# Properties and Performance of Space-Time Bit-Interleaved Coded Modulation Systems in Fast Rayleigh Fading Channels

Daeyoung Park, Myung-Kwang Byun, and Byeong Gi Lee

*Abstract:* **In this paper, we investigate the properties and performance of *space-time bit-interleaved coded modulation* (ST-BICM) systems in fast Rayleigh fading channels. We first show that ST-BICM with QPSK signaling in fast fading channels possesses the uniform distance property, which makes performance analysis tractable. We also derive the probability distribution of the squared Euclidean distance between space-time symbols assuming uniform bit-interleaving. Based on the distribution, we show that the diversity order for each codeword pair becomes maximized as the frame length becomes sufficiently long. This maximum diversity order property implies that the bit-interleaver transforms an ST-BICM system over transmit diversity channels into an equivalent coded BPSK system over independent fading channels. We analyze the performance of ST-BICM in fast fading channels by deriving an FER upper bound. The derived bound turns out very accurate, requiring only the distance spectrum of the binary channel codes of ST-BICM. Numerical results demonstrate that the bound is tight enough to render an accurate estimate of performance of ST-BICM systems.**

*Index Terms:* **Space-time codes, bit-interleaved coded modulation, fast fading, distance spectrum.**

## I. INTRODUCTION

Space-time codes have been attracting much interest of researchers because they support high data rates with moderate complexity in wireless communication environments [1]. This technique enables to integrate channel coding, modulation, and multiple transmit antennas at the base station, with optional receive diversity incorporated at the mobile station. Independently of this, *bit-interleaved coded modulation* (BICM) was introduced as a means of improving the performance of coded modulation over fading channels [2]. It makes the code diversity equal to the smallest number of distinct bits (rather than channel symbols), and offers much better trade-offs between code diversity and trellis complexity than *trellis coded modulation* (TCM) does. The concept of BICM, when it is applied to multiple transmit antenna environment, yields *space-time bit-interleaved coded modulation* (ST-BICM) [3]–[5]. As BICM divides the code design process into encoder selection and modulation scheme selection processes, the design becomes simpler than the standard space-time code design.

Biglieri *et al.* [3] analyzed the information-theoretic limit of ST-BICM and showed that the ergodic and outage capacities are close to the general coded modulation case. Tonello [4], [5] presented an ST-BICM encoder structure and an iterative decoding method that follows the turbo decoding principle. The author also presented a code construction criterion in fast fading channels that the channel codes should be designed to have a large free Hamming distance, but did not present analytic bounds for *frame error rate* (FER).

Recently, there have been several attempts to analyze the performance of space-time codes in fast fading channels. Uysal *et al.* [6], [7] derived the exact *pairwise error probability* (PEP) of a residue integration form for fast fading channels and Simon [8] derived the exact PEP in numerical integration form for fast and slow fading channels. The evaluation of those FER estimates required to know all the Euclidean distances of each error event, which is a considerable burden of memory and computation. Recently in [9], we have analyzed the performance of space-time codes in fast Rayleigh fading channels by deriving a new FER upper bound and proposing a distance spectrum computation method. The derived bound required only the product distances of error events and turned out to be very accurate.

In this paper, we investigate the properties and effects of bit-interleaving to understand the outstanding performance of ST-BICM. We first show that ST-BICM with QPSK signaling in fast fading channels possesses the uniform distance property and that the error probability is independent of the transmitted codeword. We also derive the probability distribution of the Euclidean distance between two space-time symbols assuming uniform interleaving, based on which, we then show that the bit-interleaving makes the diversity order maximized with the probability close to 1. This maximum diversity order property holds irrespectively of the types of the binary channel codes, as it is inherited by the bit-interleavers. This property enables us to predict the performance of ST-BICM using that of the equivalent coded BPSK system over independent fading channels. We can analyze the performance of ST-BICM by deriving a tight FER upper bound.

This paper is organized as follows: To begin with, we describe the system model and the PEP in Section II. Then, we show that ST-BICM in fast fading possesses the uniform distance property, and also derive the probability distribution of the Euclidean distance assuming uniform interleaving, and based on this, we establish the maximum diversity order property of ST-BICM in Section III. Finally, in Section IV, we derive a new upper bound

