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Differences in the Use of Heuristics When a Sixth Grader Solves a Problem

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ABSTRACT

The purpose of this study is to look at the use of heuristics when a sixth grader solves a problem. Two research questions have been formulated: The similarities and differences in the use of heuristics when a student solves two problems that are science-knowledge-based and not science-knowledge-based, and the different types of prompts. A male sixth grade student participated in this study. All of the information for the study was collected in three interviews. The interviews began with observing the student's solving problems. The student was asked how and why he solved problem that way. There were some interactions between the researcher and the student during the interview procedures. As results of this study, eight general heuristics were used in both solutions: Check examples for support of an idea; check examples for exceptions to an idea; restate the problem; compare to known examples or patterns; make a hypothesis; check the relevance of other information present; use analogy; and recognize patterns/similarity. There seemed to be more similarities than differences in the type of general heuristic that were used in the two problem solutions. The student was systematic and consistent in his use of the general use of heuristics. Five types of interviewer prompts were detected in the two problem solutions, directional cues, modeling, clarity, problem posing, metacognition and validation.

Key words: heuristics, problem solving, prompts, problem solutions

I. Introduction

Deductive reasoning is to take information and sort it according to an existing arrangement. This can be referred to as 'an information-rearranging process'. On the other hand, inductive reasoning can be referred to as 'an information-extending process'. This is the process of taking known information, drawing out the similarities, and making general statements based on the similarities(Hunt, 1982). Dimitri Medeleev used inductive reasoning to develop the periodic table of elements, and children use inductive reasoning to acquire grammar when learning how to speak(Hunt, 1982). It is a natural process that is even instinctual in simpler organisms(Hunt, 1982). Inductive reasoning seems to be the extension of a natural tendency.

There is some development of these tendencies as one matures. Millar and Barg(1982) used inclusion problems in a study to show that child development of inductive reasoning ability as

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s/he matures. The result was that seven and nine year olds solve problems more consistently than five year olds. Another study by Ferrara, Brown, and Campions points out that there is a larger difference in the general problem solving abilities of third graders and fifth graders, as the transfer distance of the problem increases(Ferrara, *et al.*, 1986). In other words, the older children were more proficient at finding out problems that were only remotely related to previous material.

Children in intermediate grades have the ability to problem solve quite well(Worth, 1982). In a study of fourth graders by Lee(Lee, 1982) it was shown that the students employed a large number of well developed heuristics to various problems. Putting the idea of intermediate students being proficient problem solvers, together with the idea of cognitive development, it would seem realistic to think that heuristics that are used by a fourth grader could be developed and used more effectively by an older child(Martinez, 1998).

The purpose of this study is to look at the use of heuristics when a sixth grader solves a problem. An interesting aspect of this problem is that it uses a limited amount of prior knowledge of chemistry, but a larger number of scientific vocabulary words. Do the use of scientific words alter the way in which an individual solves a problem? The comparison to another problem that did not use scientific knowledge or science vocabulary was enticing. The science vocabulary in this problem gives the appearance of an exclusively science-knowledge-based problem. In reality, it is more a classification problem than a science knowledge problem. I wondered if the use of science vocabulary, the apparent use of content, would make a difference in an individual' s problem solving techniques.

From this, two research questions have been formulated:

First, what are the similarities and differences in the use of heuristics when a student solves two problems: one appearing to be based heavily science-knowledge-based, the other obviously not science-knowledge-based?

Second, what prompts were used to help the student, and how effective were the different types of prompts?

Ⅱ. Research Method

1. Method and Procedure

A male sixth grade student (12 years, 2 months in age) participated in this study. The student was a "B" science student at a foreign school in Seoul and consistently performed above average in all classes. His teacher selected the student for the purpose of this study. His teacher asked him to participate in this study, and both he and his mother agreed to do this. All of the information for the study was collected in three interviews, and each interview lasted approximately one hour and fifteen minutes. The interviews began with observing the student's solving problems. The student was asked how and why he solved problem that way. There were some interactions between the researcher and the student during the interview procedures.

It was explained to the student that he would be given several logic problems, and that the problems would all be examples of something that exhibited a pattern. After figuring out the rule, he would be asked to write all the rule(s) that he thought were necessary to define the pattern. He would then be asked to fill in the missing parts of the pattern. The student was allowed to make any notes, do any scratch work, and use a calculator if desired.

The student was given two letter series and two number series problems, one at a time. Each time the student was asked to determine the pattern, figure out the rules that guide the pattern, write down those rules, and then fill in the missing parts of the pattern. The series problems are shown below(Figure 1).

