

## 카오스 무인 비행체에서의 장애물 회피 방법

# Obstacle Avoidance Method in the Chaotic Unmanned Aerial Vehicle

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### Abstract

In this paper, we propose a method to avoid obstacles that have unstable limit cycles in a chaos trajectory surface. We assume all obstacles in the chaos trajectory surface have a Van der Pol equation with an unstable limit cycle. When a chaos UAVs meet an obstacle in an Arnold equation, Chua's equation and hyper-chaos equation trajectory, the obstacle reflects the UAV(Unmanned Aerial Vehicle).

**Key words :** UAV, Chaos, Arnold equation, Chua's equation, Obstacle avoidance

## 1. INTRODUCTION

Chaos theory has been drawing a great deal of attention in the scientific community for almost two decades. Remarkable research efforts have been spent in recent years, trying to export concepts from Physics and Mathematics into real world engineering applications. Applications of chaos are being actively studied in such areas as chaos control [1]-[2], chaos synchronization and secure/crypto communication [3]-[7], Chemistry [8], Biology [9] and robots and their related themes [10].

Recently, Nakamura, Y. et al [10] proposed a chaotic mobile robot where a mobile robot is equipped with a controller that ensures chaotic motion and the dynamics of the mobile robot are represented by an Arnold equation. They applied obstacles in the chaotic trajectory, but they did not mention obstacle avoidance methods.

In this paper, we propose a method to obstacle avoidance using unstable limit cycles in the chaos trajectory surface. We assume that all obstacles in the chaos trajectory surface have a Van der Pol equation with an unstable limit cycle. When chaos UAV(Unmanned Aerial Vehicles) meet obstacle among their arbitrary wandering in the chaos trajectory, which is derived using chaos circuit equations such as the Arnold equation, Chua's equation, hyper chaos equation, the obstacle reflective the chaos UAVs

## 2. Chaotic UAV equation

### 2.1 UAV(Unmanned Aerial Vehicle)[24]

We assume that each UAV is equipped with standard autopilots for heading hold and mach hold. In order to focus on the essential issues, we will assume that altitude is held constant. Let  $(x, y), \psi, v$  denote the inertial position, heading angle, and velocity for the UAV respectively. Then the resulting kinematics equations of motion are

$$\begin{aligned} \dot{x} &= v \cos(\psi) \\ \dot{y} &= v \sin(\psi) \\ \dot{\psi} &= \alpha_{\psi} (\xi^c - \psi) \\ \dot{v} &= \alpha_v (v^c - v) \end{aligned} \tag{1}$$

where  $\psi^c$  and  $v^c$  are the commanded heading angle and velocity to the autopilots,  $\alpha_{\psi}$  and  $\alpha_v$  are positive constraints[22,23].

Assuming that  $\alpha_v$  is a large compared to  $\alpha_{\psi}$ , Eq. (1) reduces to

$$\begin{aligned} \dot{x} &= v \cos(\psi) \\ \dot{y} &= v \sin(\psi) \\ \dot{\psi} &= \alpha_{\psi} (\xi^c - \psi) \end{aligned} \tag{2}$$

Letting  $\psi^c = \psi + (1/\alpha_{\psi})\omega$   $v^c \approx v$ , Eq. (2) becomes and

$$\begin{aligned} \dot{x} &= v \cos(\psi) \\ \dot{y} &= v \sin(\psi) \\ \dot{\psi} &= \omega \end{aligned} \tag{3}$$

Eq.(3) rewritten as follows,

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix} \quad (4)$$

Eq. (3) is similar to two wheel mobile robot equation Fig 1 and Eq. (5).

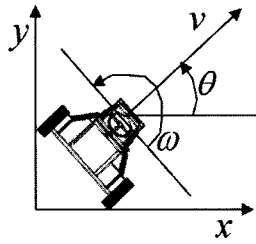


Fig. 1. Two wheel robot

$$\begin{pmatrix} \dot{\tilde{x}}_1 \\ \dot{\tilde{y}}_1 \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix} \quad (5)$$

where(  $x,y$ ) is the position of the robot and  $\theta$  is the angle of the robot.

