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# Estimation of Collapse Moment for Wall Thinned Elbows Using Fuzzy Neural Networks

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**Abstract** In this work, the collapse moment due to wall-thinning defects is estimated by using fuzzy neural networks. The developed fuzzy neural networks have been applied to the numerical data obtained from the finite element analysis. Principal component analysis is used to preprocess the input signals into the fuzzy neural network to reduce the sensitivity to the input change and the fuzzy neural networks are trained by using the data set prepared for training (training data) and verified by using another data set different (independent) from the training data. Also, two fuzzy neural networks are trained for two data sets divided into the two classes of extrados and intrados defects, which is because they have different characteristics. The relative 2-sigma errors of the estimated collapse moment are 3.07% for the training data and 4.12% for the test data. It is known from this result that the fuzzy neural networks are sufficiently accurate to be used in the wall-thinning monitoring of elbows.

**Keywords:** Collapse moment, fuzzy neural networks, principal component analysis, wall thinning defect

## 1. Introduction

The pipe bends and elbows are regarded as critical components in piping systems because they are incorporated into piping systems to allow modification of the isometric routing and more importantly pipe bends are usually incorporated to reduce anchor reaction forces. Also, the pipe bends and elbows are capable of absorbing considerably large thermal expansion and seismic movement through the energy dissipation as a result of local plastic deformation so that they maintain the integrity of piping system under transiently loading conditions[1]. However, care must be taken to ensure that the collapse load is avoided. Therefore, it is important to accurately estimate the safety margin for a collapse of elbows under various operating conditions.

However, the pipe elbows in nuclear power plants are subjected to various degradation mechanisms. Especially, the wall thinning is considered as an important degradation mechanism in carbon steel elbows[2]. The wall-thinning defect is mainly caused by flow-accelerated corrosion, and it results in reducing failure pressure, load-carrying capacity, deformation ability, and fatigue resistance of pipe elbows. Therefore, it is necessary to investigate the effect of thinning defect on the failure behavior of pipe elbows and to accurately estimate the collapse loads of wall thinned elbows under various loading conditions.

The objective of this work is to predict the collapse moment under a variety of loading conditions by fuzzy neural networks by measuring

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the defect geometry using laser technique. Many artificial intelligence techniques including neural networks and fuzzy inference methods have been successfully applied to a lot of nuclear engineering problems such as signal validation [3-5], plant diagnostics [6-7], optimal fuel loading [8-9], control [10], event identification [11-12], and so forth. Also, the neural networks have largely and successfully been applied to function approximation problems that will be used in this work. Therefore, the fuzzy neural networks which are characterized by the neuronal improvements of fuzzy systems as well as the fuzzification of neural network systems are applied in this work. The fuzzy neural networks aim at exploiting the complementary nature of the two approaches; the fuzzy and neural network systems.

The number of the input signals to a fuzzy neural network had better be reduced to save the time necessary to train the fuzzy neural network. Principal component analysis (PCA)[13,14] is used to reduce the dimension of an input space without losing a significant amount of information. Also, PCA has the characteristics to reduce the excessive sensitivity of the fuzzy neural networks to input parameter change. Fuzzy system parameters such as membership functions and the connectives between layers in neural networks are trained by two methods to minimize the errors (root mean squared error and/or maximum error) between the target values and the estimated values. A back-propagation algorithm is applied to optimize the membership function parameters and a least-squares algorithm to optimize the connectives between network layers.

To train and test the fuzzy neural networks, the collapse moment-related data should be provided. These data are obtained by performing a finite element analysis for various loading conditions and defect geometries such as the thinning defect locations of extrados and intrados, bend radius, wall thickness at the thinning defect, thinning length, thinning angle, internal pressure, and bending modes of closing and opening. The collapse moment is predicted using these loading conditions and defect geometries as the inputs into the fuzzy neural networks.

## 2. Calculation of Collapse Moment Using Finite Element Analysis

### 2.1. Analysis Condition

In order to evaluate the collapse load of wall thinned elbows, the carbon steel elbow that has outer radius ($D_o$) of 400 mm and nominal thickness ($t_{nom}$) of 20 mm was selected (refer to Fig. 1). Yield stress and ultimate tensile stress of selected material of elbow are 302 MPa and 450 MPa, and the elastic modulus and the Poisson ratio are 206 GPa and 0.3, respectively.

