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ON CLOSURE GAMMA-SEMIGROUPS

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Abstract. We introduce the notion of closure Γ-semigroups. We give a condition for a closure Γ-semigroup to be Γ-central, and we show that the Γ-centralizer of a closure Γ-semigroup is a Γ-subsemigroup.

1. Introduction

In 1986, M. K. Sen and N. K. Saha [1] introduced the notion of gamma-semigroups. They studied Γ-group and Γ-regular semigroup, and established a relation between Γ-group and Γ-regular semigroup. The aim of this paper is to introduce the notion of closure Γ-semigroups, and to investigate some properties.

2. Preliminaries

Let $M = \{x, y, z, \cdots\}$ and $\Gamma = \{\alpha, \beta, \gamma, \cdots\}$ be two non-empty sets. Then M is called a Γ-*semigroup* if

(1) $x\alpha y \in M$,

(2) $(x\alpha y)\beta z = x\alpha(y\beta z)$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.

A nonempty subset S of a Γ-semigroup M is called a Γ-*subsemigroup* of M if $S\Gamma S \subseteq S$.

3. Closure Γ-semigroups

Definition 3.1. A Γ-semigroup M is called a *right closure* Γ-*semigroup* if there exist a unary operation "$\tilde{\ }$" satisfying

(U1) $x\gamma\tilde{x} = x$,

(U2) $\tilde{x}\gamma\tilde{y} = \tilde{y}\gamma\tilde{x}$,

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(U3) $\tilde{\tilde{x}} = \tilde{x}$,
(U4) $\widetilde{x\gamma y\gamma}\tilde{y} = \widetilde{x\gamma y}$
for all $x, y \in M$ and $\gamma \in \Gamma$, and in such case we call "$\tilde{}$" a *right closure* on M.

If (U1) and (U4) are replaced by
(U5) $\tilde{x}\gamma x = x$,
(U6) $\widetilde{x\gamma y\gamma}\tilde{x} = \widetilde{x\gamma y}$,
respectively, we say that M is a *left closure* Γ*-semigroup*, and "$\tilde{}$" is a *left closure* on M.

In what follows a closure Γ-semigroup means a right closure Γ-semigroup unless otherwise specified.

Let M be a closure Γ-semigroup. Denote $\widetilde{M} := \{\tilde{x} \mid x \in M\}$, and

$$C_\Gamma(M) := \{y \in M \mid \tilde{x}\gamma y = y\gamma\tilde{x} \text{ for all } x \in M \text{ and } \gamma \in \Gamma\},$$

which is called the Γ*-centralizer* of M. A closure Γ-semigroup M is said to be Γ*-central* if $C_\Gamma(M) = M$.

PROPOSITION 3.2. *If M is a Γ-central closure Γ-semigroup, then the condition* (U2) *is superfluous.*

PROOF. The proof is straightforward. □

PROPOSITION 3.3. *For any elements x and y of the Γ-centralizer of a closure Γ-semigroup M, we have $\widetilde{x\gamma y} = \widetilde{x\gamma y\gamma}\tilde{x}$ for every $\gamma \in \Gamma$.*

PROOF. Let $x, y \in C_\Gamma(M)$ and $\gamma \in \Gamma$. Then

$$\widetilde{x\gamma y} = x\gamma y\gamma\tilde{x} = \widetilde{x\gamma y\gamma\tilde{x}}\gamma\tilde{\tilde{x}} = \widetilde{x\gamma y\gamma\tilde{x}}\gamma\tilde{x} = \widetilde{x\gamma y\gamma}\tilde{x}.$$

This completes the proof. □

Using Proposition 3.3, we know that if M is a Γ-central closure Γ-semigroup then the operation "$\tilde{}$" is also a left closure on M.

THEOREM 3.4. *Let M be a closure Γ-semigroup. If the operation "∼" is a left closure on M, then M is Γ-central.*

PROOF. Let $x, y \in M$ and $\gamma \in \Gamma$. Then

$$\begin{aligned}
\tilde{x}\gamma y &= \tilde{x}\gamma y\gamma\widetilde{\tilde{x}\gamma y} && \text{by (U1)} \\
&= \tilde{x}\gamma y\gamma\widetilde{\tilde{x}\gamma y\gamma}\tilde{y} && \text{by (U4)} \\
&= \tilde{x}\gamma y\gamma\widetilde{\tilde{x}\gamma y\gamma}\tilde{\tilde{x}}\gamma\tilde{y} && \text{by (U6)} \\
&= \tilde{x}\gamma y\gamma\tilde{x}\gamma\tilde{y} && \text{by (U1) and (U3)} \\
&= \tilde{x}\gamma y\gamma\tilde{y}\gamma\tilde{x} && \text{by (U2)} \\
&= \tilde{x}\gamma y\gamma\tilde{x}. && \text{by (U1)}
\end{aligned}$$

Similarly, $y\gamma\tilde{x} = \tilde{x}\gamma y\gamma\tilde{x}$, and so $\tilde{x}\gamma y = y\gamma\tilde{x}$, that is, $y \in C_\Gamma(M)$. Hence M is Γ-central. □

THEOREM 3.5. *The Γ-centralizer of a closure Γ-semigroup M is a Γ-subsemigroup of M.*

PROOF. Let $y, z \in C_\Gamma(M)$ and $\gamma \in \Gamma$. Then

$$\tilde{x}\gamma(y\gamma z) = (\tilde{x}\gamma y)\gamma z = (y\gamma\tilde{x})\gamma z = y\gamma(\tilde{x}\gamma z) = y\gamma(z\gamma\tilde{x}) = (y\gamma z)\gamma\tilde{x}$$

for all $x \in M$, and so $y\gamma z \in C_\Gamma(M)$. Hence $C_\Gamma(M)$ is a Γ-subsemigroup of M. □

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References

[1] M. K. Sen and N. K. Saha, *On Γ-semigroup-I,* Bull. Calcutta Math. Soc. **78** (1986), 180–186.

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