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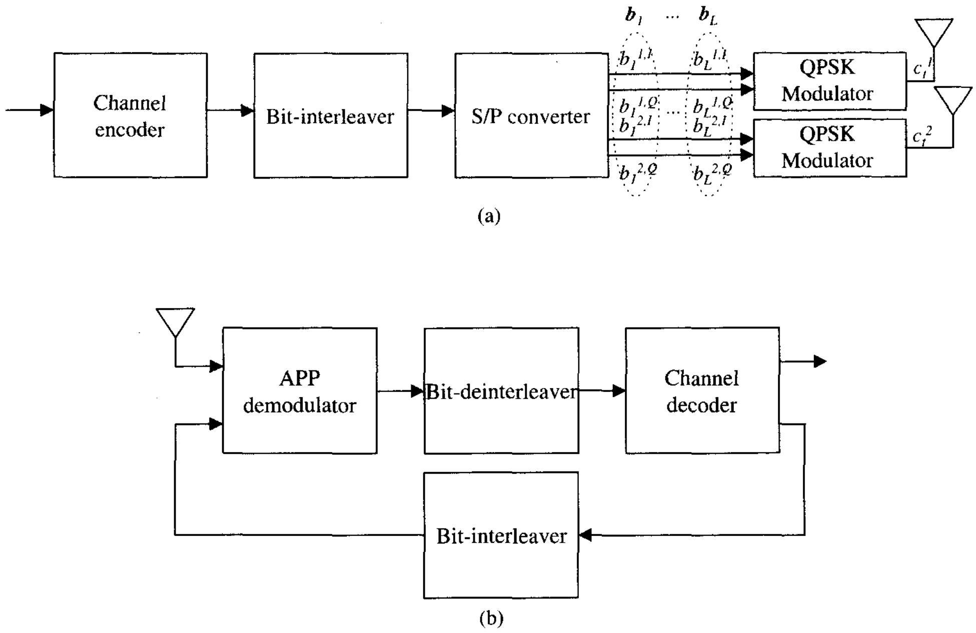



Fig. 1. The block diagram of ST-BICM: (a) transmitter, (b) receiver.

of the frame error probability and demonstrate by simulations that the bound is tight enough to estimate the performance of ST-BICM accurately.

## II. PRELIMINARIES

To begin with, we describe the system model of ST-BICM systems and introduce some key definitions and equations related to PEP bounds in support of the analytical discussions to follow in the subsequent sections.

### A. System Model

We consider a baseband communication system with $n_T$ transmit antennas and $n_R$ receive antennas. Fig. 1(a) shows the block diagram of the transmitter of ST-BICM for the case of $n_T = 2$. The transmitted data are encoded by a binary linear channel code, such as a convolutional code, a turbo code, etc. The encoded sequence is bit-interleaved and is applied to a *serial-to-parallel* (S/P) converter that produces $2 \times n_T$ parallel data sequences. The data sequences are then mapped into QPSK symbols based on the Gray mapping rule. We assume that the frame length is $L$ and the elements of the signal constellation are contracted such that the average energy of the constellation becomes 1.

We define a *space-time codeword matrix* of size $n_T \times L$, obtained by arranging the transmitted sequence in an array, as

$$\boldsymbol{c} \equiv \begin{bmatrix} c_1^1 & c_2^1 & \cdots & c_L^1 \\ c_1^2 & c_2^2 & \cdots & c_L^2 \\ \vdots & \vdots & \ddots & \vdots \\ c_1^{n_T} & c_2^{n_T} & \cdots & c_L^{n_T} \end{bmatrix}, \tag{1}$$

for which the $i$-th row $\boldsymbol{c}^i \equiv [c_1^i \; c_2^i \; \cdots \; c_L^i]$ is the data sequence transmitted from the $i$-th transmit antenna, and the $t$-th column $\boldsymbol{c}_t \equiv [c_t^1 \; c_t^2 \; \cdots \; c_t^{n_T}]^T$ is the space-time symbol at time $t$, where $[\cdot]^T$ denotes the matrix transpose operation.

At time $t$, the received signal at receive antenna $j$, $j = 1, 2, \cdots, n_R$, is given by

$$r_t^j = \sqrt{E_s} \sum_{i=1}^{n_T} \alpha_t^{i,j} c_t^i + \eta_t^j, \tag{2}$$

where $E_s$ denotes the energy per symbol and $\eta_t^j$ the noise component of the receive antenna $j$ at time $t$, which is an independent sample of the zero-mean complex Gaussian random variable with independent real and imaginary parts, each with variance $\frac{N_o}{2}$. Coefficient $\alpha_t^{i,j}$ is the fading attenuation for the path from transmit antenna $i$ to receive antenna $j$ at time $t$.

In this paper, we assume that the signals received at different antennas experience independent fading, which means that the fading coefficients $\alpha_t^{i,j}$ are independent zero-mean complex Gaussian random variables with independent real and imaginary parts, each with variance $1/2$. We consider that the path coefficients can be modeled as fast Rayleigh fading. For fast fading, it is assumed that the fading coefficients vary symbol to symbol.

For decoding, we adopt the iterative demodulation-decoding method in [10] (see Fig. 1(b)). The *a posteriori probability* (APP) demodulator generates the *log likelihood ratio* (LLR) of channel-encoded bits using noise statistics. The LLR's are deinterleaved and transferred to the channel decoder, such as BCJR decoder [11] or SOVA decoder [12]. The decoder outputs are re-interleaved and fed back to the APP demodulator. LLR's are iteratively interchanged between the demodulator and the decoder to successively improve the error performance.

### B. Pairwise Error Probability

The PEP is the probability that the decoder selects the sequence $\hat{\boldsymbol{c}}$ as an estimate of the transmitted sequence $\boldsymbol{c}$. If an ideal *channel state information* (CSI) is available at the receiver, the PEP takes the expression [13]

$$P_e(\boldsymbol{c},\hat{\boldsymbol{c}}) = E\left[Q\left(\sqrt{\frac{\gamma_s}{2}d^2(\boldsymbol{c},\hat{\boldsymbol{c}})}\right)\right], \tag{3}$$

for the tail probability of the Gaussian probability density function $Q(y) \equiv \frac{1}{\sqrt{2\pi}}\int_y^\infty e^{-\frac{1}{2}x^2}dx$, the SNR per symbol $\gamma_s \equiv \frac{E_s}{N_0}$, and the squared modified Euclidean distance $d^2(\boldsymbol{c},\hat{\boldsymbol{c}})$.