AABCCDEEFG_ _ _ _
_ _ AZBCBYCDCXDE_ _ _ _
2,3,5,6,8,9,_,_,_,_
0,1,1,2,3,5,8,13,_,_

Fig. 1. A problem as practice for writing rules

Although these problems were initially intended to give some idea of basic problem solving heuristic that the student would use in subsequent problems, they ended up serving well as practice for writing rules.

Following these single series problems, the student was given another number problem that used a system to govern several short series(see figure 2). Again, the student was asked to figure out and write down the rules that govern the pattern. Following this, the student was asked to fill in the missing parts of the pattern.

Number problem
a. 127, 10, 1
b. 96, 15, 6
c. 2056, 13, 4
d. 194, 14, __
e. 227, __, __

Student rule: Add all the first digits in the first number together to get 2nd # subtract nine from the 2nd # to get the 3rd #.

Fig. 2. A number problem that used a system to govern several short series

The final phase of data collection involved another system problem. This problem involved the system for naming some of the binary chemical compounds. The student was asked some questions to determine his chemistry knowledge base, and then given some pertinent information. The student was told he could take notes and then use those notes and the periodic table any time during the session. The pertinent information that the student needed to understand prior to solving the problem was, how to read the periodic table, the connection between the symbol and the name of the chemical, the difference between where metals and nonmetals are located on the periodic table, and Greek prefixes(mono, di, tri, tetra, and penta). When the interviewer was convinced that the student had a complete understanding of the pertinent information, the student was given a list of chemical formulas for selected binary compounds with their respective names(see figure 3).

Naming problem

CO_2	carbon dioxide
CaO	calcicum oxide
CF_4	carbon tetrafluoride
K_2S	potassium sulfide
Sc_2O_3	scandium oxide
$ScCl_3$	scandium chloride
N_2O_5	dinitrogen pentoxide
NaBr	sodium bromide
P_2S_3	diphosphorous trisulfide
NCl_3	nitrogen trichloride
LiCl	lithium chloride
BaO	barium oxide
KI	potassium iodide
CO	carbon monoxide
Li_2O	lithium oxide

Student rules:

(name) used in compound, used in name
nonmetals are the only ones to use prefixes, but they only use them with other nonmetals.
the little number tells you what prefix to use
you ide the last element of the compound.

Fig. 3. A list of chemical formulas for selected binary compounds

Two types of binary compounds were mixed together on the list, some formulas represented compounds where a nonmetal bonded to a metal with one oxidation number, some where a nonmetal bonded to another nonmetal. The student was asked to determine and write the rules that govern the system that was used to name the compounds on the list. When the student was convinced that he had found all the pertinent rules, he was given a list of formulas for binary compounds, and asked to name them using his rules. He was not prevented from revising his rules as he worked on naming the compounds(see figure 4).

Checking For Rule Use	Student answers
CS_2	carbon disulfide
N_2O_3	dinitrogen trioxide
Na_2O	sodium oxide
CaF_2	calcium difluoride
Sc_2S_3	scandium disulfide
SO	sulfur oxide

Fig. 4. A list of name the compounds

2. Data Analysis

The number system problem and the naming problem were selected for comparison because they were most similar. Both problems give several examples that are governed by a system.

The transcript of both problems was analyzed for use of heuristics by the student and prompts by the interviewer. For each problem, the transcript was broken down into sections that represented a single heuristics or prompts. The summary descriptions of those single heuristics and prompts were then mapped out in a flow chart-like arrangement. Following this, the heuristic and prompts mapped out on the flow chart were categorized by the general method each represented or was most closely related to. These general methods were transferred, with some details attached, to a written format in short chains of logic. Each time the descriptions were transferred from one form to another, the transcript was consulted. Checking the transcript resulted in confirmation or slight alteration of the recorded interpretation.

The logic chains were color-coded according to a general method to make the heuristic patterns and the prompts easier to analyze. Heuristics that were used in both problem solutions were noted and the phrases were highlighted. Heuristics that only appeared in one problem solution were noted and the step number was highlighted instead of the phrase. The prompts used by the interviewer were highlighted on a separate copy. Heuristics were analyzed by the number of times used, the situational use, symmetry of use in both problem solutions, and large patterns of use. Prompts were analyzed by number of uses and effectiveness of use. Finally, the overall difficulty of the problem was analyzed by the combination of heuristic and prompts, and student successes and failures.

