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불안정 비선형 시불변 시스템을 위한 퍼지제어기가 결합된 적응제어기

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An Adaptive Controller Cooperating with Fuzzy Controller for Unstable Nonlinear Time-invariant Systems

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Abstract : A new adaptive controller which combines a model reference adaptive controller (MRAC) and a fuzzy controller is developed for unstable nonlinear time-invariant systems. The fuzzy controller is used to analyze and to compensate the nonlinear time-invariant characteristics of the plant. The MRAC is applied to control the linear time-invariant subsystem of the unknown plant, where the nonlinear time-invariant plant is supposed to comprise a nonlinear time-invariant subsystem and a linear time-invariant subsystem. The stability analysis for the overall system is discussed in view of global asymptotic stability. In conclusion, the unknown nonlinear time-invariant plant can be controlled by the new adaptive control theory such that the output error of the given plant converges to zero asymptotically.

Key words : Nonlinear Control(비선형제어), Fuzzy Control(퍼지제어), Adaptive Control(적응 제어)

1. Introduction

If the plant is a unknown nonlinear time- invariant system which cannot be approximated as a linear time-invariant model, most adaptive control theories could not be applied because of mathematical restrictions[1],[2]. A few applicable adaptive control theories deal with the robust stability problem[3]-[5] to compensate the nonlinear time-invariant characteristics of the plant and can achieve stability of overall system. However their performance cannot be satisfied because they have no specific compensation tools for the nonlinear time-invariant characteristics. To solve this problem fuzzy control theories[6],[7]

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can be introduced. They could be especially useful where the plant is unknown or too complex to be analyzed by model-based control theories. If these fuzzy control theories, however, need input/output data in order to establish fuzzy logic control structures, they could not be applied to control an unstable nonlinear time- invariant system.

A new adaptive control theory which combines a fuzzy controller and the MRAC is developed. The fuzzy controller within the new control theory is used to analyze and to compensate the nonlinear time-invariant characteristics of the plant. It is supposed that a given nonlinear time-invariant plant comprises a nonlinear time-invariant subsystem and a linear time-invariant subsystem. The fuzzy identification method suggested by Takaki and Sugeno[8] is adopted to model the nonlinear time-invariant characteristic which is considered as the steady state output error. Here the error generator is assumed to generate the output error of the MRAC system in steady state and is modeled as a fuzzy model. A fuzzy control system with state feedback is then designed in order to make the output of the error generator converge to zero asymptotically. In conclusion, the unknown nonlinear time-invariant plant can be controlled by the new adaptive control theory such that the output error of the given plant converges to zero asymptotically.

2. Model Reference Adaptive Control - Ideal Case (relative degree n*=1)

When a plant is linear time-invariant, the standard structure of the MRAC system is shown in Fig. 1. The plant is represented by linear time-invariant differential equations

$$\dot{\boldsymbol{x}}_p = \boldsymbol{A}_p\boldsymbol{x}_p + \boldsymbol{b}_p u \qquad (1)$$
$$y_p = \boldsymbol{h}_p^T\boldsymbol{x}_p$$

where $x_p : R^+ \rightarrow R^n$ is the n-dimensional state vector, $u : R^+ \rightarrow R$ is the input, $y_p : R^+ \rightarrow R$ is the output. The transfer

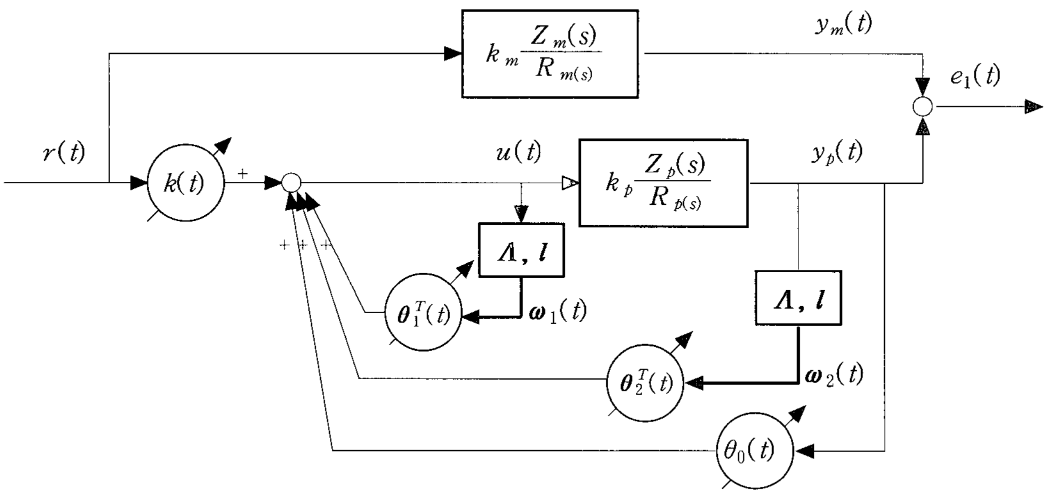


Fig. 1 The standard structure of the model reference adaptive control

function $W_p(s)$ of the plant is represented as

$$W_p(s) = \boldsymbol{h}_p^T(s\boldsymbol{I}-\boldsymbol{A}_p)^{-1}\boldsymbol{b}_p \triangleq k_p\frac{Z_p(s)}{R_p(s)} \quad (2)$$