In fast fading channels, the distance $d^2(\boldsymbol{c},\hat{\boldsymbol{c}})$ can be written as [1]

$$d^2(\boldsymbol{c},\hat{\boldsymbol{c}}) = \sum_{t=1}^{L}\sum_{j=1}^{n_R} D_t\left|\beta_t^j\right|^2, \tag{4}$$

where

$$D_t \equiv \|\boldsymbol{c}_t - \hat{\boldsymbol{c}}_t\|^2 = \sum_{i=1}^{n_T}\left|c_t^i - \hat{c}_t^i\right|^2, \tag{5}$$

denotes the squared Euclidean distance between the two space-time symbols $\boldsymbol{c}_t$ and $\hat{\boldsymbol{c}}_t$. Also, $\beta_t^j$'s in (4) are independent zero-mean complex Gaussian random variables with independent real and imaginary parts, each with variance $1/2$. Let $\delta_H$ denote the *symbol-wise Hamming distance*, i.e., the number of time instances $t = 1, 2, \cdots, L$, such that $D_t \neq 0$. Then, the right-hand side of (4) has $\delta_H n_R$ independent random variables, so the diversity order of $\delta_H n_R$ is achieved.

The PEP in (3) has been evaluated in [8] and [14] and expressed as

$$P_p(D) = \frac{1}{\pi}\int_0^{\pi/2}\prod_{t=1}^{L}\left(1+\frac{D_t\gamma_s}{4\sin^2\theta}\right)^{-n_R}d\theta, \tag{6}$$

where $D \equiv \{D_1, \cdots, D_L\}$ denotes the set of the squared Euclidean distances corresponding to a pairwise error event. If $D$ is given, we can obtain the exact value of the PEP through numerical integration of (6). However, to evaluate the FER, we should enumerate a number of sets of all distances of simple error events, which requires a large amount of memory and computation. This problem can be resolved by employing a new upper bound of the PEP that relies only on the *product distance*

$$\delta_p \equiv \prod_{\substack{t=1 \\ D_t\neq 0}}^{L} D_t,$$

i.e., the product of all nonzero squared Euclidean distances.[1] Recently, Byun *et al.* presented a new tight PEP upper bound of the form [14]

$$P_B(\delta_H,\delta_p) = J_{(\delta_H n_R)}\left(\frac{\delta_p^{1/\delta_H}\gamma_s}{4}\right), \tag{7}$$

where

$$J_m(c) \equiv [P(c)]^m \sum_{k=0}^{m-1}\binom{m-1+k}{k}[1-P(c)]^k, \tag{8}$$

for the positive integer $m$ and $P(x) \equiv \frac{1}{2}\left(1-\sqrt{\frac{x}{1+x}}\right)$, $x \geq 0$.[2] The bound in (7) depends only on $\delta_p$, and it is uniformly tighter than the Fitz's bound in [15] and the tightest upper bound for each given $\delta_p$ [14].

## III. PROPERTIES OF ST-BICM

Based on the system model above, we now investigate the properties of ST-BICM systems: We first show that ST-BICM in fast fading channels has the uniform distance property. We then derive the probability distribution of $\delta_H$ and $\delta_p$, and based on this, we present that ST-BICM in fast fading channels has the maximum diversity order property.

### A. Uniform Distance Property

As shown in Fig. 1(a), the encoded sequence is bit-interleaved and applied to an S/P converter that produces $2 \times n_T$ parallel bit sequences $b_t^{i,I}$ and $b_t^{i,Q}$, $i = 1, 2, \cdots, n_T$. So the S/P-converted bit sequences may be rearranged in matrix form by

$$\boldsymbol{\beta} = \left[\begin{array}{cccccc} \boldsymbol{b}_1 & \boldsymbol{b}_2 & \cdots & \boldsymbol{b}_t & \cdots & \boldsymbol{b}_L \end{array}\right] \in \mathbf{Z}_2^{2n_T\times L}, \tag{9}$$

for

$$\boldsymbol{b}_t = \left[\begin{array}{ccccccc} b_t^{1,I} & b_t^{1,Q} & b_t^{2,I} & b_t^{2,Q} & \cdots & b_t^{n_T,I} & b_t^{n_T,Q}\end{array}\right]^T \in \mathbf{Z}_2^{2n_T},\ t = 1, 2, \cdots, L, \tag{10}$$

where $\mathbf{Z}_2$ denotes a binary group comprised of 0 or 1. The modulator at the last stage maps $\boldsymbol{b}_t$ into a space-time symbol $\boldsymbol{c}_t$ based on the Gray mapping rule, i.e.,

$$c_t^i = \frac{1}{\sqrt{2}}\left\{(-1)^{b_t^{i,I}} + j(-1)^{b_t^{i,Q}}\right\}. \tag{11}$$