## 2.2 Chaos equations

In order to generate chaotic motions for the UAV, we apply chaos equations such as an Arnold equation or Chua's circuit equation.

### 2.2.1 Arnold equation [10]

We define the Arnold equation as follows:

$$\begin{aligned} \dot{\tilde{x}}_1 &= A \sin x_3 + C \cos x_2 \\ \dot{\tilde{x}}_2 &= B \sin x_1 + A \cos x_3 \\ \dot{\tilde{x}}_3 &= C \sin x_2 + B \cos x_1 \end{aligned} \quad (6)$$

where A, B, C are constants.

### 2.2.2 Chua's circuit

Chua's circuit is one of the simplest physical models that has been widely investigated by mathematical, numerical and experimental methods. We can derive the state equation of Chua's circuit from Fig. 2.

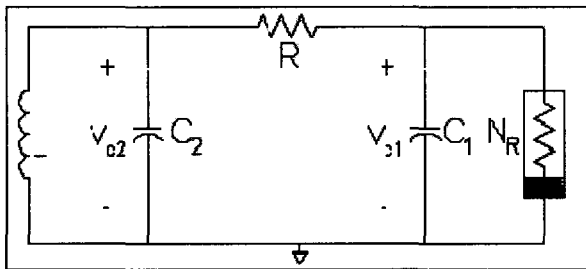


Fig. 2. Chua circuit

$$\begin{aligned} \dot{\tilde{x}}_1 &= a(x_2 - g(x_1)) \\ \dot{\tilde{x}}_2 &= x_1 - x_2 + x_3 \\ \dot{\tilde{x}}_3 &= -\beta x_2 \end{aligned} \quad (7)$$

where .

$$g(x) = m_{2n-1}x + \frac{1}{2} \sum_{k=1}^{2n-1} (m_{k-1} - m_k)(|x + c_k| - |x - c_k|)$$

### 2.2.3 Hyper-chaos equation

Hyper-chaos equation is one of the simplest physical models that have been widely investigated by mathematical, numerical and experimental methods for complex chaotic dynamic. We can easily make hyper-chaotic equation by using some of connected N-double scroll. We can derive the state equation of N-double scroll equation as followings.

$$\begin{aligned} \dot{x} &= a[y - h(x)] \\ \dot{y} &= x - y + z \\ \dot{z} &= -\beta y \end{aligned} \quad (8)$$

Where,

$$h(x) = m_{2n-1}x + \frac{1}{2} \sum_{i=1}^{2n-1} (m_{i-1} - m_i)(|x + c_i| - |x - c_i|)$$

In order to make a hyper-chaos, we have compose to 1 dimensional CNN(Cellular Neural Network) which are identical two N-double scroll circuits and then we have to connected each cell by using unidirectional coupling or diffusive coupling. In this paper, we used to diffusive coupling method. We represent the state equation of x-diffusive coupling and y-diffusive coupling as follows.

x-diffusive coupling

$$\begin{aligned} \dot{\tilde{x}}^{(j)} &= a[y^{(j)} - h(x)^{(j)}] + D_x(x^{(j-1)} - 2x^{(j)} + x^{(j+1)}) \\ \dot{\tilde{y}}^{(j)} &= x^{(j)} - y^{(j)} + z^{(j)} \\ \dot{\tilde{z}}^{(j)} &= -\beta y^{(j)}, \quad j=1, 2, \dots, L \end{aligned} \quad (9)$$

y-diffusive coupling

$$\begin{aligned} \dot{\tilde{x}}^{(j)} &= a[y^{(j)} - h(x)^{(j)}] \\ \dot{\tilde{y}}^{(j)} &= x^{(j)} - y^{(j)} + z^{(j)} + D_y(y^{(j-1)} - 2y^{(j)} + y^{(j+1)}) \\ \dot{\tilde{z}}^{(j)} &= -\beta y^{(j)}, \quad j=1, 2, \dots, L \end{aligned} \quad (10)$$

where, L is number of cell.

