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A Study on Fuzzy Wavelet Basis Function for Image Interpolation

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Abstract

The image interpolation is one of an image preprocessing process to heighten a resolution. The conventional image interpolation used much to concept that it put in other pixel to select the nearest value in a pixel simply, and use much the temporal object interpolation techniques to do the image interpolation by detecting motion in a moving picture presently.
In this paper, it is proposed the image interpolation techniques using the fuzzy wavelet base function. This is applied to embody a correct edge image and a natural image when expand part of the still image by applying the fuzzy wavelet base function coefficient to the conventional B-spline function. And the proposal algorithm in this paper is confirmed to improve about 1.2831 than the image applying the conventional B-spline function through the computer simulation.

Key Words : Interpolation, Fuzzy Wavelet, B-spline, temporal object

1. Introduction

Geometric image processes modify the arrangement of pixels based on some geometric transformation. The idea is to merely move pixels around within the image. Ideally the pixel value are not altered. If, however, the geometric process attempts to source a pixel from a location that does not exist, a new pixel will be generated. This pixel generation method is the interpolation. And the image interpolation is one of a basic techniques that process a digital image, it is an important role in many application of the image processing field that a rotate of a medical image, a slice interpolation and 3 dimensions reconfiguration etc. Recently, the tendency of a display is grown, and cleared more, and gone forward for the direction that support a high resolution gradually.

The correct meaning of the interpolation used to wish to heighten a resolution, or get a natural image by doing the restoration of the data damage part. And it is using mainly in the restoration following data compression and compression damage presently. This is used to heighten a resolution when it is compressed and reduced by changing a size of a digital image in an image system and an image coding[1,2]. Also, research for a compression technology efficiently is consisting as the request for the storage and transmission of the image information grows. Digital camera that is giving a digital zoom function based on an image interpolation is sending to the goods at present[3,4].

Recently, the wavelet transform is getting superior result being applied to a multi-dimension analysis, a time-frequency signal analysis, a function approximation. The wavelet transform is the technology of a signal processing and it is applied in the several fields because of having the advantage of a time-frequency localization for a signal. It is regarded in a different case about the conventional Fourier transform, and it can do hard analysis to expect in Fourier transform and it is getting into the spotlight as the important application field in the signal processing and the image processing. It can detect easily the characteristic of a signal by using the wavelet base function of a wavelet transform. So, it is applied in field that is not 1 dimension but 2 dimensions image processing.

This paper embodies the algorithm that heighten a resolution by doing the image scale extension or reduction, a pixel restoration that is a function originally using B-spline function, and wish to get a natural image by removing a smoothing phenomenon and Blurring phenomenon applying the fuzzy wavelet transform base function coefficient to it. Two chapter introduces the conventional algorithm and the proposed method applying the fuzzy wavelet base function coefficient to B-spline function. And 3 chapter explains the result of the proposal algorithm and the conventional algorithm through simulation, and composes to the conclusion finally.

2. B-Spline Interpolation Function

In many signal processing applications, the sample of a continuous time signal $f_c(t)$ take the place of a discontinuous signal $f(k)$. This uses $f_c(t)$ as an ordinary smooth function because of being not a unique expression method. Many investigators are proposed the method of a spline function, here describes a basic mathematical equation. The spline of n degree is same with n order polynomial equation in each region between 2 knot, and this polynomial equation is related to a different (n-1) time continuously in a general function of knot. All n times degree spline with a same distance note is known by Schoenberg[5]. $f_c^n(t)$ is expressed with (1).

$$f_c^n(t) = \sum_{k=-\infty}^{\infty} c(k)\beta^n(t-k), \quad \forall t \in R \qquad (1)$$

Here, $c(k)$ is a real number l_2, $\beta^n(t)$ is arranged with (2) in B-spline getting by n layer convolution of a single pulse in a center of n degree.