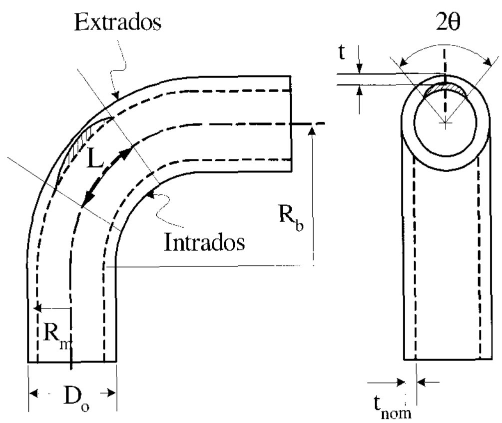



Fig. 1 Definition of dimensions of wall thinning defect in elbows

In the analysis, various loading conditions and defect geometries were considered as summarized in Table 1. In the loading condition, the moment was applied by end point displacement under a constant internal pressure until the stress of wall thinned area exceeds the ultimate tensile stress or the reaction force is reduced.

A three dimensional quarter model with

20-node brick element was considered in the finite element analysis. The quarter model represents a quarter portion of an elbow that consists of the semicircle at the circumference and a half long in length by cutting out a plane perpendicular to the flow direction at the center bending position of elbow (refer to Fig. 2). The analysis was performed using a commercial finite element program, ABAQUS [15].

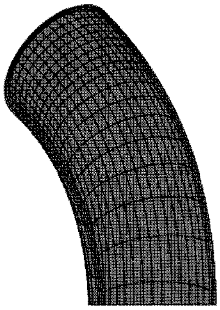


Fig. 2 A quarter model for the finite element analysis

2.2. Collapse Moment

In this analysis, both local and global stress criteria were employed as failure criteria for a collapse moment of wall thinned elbow subjected to combined internal pressure and bending moment. Firstly, thus, the collapse moment was determined by local stress criterion assuming that the collapse occurs when the local stress at thinning defect exceeds the ultimate tensile stress of material. Additionally, the collapse moment was determined by the global stress criterion that determines the collapse moment by twice elastic slope method[16]. Then the lower value of these collapse moments was selected as collapse moment.

## 3. Principal Component Analysis

PCA is usually used to reduce the number of input variables into the fuzzy neural networks. The lower dimensional input space has a merit to reduce the time necessary to train the fuzzy neural networks. PCA can facilitate the selection of the input signals to the neuro-fuzzy inference system. Also, PCA has the characteristics to reduce the excessive sensitivity of the fuzzy neural networks to input parameter change because it has the characteristics of smoothing signals and eliminating noises. In this work, the main purpose of PCA application is to reduce the sensitivity.

PCA is to map a multi-dimensional data set into a lower dimensional space while minimizing the loss of information. The basic idea is to project the original space $X$ onto a lower dimensional linear subspace $Z$ spanned by the eigenvectors of the covariance matrix corresponding to the largest eigenvalues. Given a set of signals $\mathbf{X} = \lfloor \mathbf{x}_1 \quad \mathbf{x}_2 \quad \Lambda \quad \mathbf{x}_p \rfloor$ where $\mathbf{X}$ is a $n \times p$ matrix of which elements consist of $n$ samples of $p$ signals, its true covariance matrix is replaced with the sample covariance matrix $\mathbf{S}$ because it is seldom known. The eigenvalues and the orthonormal eigenvectors of the covariance