## 2.3. Embedding of Chaos circuit in the UAV

In order to embed the chaos equation into the UAV, we define and use the Arnold equation, Chua's circuit equation and Hyper-chaos equation as follows.

### 2.3.1 Arnold equation

We define and use the following state variables:

$$\begin{aligned} \dot{x}_1 &= D \dot{y} + C \cos x_2 \\ \dot{x}_2 &= D \dot{x} + B \sin x_1 \\ \dot{x}_3 &= \theta \end{aligned} \quad (11)$$

where B, C, and D are constant.

Substituting (4) into (6), we obtain a state equation on

$\dot{x}_1$ ,  $\dot{x}_2$ , and  $\dot{x}_3$  as follows:

$$\begin{aligned} \dot{x}_1 &= Dv + C \cos x_2 \\ \dot{x}_2 &= Dv + B \sin x_1 \\ \dot{x}_3 &= \omega \end{aligned} \quad (12)$$

We now design the inputs as follows [10]:

$$\begin{aligned} v &= A / D \\ \omega &= C \sin x_2 + B \cos x_1 \end{aligned} \quad (13)$$

Finally, we can get the state equation of the UAV as follows:

$$\begin{aligned} \dot{x}_1 &= A \sin x_3 + C \cos x_2 \\ \dot{x}_2 &= B \sin x_1 + A \cos x_3 \\ \dot{x}_3 &= C \sin x_2 + B \cos x_1 \\ \dot{x} &= V \cos x_3 \\ \dot{y} &= V \sin x_3 \end{aligned} \quad (14)$$

Equation (14) includes the Arnold equation.

### 2.3.2 Chua's equation

Using the methods explained in equations (11)-(14), we can obtain equation (15) with Chua's equation embedded in the UAV.

$$\begin{aligned} \dot{x}_1 &= \alpha (x_2 - g(x_1)) \\ \dot{x}_2 &= x_1 - x_2 + x_3 \\ \dot{x}_3 &= -\beta x_2 \\ \dot{x} &= V \cos x_3 \\ \dot{y} &= V \sin x_3 \end{aligned} \quad (15)$$

Using equation (15), we obtain the embedding UAV trajectories with Chua's equation.

### 2.3.2 Hyper-chaos equation

Combination of equation (1) and (9) or (10), we define and use the following state variables (16) or (17).

$$\begin{pmatrix} \dot{\bar{x}}_1 \\ \dot{\bar{x}}_2 \\ \dot{\bar{x}}_3 \\ \dot{\bar{x}} \\ \dot{\bar{y}} \end{pmatrix} = \begin{pmatrix} a[y^{(j)} - h(x^{(j)}) + D_x(x^{(j-1)} - 2x^{(j)} + x^{(j+1)})] \\ x^{(j)} - y^{(j)} + z^{(j)} \\ -\beta y^{(j)} \\ v \cos x_3 \\ v \sin x_3 \end{pmatrix} \quad (16)$$

$$\begin{pmatrix} \dot{\bar{x}}_1 \\ \dot{\bar{x}}_2 \\ \dot{\bar{x}}_3 \\ \dot{\bar{x}} \\ \dot{\bar{y}} \end{pmatrix} = \begin{pmatrix} a[y^{(j)} - h(x^{(j)}) + D_y(x^{(j-1)} - 2x^{(j)} + x^{(j+1)})] \\ x^{(j)} - y^{(j)} + z^{(j)} \\ -\beta y^{(j)} \\ v \cos x_3 \\ v \sin x_3 \end{pmatrix} \quad (17)$$

Using equation (16) and (17), we obtain the embedding chaos robot trajectories with Hyper-chaos equation.

## 3. VDP( Van der Pol) obstacle.

### 3.1 VDP equation

In this section, we will discuss the mobile robot's avoidance of Van der Pol(VDP) equation obstacles. We assume the obstacle has a VDP equation with an unstable limit cycle, because in this condition, the mobile robot can not move close to the obstacle and the obstacle is avoided.