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$$\beta^n(t) = \beta^{n-1} * \beta^0(t) = \beta^0 * \beta^0 * \cdots * \beta^0(t) \tag{2}$$
$$\beta^0(t) = \begin{cases} 1, & \text{for } t \in [-\frac{1}{2}, \frac{1}{2}) \\ 0, & \textit{otherwise} \end{cases}$$

B-spline is applied perfectly by (2), and a function gets into a symmetry by the center on 0. For n, $m \in N$, discontinuous B-spline $b_m^n(k)$ proved by $b_m^n(k) = \beta^n(k/m)$ that is array of the integral sample connected with a continuous B-spline(n times degree) that is extended by m element[6]. z-transforms of $f(k)$, $b_m^n(k)$ and $c(k)$ are defined by each $F(z)$, $B_m^n(z)$ and $C(z)$. If it define by $f_c^n(k) = f(k)$, $\forall k \in Z$ in a signal expression, equation (2) can define with (3) again.

$$F(z) = C(z)B_1^n(z) \tag{3}$$

All zeros of $B_1^n(z)$ for all $n \in N$ are a real number and a negative, and are not -1. Moreover, if it is no $B_1^n(z)$ in $z = z_i$, also there is not existed to $z = z_i^{-1}$ because it is a symmetrical polynomial equation. Therefore, if it chooses carefully the region of a convergence, it keeps $[B_1^n(z)]^{-1}$ stability except IIR that cause does not happen. The B-spline coefficient $c(k)$ is obtained as running that is direct B-spline transform in z plane by multiplying $F(z)$ to $[B_1^n(z)]^{-1}$. A spline interpolation by integral element m of a signal $f(k)$ is given by $f_c^n(k/m)$ defined by $f_m^n(k)$.

$$f_m^n(k) = \sum_{l=-\infty}^{\infty} c(l) b_m^n(k - lm) \tag{4}$$

The total system for an interpolation is same with Fig. 1.

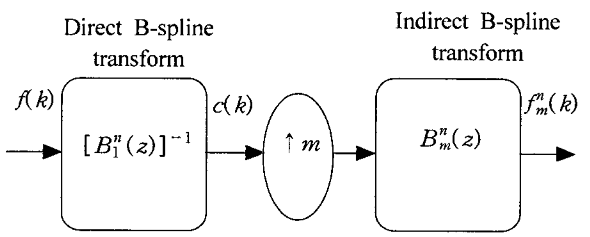

Fig. 1 Block diagram of B-spline signal interpolation

The operation of an interpolation signal reconfiguration in equation 4 is known as a indirect B-spline transform, and the digital filter $B_m^n(z)$ is a symmetry FIR. If m and n are not even both, it is analyzed with (5).

$$\begin{aligned} B_m^n(z) &= \frac{z^a}{m^n}\left(\frac{1-z^{-m}}{1-z^{-1}}\right)^{n+1} B_1^n(z) \\ &= z^a m[M_m(z)]^{n+1} B_1^n(z) \end{aligned} \tag{5}$$

Here, a is $(m-1)(n+1)/2$, and filter $M_m(z) = (1/m)(1 + z^{-1} + \cdots + z^{-(m-1)})$ expresses sum. By (5), the analysis equation for $m \geq 3$ can solve a complexity as calculative. Digital filter $B_m^n(z)$ acts in a high ratio because an input signal has m times doing a sampling more. But, this does the filtering operation with much errors.