Table 1 Analysis conditions for wall-thinning elbow

| Thinning Location | Bend Radius ($R_b/R_m$) | Defect Geometry | | | Load | |
|---|---|---|---|---|---|---|
| | | $L/D_o$ | $t/t_{nom}$ | $\theta/\pi$ | Bending Mode | Pressure[MPa] |
| Extrados | 3.0 | 0.25 | 0.233 | 0.0626 | Closing | 0 |
| Intrados | 6.0 | 0.5 | 0.466 | 0.125 | Opening | 5 |
| | | 1.0 | 0.699 | 0.25 | | 10 |
| | | 2.0 | | 0.50 | | 15 |

matrix **S** are calculated, and then the eigenvalues are arranged according to their magnitude, $\lambda_1 \geq \lambda_2 \geq \Lambda \geq \lambda_p$. The respective eigenvectors $\mathbf{p}_1, \mathbf{p}_2, \cdots, \mathbf{p}_p$ are called the principal components. The eigenvalues are proportional to the amount of variance (information) represented by the corresponding principal component. The transformation to the principal component space can be written as:

$$\mathbf{Z} = \mathbf{X}\,\mathbf{P} \qquad (1)$$

where $\mathbf{Z} = [\mathbf{z}_1\ \mathbf{z}_2\ \Lambda\ \mathbf{z}_m\ ]$ and $\mathbf{P} = [\mathbf{p}_1\ \mathbf{p}_2\ \Lambda\ \mathbf{p}_m]$

The feature matrix **Z** can be transformed back into the original data **X** without a loss of information as long as the number of features, $m$, is equal to the dimension of the original space, $p$. For $m < p$, some information is usually lost. The objective is to choose a small $m$ that does not lose much information. In this work, the feature vector that is calculated by Eq. (1) is used as inputs to the fuzzy neural networks.

## 4. Fuzzy Neural Networks

The fuzzy inference system is constructed from a collection of fuzzy *if-then* rules. An artificial neural network is usually defined as a network composed of a large number of simple processors (neuron) that are massively interconnected, operate in parallel and learn from experience. A system that consists of a fuzzy inference system implemented in the framework of neural network is usually called an adaptive network-based fuzzy inference system (ANFIS) or fuzzy neural networks [17] In this work, the fuzzy neural network is used to predict the collapse moment of the wall-thinned elbows and the training of the fuzzy neural network is accomplished by a hybrid method combined with a back-propagation algorithm and a least-squares algorithm. Also, a Sugeno-Takagi type [18] fuzzy inference system is used where the $i$-th rule can be described as follows:

$$\text{If } x_1 \text{ is } A_{i1}\ AND\ \Lambda\ \ AND\ x_m \text{ is } A_{im}, \text{ then } \hat{y}^i \text{ is } f^i(x_1, \Lambda\ , x_m), \qquad (2)$$

where $x_j$ is the input variables to the fuzzy neural network ($j = 1, 2, ..., m$ ; $m$ = the number of input variables), $A_{ij}$ the membership functions for the antecedent of the $i$-th rule and $j$-th input ($i = 1, 2, ..., n$ ; $n$ = the number of rules), and $\hat{y}^i$ the output of the $i$-th rule.

In Eq. (2), the *if* part is fuzzy linguistic, while the *then* part is crisp. Usually $f^i(x_1, \Lambda\ , x_m)$ is a polynomial in the input variables but it can be any function as long as it can appropriately describe the output of the fuzzy inference system within the fuzzy region specified by the antecedent of the rule. In this work, the symmetric Gaussian membership function is used. The output of an arbitrary $i$-th rule, $f^i$, consists of the first-order polynomial of inputs as given in Eq. (3).

$$f^i(x_1 \Lambda\ , x_m) = \sum_{j=1}^{m} q_{ij} x_j + r_i, \qquad (3)$$

where $q_{ij}$ is the weighting value of the $j$-th input on the $i$-th rule output and $r_i$ is the bias of the $i$-th output. So the fuzzy inference rule expressed by Eqs. (2) and (3) is called a first-order Sugeno-Takagi type fuzzy rule.

The estimated output of a fuzzy inference system for a specific case $k$ is expressed by a weighted sum of the consequent of all the fuzzy rules:

$$\hat{y}_k = \sum_{i=1}^{n} \overline{w}^i f^i = \mathbf{w}_k^T \mathbf{q} \qquad (4)$$

where

$$\overline{w}^i = \frac{w^i}{\sum_{i=1}^{n} w^i},$$

$$w^i = \prod_{j=1}^{m} A_{ij}(x_j),$$

$$\mathbf{q} = [q_{11} \Lambda\ q_{n1} \Lambda\ \Lambda\ q_{1m} \Lambda\ q_{nm}\ r_1 \Lambda\ r_n]^T,$$

$$\mathbf{w}_k = [\overline{w}^1 x_1 \Lambda\ \overline{w}^n x_1 \Lambda\ \Lambda\ \overline{w}^1 x_m \Lambda\ \overline{w}^n x_m\ \overline{w}^1 \Lambda\ \overline{w}^n]^T.$$