Ⅲ. Results and Discussions

1. Context

The student did not seem to have much trouble classifying the compounds. In fact, he was not informed anytime during the solution process that there were different types of compounds on the list. He was able to figure out by writing rules about the name classifications, that some compounds contained metals and nonmetals, while others contained only nonmetals. It also seemed apparent that he was able to understand the patterns, like "ide" was on the end of every name, even if he could not understand why it was there. The most difficult part seemed to be determining what constituted a rule. In the end, the only rule that the student did not recognize was that the mono prefix was only used on the second element in the name, while other prefixes were used on both elements. However, when it came to using the rule, he used that rule correctly, even though it was not stated.

The disappointment was that the student named three out of six formulas incorrectly on the completion problems. The first three compounds on the sheet were name correctly. The last three were named incorrectly. He was careful naming the first three, but did not check the rules he had written while working on the second three. This was the same thing I witnessed in class. Students would forget about the rules, and would seemingly use them at random. When the incorrect answers were pointed out to the student in the study, he quickly corrected them. However, the correction process could have simply used the thought process, that if the names were wrong with the prefixes, take them off, and if they were wrong without the prefixes, put them on.

2. Heuristic Analysis Results

There seemed to be a great deal of similarity between the two problem solutions in terms of the heuristics that were used. Eight general heuristics were used in both solutions: Check examples for support of an idea; check examples for exceptions to an idea; restate the problem; compare to known examples or patterns; make a hypothesis; check the relevance of other information present; use analogy; and recognize patterns/similarity.

These similar heuristics were used in a total of 54 separate incidences during the solution of both problems. The heuristics that were used in the number problem solution only seemed to reflect a different type of thinking involved with a math problem. Having basic computation strategies already available(add, subtract, multiply and divide) seemed to guide parts of the solution process. These known strategies were initially tried in a very simplistic way, assuming a one step process. The student tried subtracting the same number, nine, from the third number in each series. The student also tried multiplying by the same number. Another type of one step computation that was used was dividing one number in the series by another number in the series. The mathematics lent itself quite well to the practice of combining methods. In two instances, the student usedone computation, followed directly by the use of another. There was not any mechanism in the naming problem that seemed similar to the available computation strategies. As a result, the heuristics that seemed to be commonly used, due to the availability of an official change mechanism or rule, were absent from the naming solution.

The use of one of the heuristics, working on unfinished examples using previously deciphered rules, could be linked to the way the problem was presented. The number problem and the naming problem had one major difference in the way they were presented. Each problem had several examples that needed to be completed by the student, presumably after a rule was found. For the naming problem, these were not given to the student until after he decided that all the rules were deciphered. In the case of the number problem, the blank examples were given to the student with the completed example. However, on the naming problem, the tasks of completing example lead to the realization of the need for another rule. Thus, working on an unfinished example could have been considered as a heuristic used in both solutions.

The other heuristics that were used in the number problem solution only, guessing and hill climbing, were less dependent on an official mechanism to trigger their use, but were not used in the naming solution. The heuristics used only in the naming solution also seemed generalizable to other types of problems, but were simply not used in the number solutions. These heuristics included taking a mind break, posing another problem, modifying example to fit an observed pattern, self-verification without support, and search for cause in a cause and effect relationship. These were all very interesting for different reasons. Each of these heuristics was used only once, with the exception of the cause and effect heuristic. The cause and effect heuristic was a different form of heuristic. It will be discussed later.

The mind break heuristic was used about 50 minutes into the session. Even though the student had to be refocused by the interviewer, it broke the tension for the student. After a 10-15 second break, the student immediately began solving the next phase of the problem. It is maybe more accurate to describe what happened as a mind clearing break.

Problem posing occurred at the time the student began the second part of the problem, when he was asked to name compounds listed by formula only. The student realized that he did not know what suffix to use on the name. He could not decide whether to use disulfide, disulfate or disulfur in conjunction with carbon to name the compound, CS_2.

The student became very entangled with the problem of determining the correct suffix. This is when the student used the cause and effect heuristic. The student realized that a pattern existed where all the example names ended in "ide." He did not accept that this was always the case. He began to try to determine the reason why the elements' names appeared in the order they did in the compound names. He assumed there was a reason for this, and went in search of the cause for this effect. The cause and effect heuristics actually describe several steps together. The simple heuristics that were used during the use of the cause and effect heuristic were counted with the other simple heuristics. In the cause and effect heuristic, the student used atomic number order and alphabetical order as organizational models to try to account for the cause and effect. He went so far as to modify the alphabetical order, by deciding that it was all right for the symbol for scandium, Sc, to come before the symbol for sulfur, S. What caused the confusion was prior knowledge. The student had learned in school about compounds that were composed of more than two elements. He remembered that these compounds used other endings. Due to this, he was not convinced that all compounds would use the "ide" ending, even though it was consistent through all the examples in the problem. Once he was informed that compounds with three or more elements did not apply to this problem, he quickly decided that all the second elements' names for this problem, regardless of why they are second, end in "ide." This ended any further cause and effect investigation.