3. Proposed Fuzzy Wavelet Transform

The membership function of L-R function is same with (6)[7].

$$\nu_{LR-I}(x) = \begin{cases} L(\frac{m_l - x}{\alpha}) & \text{for } x \leq m_l \\ 1 & \text{for } m_l \leq x \leq m_r \\ R(\frac{x - m_r}{\beta}) & \text{for } x \geq m_r \end{cases} \tag{6}$$

Here, L and R are a function of a steep shape, and α, β, ν are parameters for a fuzzy region. If L and R have the gaussian shape, it is applied to (7).

$$\nu_{LR-I}(x) = LR\left[\frac{m_l \nabla x}{\alpha} + \frac{x \nabla m_r}{\beta}\right] \tag{7}$$

Here, the operation that a boundary is different is a∇b=max(a-b, 0), the fuzzy region represents as I=[ml, mr, α, β]LR. To do fuzziness the function of a general L-R triangle shape, it used a fuzzy set number with Fig. 2. Fig. 2 made the 3 fuzzy membership functions, the fuzzy level uses mainly the fuzzy membership function that is DARK, MIDDLE, BRIGHT to do fuzziness by [0, 1] some pixel value between 0~255.

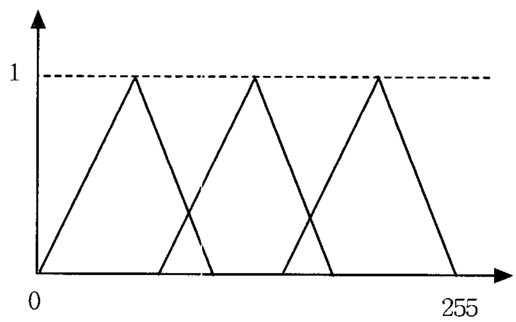

Fig. 2 Triangularity fuzzy membership function

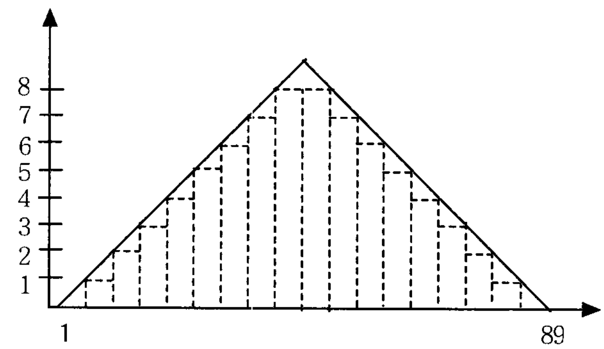

Fig. 3 Quantization of triangularity fuzzy function

Also, the area of an one fuzzy membership function subdivides into each five units to 1~89 to do digital. And it quantizes a fuzzy function so that a pixel value of a gray scale may exist between 0~8, and it digitalises a fuzzify part so that the parallel arithmetic processing is available.

Wavelet of one dimension signal fuzzy function $f(t)$ is calculated by (8)[8].

$$\begin{aligned} W_s f(k) &= \int_{-\infty}^{\infty} \psi_s(k-t)\,dt \\ &= \sum_{l\in Z}\int_{l-1}^{l} f(l)\psi_s(k-t)dt \\ &= \sum_{l\in Z} f(l)\Big[\int_{-\infty}^{l}\psi_s(k-t)\,dt - \int_{-\infty}^{l-1}\psi_s(k-t)\,dt\Big] \\ &= \sum_{l\in Z} f(l)\Big[\int_{(k-l)s^{-1}}^{\infty}\psi(t)\,dt - \int_{(k-l+1)s^{-1}}^{\infty}\psi(t)\,dt\Big] \\ &= \sum_{l\in Z} f(l)\int_{(k-l)s^{-1}}^{\infty}\psi(t)\,dt - \sum_{l\in Z} f(l+1)\int_{(k-l)s^{-1}}^{\infty}\psi(t)\,dt \\ &= \sum_{l\in Z}[f(l)-f(l+1)]\psi^s_{k-l} \end{aligned} \tag{8}$$

Here, $\psi^s_k = \int_{k/s}^{\infty}\psi(t)dt$, it is expressed as following because $\psi(t)$ is odd function.

$$\psi^s_{-k} = \int_{-k/s}^{k/s}\psi(t)dt + \int_{k/s}^{\infty}\psi(t)dt = \psi^s_k \tag{9}$$