where $W_p(s)$ is strictly proper with $Z_p(s)$ a monic Hurwitz polynomial of degree $m(\le n-1)$, $R_p(s)$ a monic polynomial of degree n, and k_p a constant parameter. Here it is assumed that only m, n and the sign of k_p are known as a priori information.

A reference model represents the behavior expected from the plant when it is augmented with a suitable controller. The model has a reference input $r(t)$ and an output $y_m(t)$. The input $r(t)$ is piecewise continuous and uniformly bounded. The transfer function of the model is defined as

$$W_m(s) = \boldsymbol{h}_m^T(s\boldsymbol{I}-\boldsymbol{A}_m)^{-1}\boldsymbol{b}_m \triangleq k_m\frac{Z_m(s)}{R_m(s)} \quad (3)$$

where $Z_m(s)$ and $R_m(s)$ are monic Hurwitz polynomials of degree n-1 and n respectively, and k_m is a positive constant.

The control structure must be chosen so that constant values of the controller parameters exist for which perfect regulation or tracking is achieved asymptotically. The controller is composed of a gain $k(t)$, the feedforward control loop with the parameter vector $\boldsymbol{\theta}_1(t)$ and the feedback control loop with the parameter $\theta_0(t)$ and parameter vector $\boldsymbol{\theta}_2(t)$. It is described completely by the following differential equations.

$$\begin{aligned} \dot{\boldsymbol{\omega}}_1(t) &= \boldsymbol{\Lambda}\,\boldsymbol{\omega}_1(t) + \boldsymbol{l}u(t) \\ \dot{\boldsymbol{\omega}}_2(t) &= \boldsymbol{\Lambda}\,\boldsymbol{\omega}_2(t) + \boldsymbol{l}y_p(t) \\ u(t) &= \boldsymbol{\theta}^T(t)\,\boldsymbol{\omega}(t) \end{aligned} \quad (4)$$

where $\boldsymbol{\omega}(t) \triangleq [r(t),\ \boldsymbol{\omega}_1^T(t),\ y_p(t),\ \boldsymbol{\omega}_2^T(t)]^T$, $\boldsymbol{\theta}(t) \triangleq [k(t),\ \boldsymbol{\theta}_1^T(t),\ \theta_0(t),\ \boldsymbol{\theta}_2^T(t)]^T$, $k: R^+\to R$, $\boldsymbol{\theta}_1, \boldsymbol{\omega}_1: R^+\to R^{n-1}$, $\theta_0: R^+\to R$, $\boldsymbol{\theta}_2, \boldsymbol{\omega}_2: R^+\to R^{n-1}$ and Λ is an (n-1 × n-1) stable matrix arbitrarily chosen by a controller designer. Therefore, the overall control system combining (1) with (4) can be represented by the following equations.

$$\begin{bmatrix} \dot{\boldsymbol{x}}_p \\ \dot{\boldsymbol{\omega}}_1 \\ \dot{\boldsymbol{\omega}}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_p & 0 & 0 \\ 0 & \Lambda & 0 \\ \boldsymbol{l}\boldsymbol{h}_p^T & 0 & \Lambda \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_p \\ \boldsymbol{\omega}_1 \\ \boldsymbol{\omega}_2 \end{bmatrix} + \begin{bmatrix} \boldsymbol{b}_p \\ \boldsymbol{l} \\ 0 \end{bmatrix} [\boldsymbol{\theta}^T(t)\,\boldsymbol{\omega}(t)]$$

$$y_p = \boldsymbol{h}_p^T\boldsymbol{x}_p\ . \quad (5)$$

When the following parameter errors are defined as

$$\begin{aligned} &\psi(t) \triangleq k(t)-k^*,\ \ \phi_0(t) \triangleq \theta_0(t)-\theta_0^*, \\ &\boldsymbol{\phi}_1(t) = \boldsymbol{\theta}_1(t)-\boldsymbol{\theta}_1^*\ \boldsymbol{\phi}_2(t) \triangleq \boldsymbol{\theta}_2(t)-\boldsymbol{\theta}_2^*, \\ &\boldsymbol{\phi}(t) \triangleq [\psi(t), \boldsymbol{\phi}_1^T(t), \phi_0(t), \boldsymbol{\phi}_2^T(t)]^T \end{aligned}$$