The similarity of the two solutions started with the student checking for support of a hypothesis or success of a heuristic after its use. The student checked for support after most steps in both solutions. The student did not check for exceptions nearly as often. There is a separation between these two heuristic because they are not the same thing. The student may check to see if there is an example that corresponds to what he predicts, but may not check to see if there are any examples that do not corresponds to his prediction. The difference in this case seems to be less linked to student consistency and more linked to the checking procedure that was used. The student checked for support for an idea first. If an example was found he usually went on to look for exceptions. If he did not find support, the idea was not applicable, and thus there were no reasons to look for exceptions. In other words, lack of supporting examples was a verification of an exception. The one borderline exception to this pattern was during an exchange regarding the formulation of a rule to explain the use of prefixes. The student hypothesized that the prefixes are used by nonmetals. He checked for support by looking for a nonmetal that used a prefix. When one was found, he checked to make sure that none of the metals used a prefix. It could be argued that the student thought he was checking for an exception, however, he should have also looked for a nonmetal that didn't use a prefix. At one point he came across an exception, acknowledged it and then ignored it. He did not seem to recognize that it was an exception to what he was thinking. The following passage illustrates the student's confusion.

> ...you see on all these, like here, P stands for phosphorus [nonmetal with no prefix], never mind and then lithium is where? And it is a gas [nonmetal], not it's solid [metal], only the gases [nonmetals] use the da, tri, mono, tetra and penta.

Phosphorus was an exception to nonmetals always using prefixes. The student only recognized that when a prefix was used, it was used on a nonmetal (support of idea). To find an exception, he went to see if any metals used prefixes. He finally makes this statement:

> Should I put that metals don't use the prefixes or is that clear enough [that only nonmetals do]? Nonmetals only have [prefixes]. (Interviewer asked, ..Are both these always true?) Yes.

Even though the student does not recognize that there are times that nonmetals don't use prefixes, it seems that he assumes at this point that he has checked for exceptions, and has not found any. This was the only time the student showed difficulty in deciding what to look for in seeking an exception to an idea.

The other general heuristic besides the search for supporting or exceptional evidence, which was used quite widely in both solutions, was the comparison to known examples or patterns. In the number problem the student used this heuristic four times. Once, he compared to the rule he had already determined for finding the third number in the series, by using the second number in the series (subtract 9). The other three times were very close together. The student started naming number patterns he was familiar with until he found one that seemed to be similar. In the following passage, he is trying to figure out how to relate the first number in the series to either or both of the other two numbers by comparing to a pattern.

> I'm thinking about like...prime, and composite numbers, and whatchamacallit...you can figure out if three will go into it by adding the...all the numbers together...Oh, I think I got it.

In the naming problem, the student used some interesting models for comparison. Twice he used the way the word sounded as a way of determining if it was correct.

> [referring to which word, first or second, has a prefix]..if it's on the first you don't, but if it's on the second you will, 'cause it won't sound very good [any other way].
>
> ...
>
> [trying to explain why a suffix is used on the second word in the compound name].. Carbon disulfide, disulfate, disulfur...how will that be worded?... I don't know how it's pronounced. How would you pronounce it? I know it's carbon di...sulfide, but I don't know how. Would you put sulfide or sulfur, but I don't know how. Would you put sulfide or sulfur, disulfur or disulfide? Disulfide. ...

The student went on to use another language model for comparison. He explained adding the "ide" suffix as being similar to making the word plural.

> [explaining the existence of the suffix] ...because that is how you would put it plural, not plural but...not like plurally, but in a way it's kind of plural.

The other two models the student made use of are organizational models. When the student was trying to explain the cause and effect of writing a given element second in the name, he tried to make sense of the order by using two ordering systems that he was already familiar with. He first hypothesized that the order was determined by order of atomic number. When this did not hold up, he tried two different schemes, one with the element names and one with the element symbols that used alphabetical order.

Another striking similarity in the two solutions was the student's reaction when he seemed to be on the verge of being stumped. In both solutions, he began to ask about the relevance of other information that was present. In the number problem, he asked about the letters labeling the separate series (see figure 1). In the naming problem, he began asking about the numbers on the periodic table representing the electron configuration for each element.