In 2 dimensions signal, a calculation of $W^i_{2^j} f(x,y)$ becomes more complex than thing which calculates 1 dimension signal. So, 2 dimensions signal f(x, y) is calculated by (10).

$$\begin{aligned} W^1_s f(n,m) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(u,v)\psi^1_s(n-u,\, m-v)\,dudv \\ &= \sum_{k,l} f(k,l)\int\int_{[k,k+1]\times[l,l+1]}\psi^1_s(n-u,\, m-v)\,dudv \\ &= \sum_{k,l} f(k,l)\int\int_{[n-k-1,\,n-k]\times[m-l-1,\,m-l]}\psi^1_s(u,v)\,dudv \\ &= \sum_{k,l} f(n-1-k,\, m-1-l)\psi^{s,1}_{k,l} \end{aligned} \tag{10}$$

Here,
$$\begin{aligned} \psi^{s,1}_{k,l} &= \int\int_{[k,k+1]\times[l,l+1]}\psi^1_s(u,v)\,dv \\ &= \int_{\frac{k}{s}}^{\frac{(k+1)}{s}} du \int_{\frac{l}{s}}^{\frac{(l+1)}{s}} \psi^1(u,v)\,dv \end{aligned}$$

To drive $\{\psi^{s,1}_{k,l}\}$ calculations in (10), $\psi^1(u,v)$ is odd in u, and even in v. So, it can get (11).

$$\begin{aligned} \psi^{s,1}_{-k,l} &= \int\int_{[-k,-k+1]\times[l,l+1]}\psi^1_s(u,v)\,dudv \\ &= \int_{k-1}^{k} du \int_{l}^{l+1}\psi^1_s(u,v)\,dudv \\ &= -\psi^{s,1}_{k-1,l} \end{aligned} \tag{11}$$

It can get (12) similarly.

$$\psi^{s,1}_{k,-l} = \psi^{s,1}_{k,l-1},\quad \psi^{s,1}_{-k,-l} = -\psi^{s,1}_{k-1,l-1} \tag{12}$$

For all $k, l \ge 0$, it must calculate $\psi^{s,1}_{k,l}$, if $\psi(r)$ is known as odd function, it can get (13).

$$\psi^1(u,v) = \psi(r)\cos\theta \tag{13}$$

Here, $r=\sqrt{u^2+v^2}$, $\theta = arctg\frac{v}{u}$. If it obtains $\phi(x)$ using $\psi(r)$, it can get (14).

$$\phi(x) = \int_{-\infty}^{\infty}\psi(r)dr \tag{14}$$

This time, $\phi(x)$ is even function, $\psi(x)$ is odd function. If (14) is defined, it is same with (15).

$$\begin{aligned} \frac{\partial}{\partial v}\phi(\sqrt{u^2+v^2}) &= \phi'(r)\frac{u}{r} = \psi^1(u,v) \\ \frac{\partial}{\partial u}\phi(\sqrt{u^2+v^2}) &= \phi'(r)\frac{v}{r} = \psi^2(u,v) \end{aligned} \tag{15}$$

Also, if (15) is applied to $\psi^{s,1}_{k,l}$ of (10), it gets a new (16).

$$\begin{aligned} \psi^{s,1}_{k,l} &= \int_{\frac{k}{s}}^{\frac{(k+1)}{s}} du \int_{\frac{l}{s}}^{\frac{(l+1)}{s}} \frac{\partial}{\partial u}\phi(\sqrt{u^2+v^2})\,dv \\ &= \int_{\frac{l}{s}}^{\frac{(l+1)}{s}}\Big[\phi\Big(\sqrt{v^2+(\tfrac{k+1}{s})^2}\Big) - \phi\Big(\sqrt{v^2+(\tfrac{k}{s})^2}\Big)\Big]dv \\ &= \phi^s_{l,k+1} - \phi^s_{l+1,k+1} - \phi^s_{l,k} + \phi^s_{l+1,k} \end{aligned} \tag{16}$$