The squared Euclidean distance between two space-time symbols is

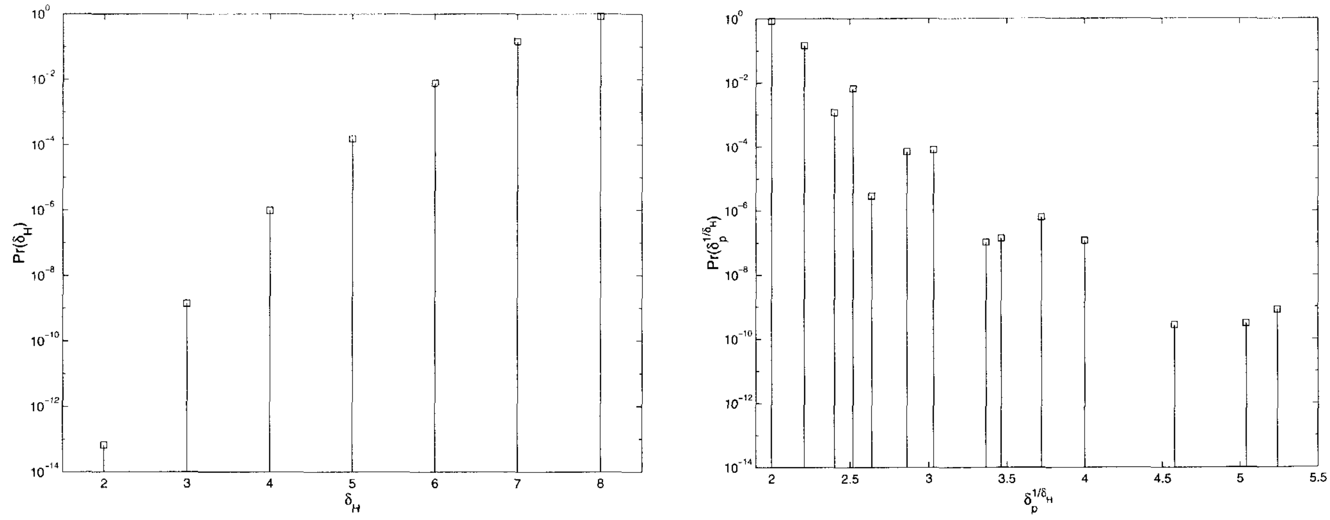
$$\begin{aligned} D_t &= \sum_{i=1}^{n_T}|c_t^i - \hat{c}_t^i|^2 \\ &= \frac{1}{2}\sum_{i=1}^{n_T}\left|1-(-1)^{b_t^{i,I}\oplus\hat{b}_t^{i,I}}\right|^2 \\ &\quad +\frac{1}{2}\sum_{i=1}^{n_T}\left|1-(-1)^{b_t^{i,Q}\oplus\hat{b}_t^{i,Q}}\right|^2 \\ &= 2d_H(\boldsymbol{b}_t,\hat{\boldsymbol{b}}_t), \end{aligned} \tag{12}$$

where $\oplus$ denotes the exclusive OR operation and $d_H(\boldsymbol{x},\boldsymbol{y})$ the Hamming distance between $\boldsymbol{x}$ and $\boldsymbol{y}$.

Equation (12) indicates that the Euclidean distance between two space-time symbols is proportional to the Hamming distance between the bit sequences corresponding to that space-time symbols in the case of the QPSK Gray mapping. So, the set of the squared Euclidean distances corresponding to a pairwise error event, $D$, depends not on the transmitted codeword but on the Hamming distance. Since we employ a linear binary

[1] As can be seen in (6), the PEP depends only on $\delta_p$, not on individual $D_t$'s, at high SNR.

[2] As can be seen in (7), $\delta_H n_R$ and $\delta_p^{1/\delta_H}$ correspond to the diversity advantage and the coding advantage, respectively [1].




(a) (b)

Fig. 2. Probability of Hamming distance $\delta_H$ and product distance $\delta_p$ ($n_T = 2$, $L$=130, $d$=8): (a) $\Pr(\delta_H)$, (b) $\Pr(\delta_p^{1/\delta_H})$.

code, the sets of Hamming distances from any codeword to all others are all identical. Therefore the uniform distance property holds for ST-BICM systems with QPSK signaling. As the uniform distance property implies that the error probability does not depend on the transmitted codewords, we may assume, in the performance analysis, that the space-time codeword matrix corresponding to an all-zero codeword of the channel code is transmitted, without loss of generality.

### *B. Probability Distribution of Euclidean Distance Between Space-Time Symbols*

The set of squared Euclidean distances between two space-time symbols, $D = \{D_t | D_t = \|c_t - \hat{c}_t\|^2,\ t = 1, 2, \cdots, L\}$, determines the PEP in (6). Note that the exact position of the nonzero bits in a space-time symbol is irrelevant to the PEP but the number of nonzero bits (i.e., Hamming distance) in the space-time symbol is important. So, it is a reasonable approach to classify the set according to the Hamming distance between $\boldsymbol{b}_t$ and $\hat{\boldsymbol{b}}_t$. Since we may assume that the space-time codeword matrix corresponding to the all-zero codeword is transmitted, we set $\boldsymbol{b}_t$=$\mathbf{0}$. Then we need to consider only the Hamming weight of $\hat{\boldsymbol{b}}_t$, $t = 1, 2, \cdots, L$.