In order to represent an obstacle of the mobile robot, we employ the VDP, which is written as follows:

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= (1 - y^2)y - x \end{aligned} \quad (18)$$

From equation (18), we can get the following limit cycle as shown in Fig. 3.

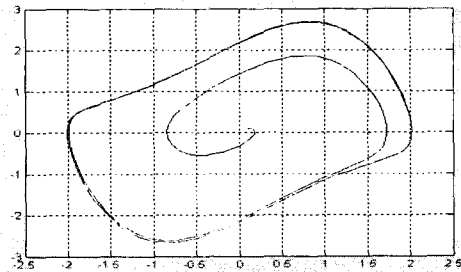


Fig. 3. Limit cycle of VDP

### 3.2 Mirror mapping

Equation (14),(15),(16) and (17) assume that the UAV moves in a smooth state space without boundaries. However, real UAV move in space with boundaries like walls or surfaces of obstacles. To avoid a boundary or obstacle, we consider mirror mapping when the UAV approach an obstacles using Eq. (19) and (20). Whenever the UAVs approach an obstacle, we calculate the UAVs' new position by using Eq. (19) or (20).

$$A = \begin{pmatrix} \cos \theta & \sin \theta \\ \sin \theta & -\cos \theta \end{pmatrix} \quad (19)$$

$$A = \frac{1}{1+m} \begin{pmatrix} 1-m^2 & 2m \\ 2m & -1+m^2 \end{pmatrix} \quad (20)$$

We can use equation (19) when the slope is infinity, such as  $\theta=90$ , and use equation (20) when the slope is not infinity.

## 4. An Obstacle Avoidance Method in the UAV

In this section, we will discuss the UAV's avoidance of Van der Pol(VDP) equation obstacles. We assume the obstacle has a VDP equation with an unstable limit cycle, because in this condition, the UAV can not move close to the obstacle and the obstacle is avoided.

**4.1 Magnitude of Distracting force from the obstacle**

We consider the magnitude of distracting force from the obstacle as follows:

$$D = \frac{0.325}{(0.2D_k + 1)e^{3(0.2D_k - 1)}} \quad (21)$$

where  $D_k$  is the distance between each effective obstacle and the UAV.

We can also calculate the VDP obstacle direction vector as follows:

$$\begin{bmatrix} \bar{x}_k \\ \bar{y}_k \end{bmatrix} = \begin{bmatrix} x_0 - y \\ 0.5(1 - y_0 - y)^2(y_0 - y) - x_0 - x \end{bmatrix} \quad (22)$$

where  $(x_0, y_0)$  are the coordinates of the center point of each obstacle. Then we can calculate the magnitude of the VDP direction vector ( $L$ ), the magnitude of the moving vector of the virtual UAV ( $I$ ) and the enlarged coordinates ( $I/2L$ ) of the magnitude of the virtual UAV in VDP ( $x_k, y_k$ ) as follows:

$$\begin{aligned} L &= \sqrt{2} \text{of} \left( \overline{x_{vdp}^2} + \overline{y_{vdp}^2} \right) \\ I &= \sqrt{\overline{x_r^2} + \overline{y_r^2}} \\ x_k &= \frac{\bar{x}_k}{L} \frac{I}{2}, \quad y_k = \frac{\bar{y}_k}{L} \frac{I}{2} \end{aligned} \quad (23)$$

Finally, we can get the Total Distraction Vector (TDV) as shown by the following equation.

$$\begin{pmatrix} \frac{\sum_k^n \left( \left(1 - \frac{D_k}{D_0}\right) \bar{x} + \frac{D_k}{D_0} \bar{x}_k \right)}{n} \\ \frac{\sum_K^N \left( \left(1 - \frac{D_k}{D_0}\right) \bar{y} + \frac{D_k}{D_0} \bar{y}_k \right)}{n} \end{pmatrix} \quad (24)$$

Using equations (21)-(24), we can calculate the avoidance method of the obstacle in the Arnold equation, Chua's equation and hyper-chaos trajectories with one or more VDP obstacles.