For a series of different cases, the estimated output is given by

$$\hat{\mathbf{y}} = \mathbf{W}\mathbf{q}, \tag{5}$$

where

$$\hat{\mathbf{y}} = [\hat{y}_1\ \hat{y}_2 \Lambda\ \ \hat{y}_N]^T$$

$$\mathbf{W} = [\mathbf{w}_1\ \mathbf{w}_2 \Lambda\ \ \mathbf{w}_N]^T.$$

$\mathbf{q}$ is called the consequent parameter vector and the matrix $\mathbf{W}$ consists of the input data and the membership function values. A series of the estimated outputs of the fuzzy neural network is represented by the $N \times (m+1)n$-dimensional matrix $\mathbf{W}$ and the $(m+1)n$-dimensional parameter vector $\mathbf{q}$.

The back-propagation algorithm that uses a gradient descent method is a general method for recursively training the fuzzy neural networks. The gradient descent method tunes the antecedent parameters (the center position of membership functions and their sharpness) so that the predefined objective function $E$ is minimized. In order to train an antecedent parameter $a_{ij}$, the following iterative calculation is used:

$$a_{ij}(t+1) = a_{ij}(t) - \eta_a \frac{\partial E}{\partial a_{ij}}\Big|_t \tag{6}$$

where $E = \sum_{k=1}^{N} (y_k - \hat{y}_k)^2$, $i = 1, 2, ..., n$, $j = 1, 2, ..., m$, $t = 0, 1, 2, ...$, and $\eta_a$ is a learning rate for a parameter $a$. The gradient descent method is very stable when the learning rate is small but susceptible to local minimum.

If the antecedent parameters of the fuzzy inference system are fixed by the back-propagation algorithm, the resulting fuzzy neural networks is equivalent to a series of expansions of some basis functions. This basis function expansion is linear in its adjustable parameters. Therefore, the least-squares method is used to determine the remaining parameters (consequent parameters $q_{ij}$ and $r_i$ ). If a total number of $N$ input-output training data are given, from Eq. (5) the consequent parameters are chosen to minimize the following cost function including the squared error between the target output and the estimated output:

$$J = \frac{1}{2}(\mathbf{y} - \hat{\mathbf{y}})^2 \tag{7}$$

where

$$\hat{\mathbf{y}} = [\hat{y}_1\ \hat{y}_2 \Lambda\ \ \hat{y}_N]^T,$$

$$\mathbf{y} = [y_1\ y_2 \Lambda\ \ y_N]^T.$$

$\hat{\mathbf{y}}$ is the estimated output and $\mathbf{y}$ is the output data vector which is used as target outputs.

The equation for minimizing the cost function is as follows:

$$\mathbf{y} = \mathbf{W}\mathbf{q}. \tag{8}$$

The parameter vector $\mathbf{q}$ in Eq. (8) is solved by using the pseudo-inverse of the matrix $\mathbf{W}$.

## 5. Application to the Collapse Moment Estimation

As described in Section 2, finite element analysis was performed to provide the training data and test data for each loading condition and defect geometry case to the fuzzy neural networks. The provided data comprise a total of 1536 input-output data pairs $(x_1, x_2, \Lambda, x_6, y_r)$. The characteristic of the collapse moment is much different according to the two thinning defect locations of extrados and intrados. Therefore, the data are classified into two classes and two fuzzy neural networks are designed for the two classes, respectively. $x_1$ through $x_6$ are the input signals that represent the bend radius, wall thickness at the thinning defect, thinning length, thinning angle, internal pressure, and bending modes of closing and opening. $y_r$ is the output signal which indicates the collapse moment. Also, the data are divided into the training and test data sets. The test data set comprise one tenth of the acquired input-output data pairs and the training data set consists of the remaining data. The ranges of the input and output signals that are used for the training and test data, in this work, are described in Table 1.