Let $n_i$ denote the number of space-time symbols whose Hamming weight is $i$ in the space-time codeword matrix. For example, there are $n_1$ symbols with Hamming weight 1, and $n_2$ symbols with Hamming weight 2, and so on. For 2-transmit antenna case, the set $D$ can be completely described by $n_i$, $i = 1, 2, 3, 4$.

We define by $\mathbf{n} = (n_1, n_2, \cdots, n_{2n_T})$ the *type* of the set $D$. Then, for a given type $\mathbf{n}$, the number of nonzero $D_t$'s, $\delta_H$, is expressed by

$$\delta_H(\mathbf{n}) = \sum_{i=1}^{2n_T} n_i. \qquad (13)$$

Since the number of $D_t$'s taking the value $2i$ is $n_i$, the product of nonzero $D_t$'s, $\delta_p$, is expressed by

$$\delta_p(\mathbf{n}) = \prod_{i=1}^{2n_T} (2i)^{n_i} = 2^{\delta_H(\mathbf{n})} \prod_{i=1}^{2n_T} i^{n_i}. \qquad (14)$$

We consider a codeword with Hamming weight $d$. The interleaver distributes the nonzero $d$ bits into $\boldsymbol{\beta}$ in (9) and the Hamming distances of the columns of $\boldsymbol{\beta}$ determine the type $\mathbf{n}$. The following theorem describes the probability to get type $\mathbf{n}$ for a given $d$.

**Theorem 1:** For a given Hamming weight $d$, the probability that the set of squared Euclidean distances between two space-time symbols is of type $\mathbf{n}$ under the assumption of uniform interleaving is given by

$$\Pr(\mathbf{n}|d) = \begin{cases} \frac{1}{\binom{2n_T L}{d}} \prod_{i=1}^{2n_T} \binom{L - \sum_{k=1}^{i-1} n_k}{n_i} \binom{2n_T}{i}^{n_i}, & \\ & \text{if } \sum_{i=1}^{2n_T} i \cdot n_i = d, \\ 0, & \text{otherwise.} \end{cases} \qquad (15)$$

*Proof:* $\Pr(\mathbf{n}|d)$ takes a nonzero value only if $\sum_{i=1}^{2n_T} i \cdot n_i = d$, because there are $\sum_{i=1}^{2n_T} i \cdot n_i$ nonzero bits in $\boldsymbol{\beta}$ of type $\mathbf{n}$. The number of possible locations of $n_1$ columns in $\boldsymbol{\beta}$ is $\binom{L}{n_1}$ and the number of different ordering of a nonzero bit in each column is $\binom{2n_T}{1}$. Likewise, the number of possible locations of $n_2$ columns in the submatrix of $\boldsymbol{\beta}$ that excludes $n_1$ columns is $\binom{L-n_1}{n_2}$ and the number of different ordering of two nonzero bits in each column is $\binom{2n_T}{2}$. Repeating this procedure, we obtain that the number of possible $\boldsymbol{\beta}$'s of type $\mathbf{n}$ is $\prod_{i=1}^{2n_T} \binom{L - \sum_{k=1}^{i-1} n_k}{n_i} \binom{2n_T}{i}^{n_i}$. Since we assume uniform interleaving, the number of all possible permutations is $\binom{2n_T L}{d}$. Therefore, by combining these, we obtain the probability in (15). □

For a given $d$, the probability of the type $\mathbf{n}$ is $\Pr(\mathbf{n}|d)$ and the Hamming distance and the product distance are $\delta_H(\mathbf{n})$ in

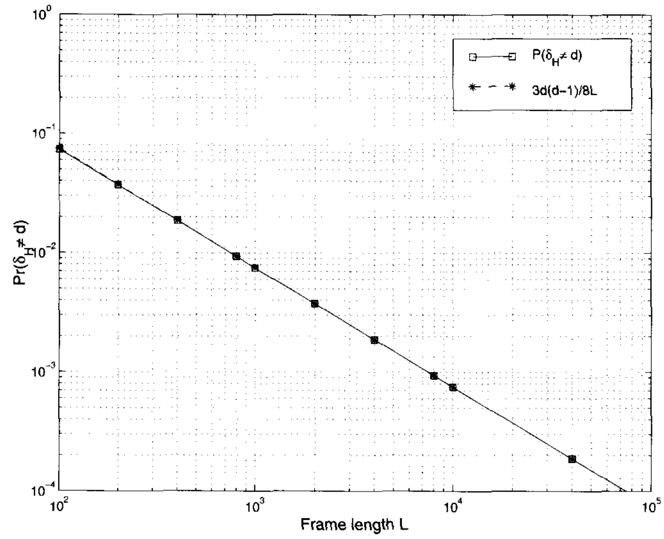



Fig. 3. Probability of $\delta_H \neq d$ ($n_T$=2, $d$=5).