Here, $\phi^s_{l,k} = \int_{\frac{l}{s}}^{\infty}\phi\Big(\sqrt{v^2+(\frac{k}{s})^2}\Big)dv$. And a calculation of $W^2_s f(n,m)$ is same with (17).

$$W^2_s f(n,m) = \sum_{k,l} f(n-1-k,\, m-1-l)\psi^{s,2}_{k,l} \tag{17}$$

Here, $\psi^{s,2}_{k,l}$ is same with a following (18).

$$\begin{aligned} \psi^{s,2}_{k,l} &= NT_{\frac{k}{s}}^{\frac{(k+1)}{s}} du \int_{\frac{l}{s}}^{\frac{(l+1)}{s}} \psi^2(u,v)\,dv \\ &= \int_{\frac{k}{s}}^{\frac{(k+1)}{s}}\Big[\phi\Big(\sqrt{\big(\tfrac{l+1}{s}\big)^2+u^2}\Big) - \phi\Big(\sqrt{\big(\tfrac{k}{s}\big)^2+u^2}\Big)\Big]dv \\ &= \phi^s_{k,l+1} - \phi^s_{k+1,l+1} - \phi^s_{k,l} + \phi^s_{k+1,l} \\ &= \psi^{s,1}_{l,k} \end{aligned} \tag{18}$$

Above all conditions calculate a filtering coefficient $\{\phi^s_{k,l}\}$ for $k, l \ge 0$. And it wishes to embody the new wavelet interpolation method taking the place of the spline coefficient $c(k)$ in the wavelet transform $f^n_c(t) = \sum_{k=-\infty}^{\infty} c(k)\,\beta^n(t-k)$, $\forall t \in R$ that explains to (1) by a coefficient of the wavelet base function that present in this paper.

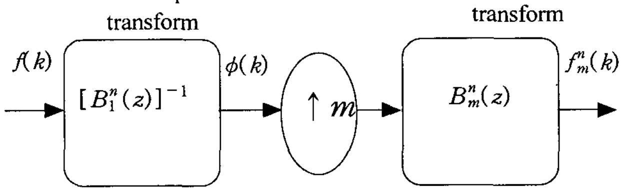

Fig. 4 Block diagram of proposal fuzzy wavelet interpolation using B-spline function

4. Simulation and Result

This simulation did the gray scale image 10 with 256×256 size, and the image quality evaluated by PSNR(Peak-to-Peak Signal to Noise Ratio) and 3 dimensions graph.

$$PSNR = 10 \times \log_{10}\left[\frac{255^2}{\frac{1}{XY}\sum_{l=0}^{X-1}\sum_{m=0}^{Y-1}(O(l,m)-I(l,m))^2}\right] \quad (19)$$

Here, X is the number of an image row, Y is the number of an image column. And 255 is the maximum with the pixel, $O(l, m)$ and $I(l, m)$ are the original image and the result image respectively. And the applied wavelet base function $\phi(x)$ is same with a following (20).

$$\phi(x) = \begin{cases} 8(x^3 - x^2) + \frac{4}{3} & 0 \le x \le 1/2 \\ -\frac{8}{3}(x-1)^3 & 1/2 < x \le 1 \\ 0 & x > 1 \end{cases} \quad (20)$$

If (20) is applied to equation 18, it can get a following (21).

$$\phi(x)^{s}_{k,l} = \begin{cases} 0 & \text{if } k^2 + l^2 \ge s^2 \\ J\left(\frac{l}{s}, \frac{k}{s}, \sqrt{1-(\frac{l}{s})^2}\right) & \text{if } s^2 \ge k^2 + l^2 \ge \frac{1}{4}s^2 \\ I\left(\frac{l}{s}, \frac{k}{s}, \sqrt{\frac{1}{4}-(\frac{l}{s})^2}\right) & \\ +J\left(\frac{l}{s}, \sqrt{\frac{1}{4}-(\frac{l}{s})^2}, \sqrt{1-(\frac{l}{s})^2}\right) & \text{if } k^2 + l^2 \le \frac{1}{4}s^2 \end{cases} \quad (21)$$