**4.2 Arnold equation case**

In Fig. 4, the computer simulation result shows that the chaos UAV has two UAVs and a total of 5VDP

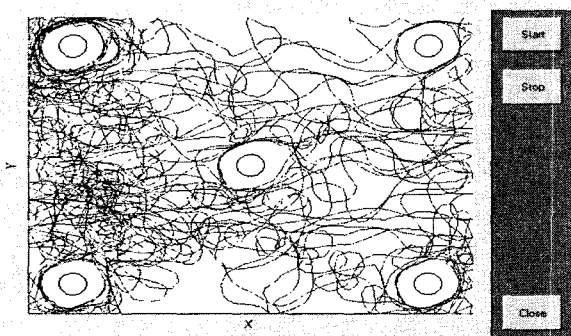


Fig. 4. Computer simulation result of obstacle avoidance with 2 UAVs and 5 obstacles in Arnold equation trajectories.

obstacles, including two VDP obstacles at the origin in the Arnold equation trajectories. We can see that the UAV sufficiently avoided the obstacles in the Arnold equation trajectories.

**4.3 Chua's equation case**

In Fig. 5, the computer simulation result shows that the chaos UAV surface has two UAVs and total of 5 VDP obstacles, including 2 VDP obstacles at the origin in the Chua's equation trajectory. We can see that the UAV sufficiently avoided the obstacles in the Chua's equation trajectory.

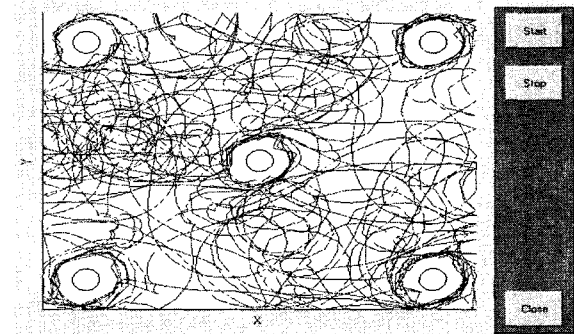


Fig. 5 Computer simulation result of obstacle avoidance with 2 UAVs and 5 obstacles in Chua's equation trajectory

**4.4 Hyper-chaos equation case**

In Fig. 6, the computer simulation result shows that the chaos UAV surface has two UAVs and total of 5 VDP obstacles, including 2 VDP obstacles at the origin in the hyper-chaos equation trajectory. We can see that the UAV sufficiently avoided the obstacles in the hyper-chaos equation trajectory.

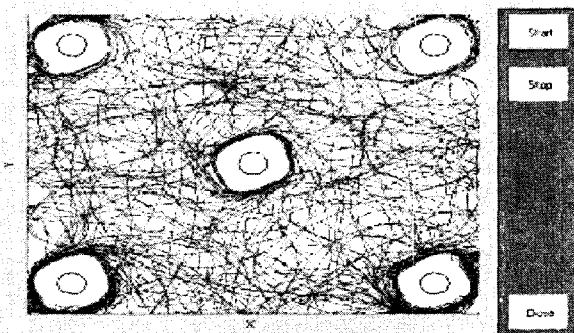


Fig. 6 Computer simulation result of obstacle avoidance with 2 UAVs and 5 obstacles in hyper-chaos equation trajectory

**5. Conclusion**

In this paper, we proposed a chaotic UAV, which

employs a UAV with Arnold equation, Chua's equation and hyper-chaos trajectories, and also proposed a obstacle avoidance method in which we assume that the obstacle have a Van der Pol equation with an unstable limit cycle.

We designed UAV trajectories such that the total dynamics of the UAV was characterized by an Arnold equation, Chua's equation and hyper-chaos equation and we also designed the UAVs trajectories to include an obstacle avoidance method. By the numerical analysis, it was illustrated that obstacle avoidance methods with a Van der Pol equation that have an unstable limit cycle gave the best performance.

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