(13) and $\delta_p(\mathbf{n})$ in (14), respectively. Fig. 2 plots the probability distribution of $\delta_H$ and $\delta_p^{1/\delta_H}$ for the case $d = 8$ when $n_T = 2$, $L = 130$. From the figure, we observe that $\delta_H$ takes on $d(= 8)$ and $\delta_p^{1/\delta_H}$ takes on 2 with a probability close to 1. In fact, $\delta_H$ takes on $d$ if and only if $\delta_p^{1/\delta_H}$ takes on 2 due to (13) and (14). The event that $\delta_H$ takes on $d$ occurs when $n_1 = d$, $n_2 = n_3 = n_4 = 0$, which implies that there is no column of $\boldsymbol{\beta}$ for which 2 or more nonzero bits are allocated. Considering that the diversity order is $\delta_H n_R$ and $d$ is the maximum value that $\delta_H$ can take, we can expect that in the case of ST-BICM we obtain the maximum diversity order in fast Rayleigh fading channels with a high probability. In the following, we will evaluate the probability that there exists at least one column where 2 or more nonzero bits are allocated.

### C. Maximum Diversity Order Property

In the above, we have shown that in the case of ST-BICM, $\delta_H$ takes on $d$ and $\delta_p^{1/\delta_H}$ takes on 2 with a high probability. Here, we take a closer look at the asymptotic behavior of the probability to determine the cases when it does not apply.

**Theorem 2:** For a given Hamming weight $d$ and a frame length $L$, the probability that $\delta_H$ does not take on $d$ is of order $O(L^{-1})$.[3]

*Proof:* By (15), the probability of the complementary event of $\delta_H = d$ becomes

$$\begin{aligned}\Pr(\delta_H \neq d) &= 1 - \Pr(n_1 = d) = 1 - \frac{\binom{L}{d}\binom{2n_T}{1}^d}{\binom{2n_T L}{d}} \\ &= 1 - \prod_{k=0}^{d-1}\left(1 - \frac{(2n_T-1)k}{2n_T L - k}\right). \qquad (16)\end{aligned}$$

[3]We write $f(x) = O(g(x))$ if there exist positive constants $M$ and $x_0$ such that $|f(x)| < Mg(x)$ for all $x > x_0$.

If we apply the following inequality[4]

$$1 - \prod_{i=1}^{N}(1 - x_i) \leq \sum_{i=1}^{N} x_i, \quad 0 \leq x_i \leq 1, \qquad (17)$$

we get

$$\begin{aligned}\Pr(\delta_H \neq d) &\leq \sum_{k=0}^{d-1}\frac{(2n_T-1)k}{2n_T L - k} \leq \sum_{k=0}^{d-1}\frac{(2n_T-1)k}{2n_T L - (d-1)} \\ &= \frac{(2n_T-1)d(d-1)}{4n_T L - 2(d-1)}, \qquad (18)\end{aligned}$$

which is of order $O(L^{-1})$. □

Theorem 2 describes the maximum diversity order property of ST-BICM in fast Rayleigh fading channels. The probability that $\delta_H$ does not take on $d$ is inversely proportional to the frame length $L$. In other words, as $L$ becomes large, the probability that ST-BICM has the maximum diversity order approaches 1.

Fig. 3 plots the probability of $\delta_H \neq d$ in comparison with its bound in (18) for the case $d$=5. We can observe that the upper bound is very close to the true probability.

If $\delta_H$ takes on $d$, then the pairwise error probability would be the same as the case when each element of $\boldsymbol{\beta}$ is mapped to a BPSK symbol $\in \{+1/\sqrt{2}, -1/\sqrt{2}\}$, and this BPSK symbol is transmitted over independent fading channels with single transmit antenna. If the frame length is sufficiently long, $\delta_H$ takes on $d$ with probability close to 1, which implies that the bit-interleaver transforms the $n_T$-antenna QPSK space-time code into the 1-antenna equivalent coded BPSK system where each coded bit is transmitted over independent fading channels. Based on this fact, we can predict the performance of ST-BICM from that of the equivalent coded BPSK system in independent fading channels.

## IV. FER PERFORMANCE OF ST-BICM

Now we analyze the FER performance of ST-BICM using the distance spectrum of the binary channel code in ST-BICM and confirm that the performance of ST-BICM is similar to that of the equivalent coded BPSK system in independent fading channels.

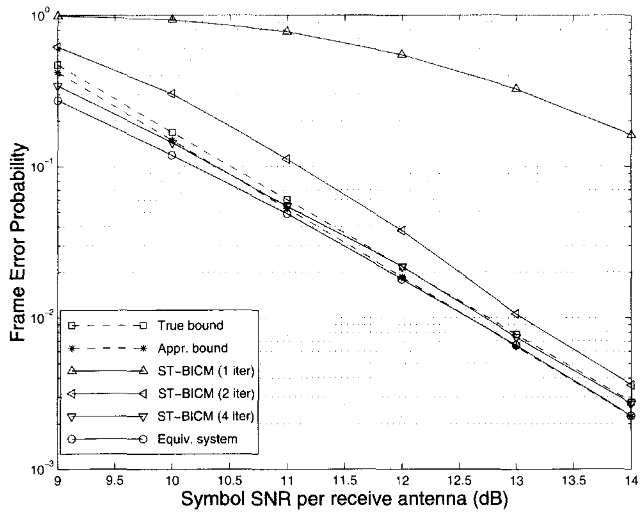
We apply the union bounding technique to obtain an upper bound of FER for *maximum likelihood* (ML) decoding of ST-BICM.[5] Then, the frame error probability of ST-BICM with QPSK signaling can be expressed by

$$P(e) \leq 2n_T r L \sum_d a_d \sum_{\mathbf{n}} P_B(\delta_H(\mathbf{n}), \delta_p(\mathbf{n})) \Pr(\mathbf{n}|d), \quad (19)$$

