to optimisation for the case of patch-based error estimation. This is probably due to lack of vertical curvature in the major part of the surface.

(g) One special capability of the optimised methods is to 'detect' a sharp edge or corner. In order to limit the HCD to user-defined values, the optimised methods tend to put one contour point in the sharp corner. In comparison, the conventional methods have no such provision to re-position their contour points. The results can generate loss of critical features in the built-up part for the conventional methods (Fig. 23). In such case, for conventional methods, an artificial density of points needs to be created around the sharp corner, to generate correct results.

(h) The total number of points obtained due to slicing gives an estimate of the NC codes file size, time expended due to file transfer and other non-cutting and non-computational file-management operations. In view of this, it is desirable that file size should be kept within limits. Further, more number of points per slice increases the chances of vibration during cutting. In this respect, optimisation shows considerable improvement over existing methods with excess number of points.

(i) Machining time is affected both by optimisation as well as number of slices. Less time is required for lower number of slices. Hence, optimisation would significantly reduce machining time in cases where it does not affect (increase) the number of slices.

(j) Choice of the value of HCD has a profound effect on the index of optimisation. High values of HCD tend to increase the index and vice versa. Tables 1 and 2 corroborate this point.

5. Conclusions

A number of new ideas have been introduced in this paper in connection with direct slicing of CAD models. These are

(a) Optimisation in the number of contour points
(b) Introduction of the idea of the triangular patch and the suggested method of cutting the same
(c) New strategy of error estimation for slice height calculations
(d) Definition of an index of optimisation

It can be concluded from the results and discussions that optimisation in the number of contour points significantly reduces the computation time for slice height determination, NC-codes file size (for cutting the slices) in the direct slicing of free form surfaces with first order approximation. Machining time is also reduced if optimisation does not affect (increase) the number of slices.

It may be observed here that the positioning of the triangular patches, as suggested by the authors in this paper, requires some standardisation. This will be the topic of another paper, which is going to be submitted within a short time.

Acknowledgement

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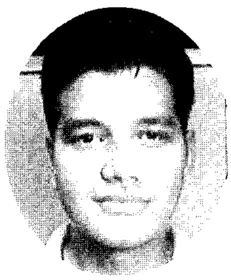
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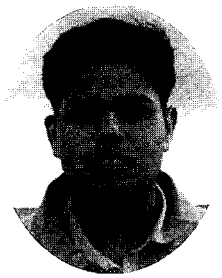
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