Here,

$$I(t,a,b) = \frac{8}{3}(a^3 - b^3) - \frac{4}{3}9a - b)[1 - 6t^2] - \sqrt{a^2+t^2}[2a^3 + 5at^2] + \sqrt{b^2+t^2}(2b^3 + 5bt^2), - 3t^4 \log\frac{a+\sqrt{a^2+t^2}}{b+\sqrt{b^2+t^2}}$$

$$J(t,c,d) = -\frac{8}{3}(c-d)(c^2+d^2+cd+3t^2+1) + \frac{1}{3}[c(2c^2+5t^2+12)\sqrt{c^2+t^2} - d(2d^2+5t^2+12)\sqrt{d^2+t^2}] + t^2(4+t^2)\log\frac{c+\sqrt{c^2+t^2}}{d+\sqrt{d^2+t^2}},$$

Table 1 could know that about 1.2831 is superior the result image applied the fuzzy wavelet base function to the result image applied the conventional B-spline function and about 0.9925 is superior to the result image applied the wavelet base function coefficient in PSNR that get using (19). And Fig. 5 is a test image, Fig. 6 is the result image applied B-spline to an interpolation. And Fig. 7 is the result image applied the wavelet base function coefficient to an interpolation, and Fig. 8 is the result image applied the basic triangularity fuzzy wavelet base function to an interpolation. Also, Fig. 9, 10, 11, 12 are 3 dimensions graph of the result image applied each algorithm.

Table. 1 PSNR applying each algorithm

알 고 리 즘	PSNR
B-Spline Function	18.9095
Wavelet base Function	19.2001
Fuzzy Wavelet base Function	20.1926

Fig. 5 Original image

Fig. 6 B-spline function result image

Fig. 7 Wavelet base function result image

Fig. 8 Fuzzy wavelet base function result image

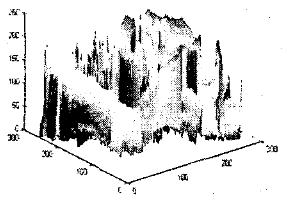

Fig. 9 Original image 3 dimensions graph

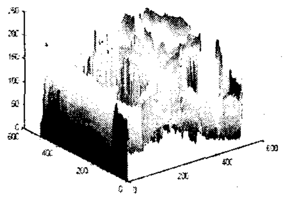

Fig. 10 B-spline function result image graph

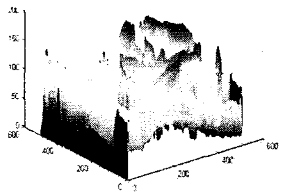

Fig. 11 Wavelet base function result image graph

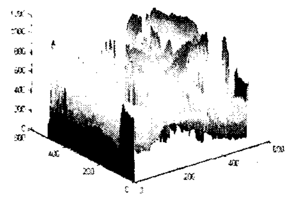

Fig. 12 Fuzzy wavelet base function result image graph

5. Conclusions

This paper proposed the improved interpolation method by applying the fuzzy wavelet base function coefficient to the conventional B-spline function in order to improve a smoothing phenomenon or Blurring phenomenon that happen in the conventional interpolation method. Pixels of the conventional image interpolation such as looking in figure 10, 11 was appearing so coarsely, but the proposal method in this paper could know that get a natural image result, a very soft image result in a rough aspect. PSNR such as table 1 confirmed to be improved each about 1.2831 and about 0.9921 through simulation. And the fuzzy function was applied to reduce the data amount of process getting the wavelet coefficient, and it reduced the arithmetic amount of coefficient being happened damage.

Hereafter, this algorithm wish to apply getting a natural image by applying to color moving picture.

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