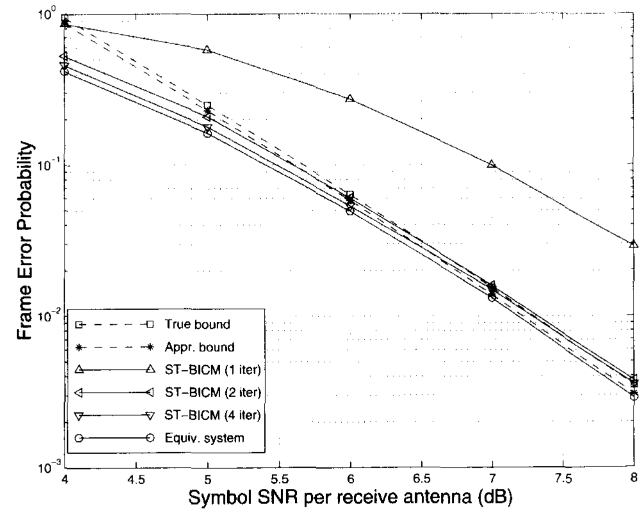
where $r$ denotes the code rate of the binary channel code, $a_d$ the number of codewords whose Hamming weight is $d$, and $P_B(\delta_H(\mathbf{n}), \delta_p(\mathbf{n}))$ the PEP upper bound in (7) for a given type

[4]See Appendix for its proof.

[5]It is not tractable to obtain the performance bound of the communication systems employing iterative decoding, so we derive the bound of ST-BICM based on ML decoding. Though the performance of iterative decoding is not guaranteed to converge to the ML performance, it has been empirically known to be close to the ML performance.




(a)

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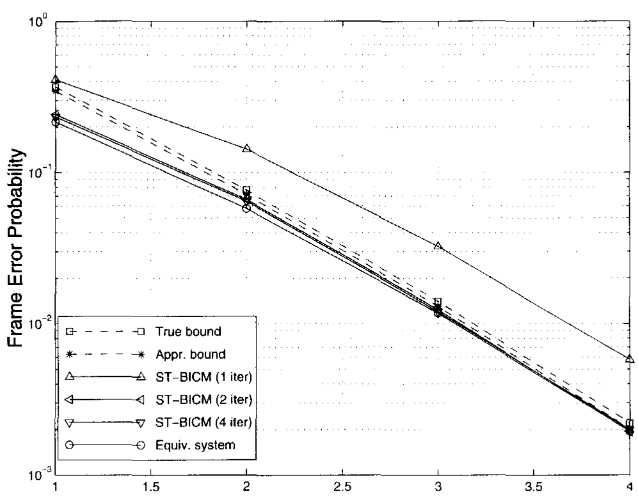


(c)

Fig. 4. Frame error probability of ST-BICM whose channel code is a 4-state convolutional code with (a) one, (b) two, and (c) four receive antennas ($n_T$=2).

n.[6] If we also use the fact that $\delta_H$ takes on $d$ and $\delta_p$ takes on $2^d$ with a probability close to 1, we get a simpler approximate bound

$$P(e) \approx 2n_T rL \sum_d a_d P_B(d, 2^d), \tag{20}$$

where

$$P_B(d, 2^d) \equiv J_{(dn_R)}\left(\frac{\gamma_s}{2}\right), \tag{21}$$

is the PEP of the 1-antenna equivalent coded BPSK system in which each coded bit is transmitted over independent fading channels with diversity order $dn_R$.

[6]For the case of time-invariant codes (e.g., convolutional codes), the sets of the simple error events that start at different times are identical if the edge effect is ignored. Since the number of input bits to the code is $2n_T rL$, there are $2n_T rL$ error events for a simple error event pattern. So, the FER bound based on union bound is $2n_T rL$ times the first error probability. In contrast, for the case of turbo codes, the sets of the simple error events that start at different times are not identical and the effect of $2n_T rL$ is incorporated in $a_d$.

Table 1. Distance spectrum of the convolutional code with generators (7,5) in octal.

| $d$ | $a_d$ | $d$ | $a_d$ |
|---|---|---|---|
| 5 | 1 | 10 | 32 |
| 6 | 2 | 11 | 64 |
| 7 | 4 | 12 | 128 |
| 8 | 8 | 13 | 256 |
| 9 | 16 | 14 | 512 |

From (19) and (20), we can observe that only the distance spectrum of the channel codes is required to evaluate the FER upper bound. Also note that the bound in (19) is the FER upper bound of ST-BICM in fast fading channels and the bound in (20) is the FER upper bound of the equivalent coded BPSK system where each coded bit is transmitted over independent fading channels.