3. Prompt Analysis Results

Five types of interviewer prompts were detected in the two problem solutions, directional cues, modeling, clarity, problem posing, metacognition and validation. In the number problem, only two prompts were used, both directional cues. An example of a directional cue would be telling the student, "*maybe you should try something besides multiplication*". In both cases, the cues caused the student to change his direction in his solution of the problem. However, in one of the cases, the student came back to the same point later in the problem solution.

In the naming problem solution, all five types of prompts were used at least once. Clarity prompts were used when the student did not understand instructions or information. These were not all cases where the student responded to the cue in the way that was expected by the interviewer. The one metacognitive prompt was also successful. The student was asked why he was using the "ide" surffix, when he had not stated any reason to be using it. In explaining, he realized that he wasn't quite sure why he was using it. Presumably he was using it because it was used on all the examples, but he was not aware of why he was using it. Perhaps he did realize why he was doing it, but did not think it was a good enough reason to state. The one problem posing prompt that the interviewer used was really related to a problem the student had previously solved, but did not write as part of a rule at the time. The question was, "*What determines which prefix you use?*" The student quickly answered, "*the number of atoms,*" as if it should have been assumed.

The modeling questions were only moderately successful. It generally took more than one attempt for a modeling prompt to be fully acknowledged. In one case, the student was having trouble recognizing that something he was stating was a rule because it was so elementary. It took two modeling prompts before he agreed that it would be a rule. He understood the concept of the problem. He didn't recognize that what he was saying was a rule. The only other successful modeling prompt was used in conjunction with the student checking for, what he thought were, exceptions to his idea that nonmetals always use prefixes and metals never use prefixes. It took three prompts before he realized what the interviewer was pointing out about his idea. The interviewer was trying to point out that his idea about nonmetals was not always true, because they do not always use a prefix. In each case of success, prompts were ignored initially. For two prompts to be successful, five had to be given. In one case, a prompt was given, ignored, and not repeated.

Another type of prompt that was used was a validation prompt. This type of prompt is used when a student asks for an idea to be validated. The interesting thing about the only validation prompt that was given was that it was ignored. The student asked if he should just write the name like he thought it should be written, even though he did not know why. The interviewer validated his request by telling him that if "*that is what you think the name is, then write it down.*" The interviewer goes on to encourage him to write it down by saying, "*That's what I want to know, is what you think the name is*". The student continued to solve the problem without ever writing anything.

Ⅳ. Conclusions

The purpose of this study is to look at the use of heuristics when a sixth grader solves a problem: The similarities and differences, and the different types of prompts. There seemed to be more similarities than differences in the type of general heuristic that were used in the two problem solutions. The student was systematic and consistent in his use of the general use of heuristics. He was consistent in the way he checked for supporting evidence of the expected result, and then when present, he continued by checking for exceptions. Although the specific applications were different, he consistently tried to compare to other models, patterns or examples to the problems. An interesting similarity was his reaction to go to other information, even if it seemed irrelevant, when he was almost stumped.

It seems likely that the differences between the problems were due in part to the difference in the context of the problem --math operations affected the problem solving procedure--, the difficulty level of the problem --one system being governed by fewer rules--, and the difference in the presentation. The similarities of heuristics used in both solutions indicate that some of the problem solving strategies were similar. Thus, the context did not seem to completely alter the student's problem solving strategies. The similarity of heuristics also may indicate that the student has a bank of heuristics that he commonly uses on all problems.

The prompts seemed to vary in success. The most valuable for shaping the path of the student problem solving, were the directional cues. These prompts helped the students change their train of thought. If the student was spending a long time doing virtually the same step over and over, or was out of ideas, the directional cue sparked the problem solving process. The modeling prompts seemed too subtle to have much effect. More than one modeling prompt was needed in order to make any type of impact on the student. The modeling prompts seem like a very natural way to make a subtle suggestion, but when a student is deep in thought, subtlety can easily be ignored.

In general, even with the 50% correct rate on the problems, it seemed like the process of classifying the compounds and finding the rules was valuable. The student was forced to think about what information was contained in the name. The main concern I had with the naming problem was the level of difficulty in classifying the compounds and identifying the rules. These did not seem to be insurmountable tasks. The student did have some trouble writing a rule once a pattern was identified, but pattern recognition was my foremost concern. Keeping in mind that the student will continue to develop cognitively over three years time (sixth grade to ninth grade), I felt that the process seemed within the realm of possibility for trial in a ninth grade classroom. This is not to say that it will work better than the deductive method, however, the experiment seems worth pursuing.

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