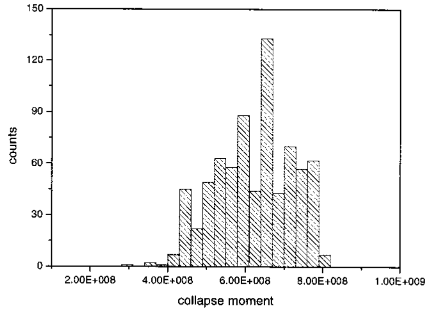
The two fuzzy neural networks are trained for two kinds of data sets divided into both the extrados and intrados defect locations, respectively, which has smaller errors compared with results using only one data set. Both the numbers of rules of the two fuzzy neural networks are 35. The antecedent parameters such as membership function parameters are optimized by the back-propagation method and the consequent parameters $q_{ij}$ and $r$ are optimized by the least-squares method. The inputs to the fuzzy neural networks are preprocessed by PCA and the transformed input signals are applied to the fuzzy neural networks.

Figure 3 shows the collapse moment and its estimation error histograms of the training and test data for *extrados* defects. The error histogram resembles the Gaussian distribution. Based on this distribution, the relative 2-sigma errors are 2.81% for the training data and 2.69% for the test data. The magnitude of these two relative errors is almost the same. Figure 4 shows the collapse moment and its estimation error histograms of the training and test data for *intrados* defects. The relative 2-sigma errors are 3.36% for the training data and 5.16% for the test data. Figure 5 shows the collapse moment and its estimation errors of the training and test data in case that both the extrados and intrados defects are considered together. The relative 2-sigma errors are 3.07% for the training data and 4.12% for the test data. From this figure, it is shown that although the maximum error is large at some defect cases, the maximum error can decrease if the defect conditions change a little during the collapse moment monitoring.

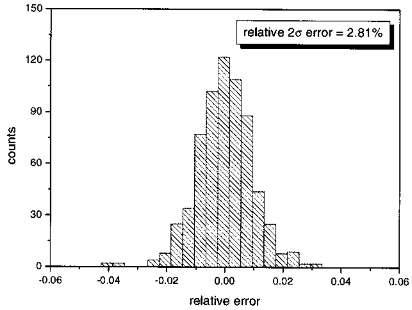
Table 2 summaries the estimation results of collapse moments by the fuzzy-neural networks. It is important to verify the fuzzy neural networks for the test data that have not been used in the training stage. It is known that the 2-sigma error of the fuzzy neural networks for the test data is similar to the 2-sigma error for the training data. Therefore, if the fuzzy neural networks are trained first using data for a variety of loading conditions and defect geometry cases, they can accurately estimate the collapse moment for any other defect cases.

## 6. Conclusions

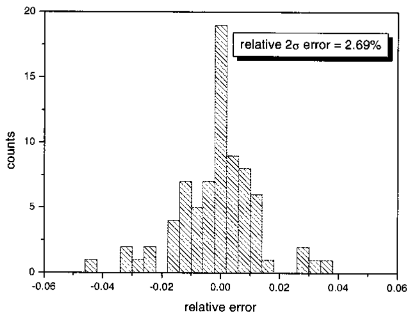
In this paper, fuzzy neural networks have been used to estimate the collapse moment due to the wall-thinning defects of elbows in piping systems. The developed fuzzy neural networks have been applied to the numerical data obtained by the finite element analysis. PCA was used to preprocess the input signals to the fuzzy neural network and the fuzzy neural networks were trained by using the data set prepared for training (training data) and verified by using another data set (test data) different (independent) from the training data. Also, two fuzzy neural networks were trained for two data sets divided into the two classes of extrados and intrados defects. The relative 2-sigma errors are 3.07% for the training data and 4.12% for the test data. The 2-sigma error of the fuzzy neural networks for the test data is similar to the 2-sigma error for the training data. Therefore, if the fuzzy neural networks are trained first by using a number of data including a variety of loading conditions and defect geometry cases, they can accurately estimate the collapse moment for any other defect cases.