We now examine some numerical FER performance of ST-BICM systems through simulations. We use the newly derived true bound in (19) and the approximate bound in (20) to evaluate the FER bounds and compare them with the simulation results. As we need only the distance spectrum of the channel code to calculate the above FER bounds, the FER bounds are applicable to any binary linear codes, such as convolutional codes, turbo codes, and others. In evaluating the FER bounds, we use the truncated distance spectrum, as the FER upper bounds may be satisfactorily approximated by taking into account the codewords whose Hamming distance is less than some predetermined value. We consider the smallest 10 Hamming distances. Table 1 lists the resulting number of simple error events of the convolutional code with the generators (7,5) in octal expression. For the simulations, we take 130 symbols per frame (i.e., $L$ = 130) and use random interleavers for bit interleavers. We employ iterative demodulation-decoding method with perfect *channel state information* (CSI) at the receiver [10]. We plot the resulting FER performance curves with respect to the symbol SNR per receive antenna, $n_T E_s/N_o$. We also evaluate through simulations the FER performance of the equivalent coded BPSK system in independent fading channels with the same generators. For this, we employ Viterbi decoding for an ML performance.

Fig. 4 plots the resulting performance of ST-BICM whose channel code is the 1/2-rate convolutional code with the generators (7,5) in octal expression. The three different graphs respectively cover ST-BICM systems with one, two, and four receive antennas. Overlaid in each graph is the performance of the equivalent coded BPSK system in independent fading channels ($n_T = 2$).

We observe from the three graphs that the performance of ST-BICM approaches that of the equivalent coded BPSK system in independent fading channels after four iterations.

We also observe from the simulation results that the FER bounds are tight enough to estimate the performance of ST-BICM with sufficient accuracy. Further, the true FER upper bound in (19) and the approximate bound in (20) nearly coincide. This testifies that the approximate bound can be used instead of the true bound, which would bring forth a significant computational reduction. Note that the approximate bound is for the equivalent coded BPSK system in independent fading and the true bound is for ST-BICM.

From the analyses and numerical results, we can confirm the following properties of ST-BICM: First, the bit-interleavers uniformly spread the $d$ bits from the channel code, so that the allocated bits in each antenna seldom coincide. Second, $\delta_H$ takes on $d$ with a high probability which approaches 1 for a large $L$ (i.e., the maximum diversity order property). Third, we can obtain the performance bound by substituting $d$ for $\delta_H$ and $2^d$ for $\delta_p$, respectively, and this bound is so tight that it can be used to predict the performance of ST-BICM directly. Fourth, ST-BICM in fast fading reduces to the equivalent coded BPSK system in independent fading channels for a large value $L$.

## V. CONCLUDING REMARKS

So far, we have studied the properties and performance of ST-BICM systems in fast Rayleigh fading channels. In particular, we have unveiled the effect of bit-interleaver by showing that it transforms an ST-BICM system in fast fading channels into an equivalent coded BPSK system in independent fading channels. We have shown that ST-BICM systems in fast fading channels have the uniform distance property and the maximum diversity order property. Based on these properties, we have analyzed the performance of ST-BICM.

The uniform distance property of ST-BICM in fast fading channels implies that the error probability is independent of the transmitted codewords. So, for the performance analysis we can assume that the transmitted codeword is the space-time codeword matrix corresponding to the all-zero codeword.

The maximum diversity order property means that for a given Hamming distance the diversity order is maximized with a high probability if the frame length becomes sufficiently long, which is attributed to uniform bit-interleavers. It also implies that the performance of ST-BICM in fast fading is equivalent to that of the coded BPSK system in independent fading channels. The bit-interleavers transform the $n_T$-antenna QPSK space-time coded system over fast fading channels into the 1-antenna equivalent coded BPSK system where each coded bit is transmitted over independent channels.

For the performance analysis of ST-BICM in fast fading channels, we have derived an FER bound taking advantage of the maximum diversity order property. Due to the maximum diversity order property, the symbol-wise Hamming distance takes the largest value with a high probability and the FER bound can be simplified to an approximate bound that does not require expectation operation with respect to the interleavers. This helps to reduce the computational complexity significantly. The derived bound is very accurate, requires only the distance spectrum of the binary channel code of ST-BICM, and can be computed without any numerical integrations.

## APPENDIX

*Proof of Inequality (17)*

We replace $1 - x_i$ with $y_i$ in (17) and prove that

$$\sum_{i=1}^{N} y_i \le N - 1 + \prod_{i=1}^{N} y_i. \tag{22}$$

We can prove it by mathematical induction.

It is trivial to show that (22) holds when $N = 2$. If we assume that (22) holds for some $K \ge 2$, we get

$$\sum_{i=1}^{K+1} y_i \le K - 1 + \prod_{i=1}^{K} y_i + y_{K+1}. \tag{23}$$

If we apply (22) with $N = 2$ to the last two terms in (23), we obtain

$$\sum_{i=1}^{K+1} y_i \le K + \prod_{i=1}^{K+1} y_i, \tag{24}$$

and this completes the proof.

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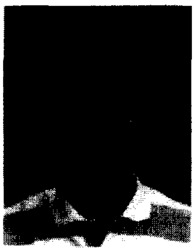

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