(a) Actual collapse moment histogram

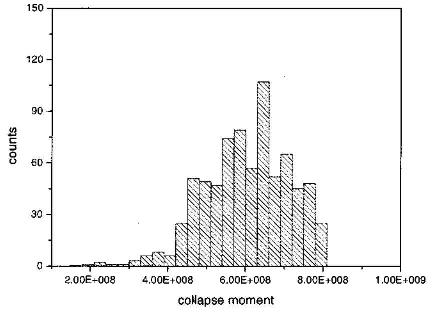



(b) Error histogram between actual collapse moment and estimated one for *training* data

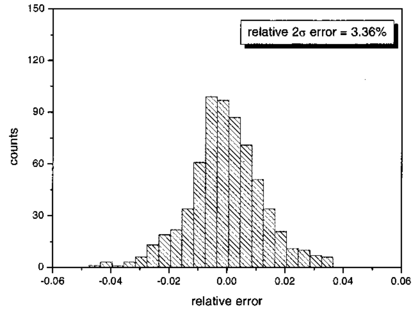



(c) Error histogram between actual collapse moment and estimated one for *test* data

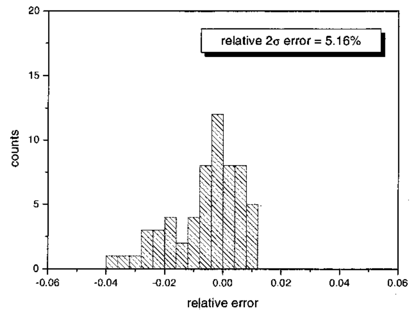
Fig. 3 Estimation performance of the fuzzy neural network for *extrados* defects




(a) Actual collapse moment histogram




(b) Error histogram between actual collapse moment and estimated one for *training* data

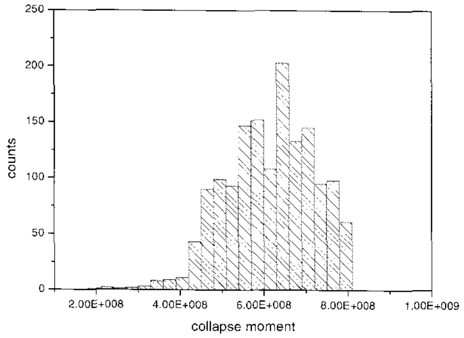



(c) Error histogram between actual collapse moment and estimated one for *test* data

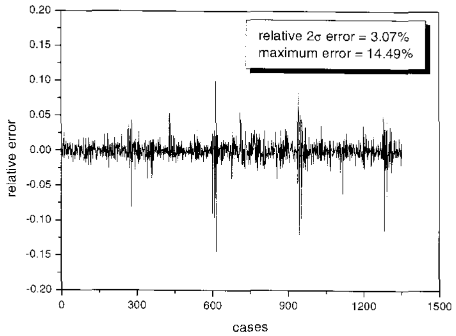
Fig. 4 Estimation performance of the fuzzy neural network for *intrados* defects

Table 2 Estimation results of collapse moment by the fuzzy-neural networks

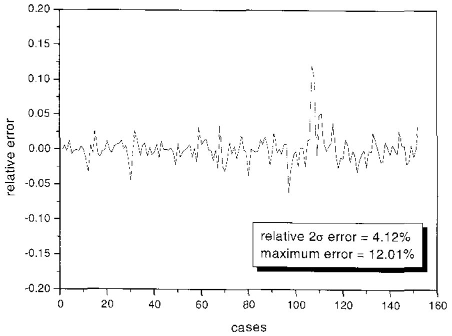
| | Training data | | Test data | |
|---|---|---|---|---|
| | Relative maximum error(%) | Relative $2\sigma$ error(%) | Relative maximum error(%) | Relative $2\sigma$ error(%) |
| Extrados defects | 14.49 | 2.81 | 4.48 | 2.69 |
| Intrados defects | 12.02 | 3.36 | 12.01 | 5.16 |
| Total | 14.49 | 3.07 | 12.01 | 4.12 |




(a) Actual collapse moment histogram




(b) Errors between actual collapse moment and estimated one for *training* data




(c) Errors between actual collapse moment and estimated one for *test* data

Fig. 5 Estimation performance of the fuzzy neural network for both *extrados* and *intrados* defects

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