the state equation (5) can also be written as

$$\dot{\boldsymbol{x}} = \boldsymbol{A}_c\boldsymbol{x} + \boldsymbol{b}_c[k^*r + \boldsymbol{\phi}^T\boldsymbol{\omega}]; \quad y_p = \boldsymbol{h}_c^T\boldsymbol{x} \quad (6)$$

where $\boldsymbol{x} = [\boldsymbol{x}_p^T,\ \boldsymbol{\omega}_1^T,\ \boldsymbol{\omega}_2^T]^T$, $\boldsymbol{h}_c = [\boldsymbol{h}_p^T, 0, 0]^T$,

$$\boldsymbol{A}_c = \begin{bmatrix} \boldsymbol{A}_p + \theta_0^*\boldsymbol{b}_p\boldsymbol{h}_p^T & \boldsymbol{b}_p\boldsymbol{\theta}_1^{*T} & \boldsymbol{b}_p\boldsymbol{\theta}_2^{*T} \\ \boldsymbol{l}\theta_0^*\boldsymbol{h}_p^T & \Lambda + \boldsymbol{l}\boldsymbol{\theta}_1^{*T} & \boldsymbol{\theta}_2^{*T} \\ \boldsymbol{l}\boldsymbol{h}_p^T & 0 & \Lambda \end{bmatrix}, \quad (7)$$

$$\boldsymbol{b}_c = [\ \boldsymbol{b}_p\ \ \boldsymbol{l}\ \ 0]^T.$$

When $\boldsymbol{\phi}(t) = \boldsymbol{0}$ that is $\boldsymbol{\theta}(t) = \boldsymbol{\theta}^*$, (6) represents the reference model nonminimaly which can be described by the $(3n-2)$th order differential equation.

$$\dot{\boldsymbol{x}}_{mc}=\boldsymbol{A}_c\boldsymbol{x}_{mc}+\boldsymbol{b}_c k^* r;\quad y_m=\boldsymbol{h}_c^T\boldsymbol{x}_{mc} \tag{8}$$

where $\boldsymbol{x}_{mc}=[\boldsymbol{x}_p^{*^T},\boldsymbol{\omega}_1^{*^T},\boldsymbol{\omega}_2^{*^T}]^T$,

$$\boldsymbol{h}_c^T(s\boldsymbol{I}-\boldsymbol{A}_c)^{-1}\boldsymbol{b}_c=\frac{k_p}{k_m}W_m(s).$$

Subtracting (8) from (6), the error equation between the reference model and the plant can be obtained as

$$\dot{\boldsymbol{e}}(t)=\boldsymbol{A}_c\boldsymbol{e}(t)+\boldsymbol{b}_c[\boldsymbol{\phi}^T(t)\boldsymbol{\omega}(t)] \tag{9}$$
$$e_1(t)=\boldsymbol{h}_c^T\boldsymbol{e}(t)$$

where $\boldsymbol{e}(t)\boldsymbol{x}(t)-\boldsymbol{x}_{mc}(t)$ is the state error and $e_1=y_p-y_m$ is the output error. The output error e_1 is expressed as the following equation.

$$e_1(t)=\frac{k_p}{k_m}W_m(s)\boldsymbol{\phi}^T(t)\boldsymbol{\omega}(t) \tag{10}$$

Furthermore, the reference model can be chosen as (3) so that its transfer function $W_m(s)$ is strictly positive real(SPR), $(\boldsymbol{A}_c,\boldsymbol{b}_c)$ is stabilizable and $(\boldsymbol{h}_c^T,\boldsymbol{A}_c)$ is detectable. Therefore, an adaptive control law can be derived from the Lyapunov stability theory using the Meyer-Kalman-Yakubovich lemma. That is, the parameter error vector $\boldsymbol{\phi}(t)$ is updated according to the following adaptive control law[1]

$$\dot{\boldsymbol{\phi}}=\dot{\boldsymbol{\theta}}=-sgn(k_p)e_1(t)\boldsymbol{\omega}(t) \tag{11}$$

and the equilibrium state $(\boldsymbol{e}=\boldsymbol{0},\boldsymbol{\phi}=\boldsymbol{0})$ of (9) and (11) is globally uniformly stable.

Since e_1 as well as the output y_m of the reference model are bounded, y_p is bounded and $\boldsymbol{\omega}(t)$ is bounded so that $\boldsymbol{e}(t)\rightarrow\boldsymbol{0}$ as $t\rightarrow\infty$ or $|e_1(t)|\rightarrow 0$ as $t\rightarrow\infty$. In conclusion, the equilibrium state of the MRAC system is globally asymptotically stable.

3. Derivation of a New Adaptive Fuzzy Controller

3.1 The Basic Analysis and Assumptions for the Plant

When a plant is modeled as linear time invariant, the standard MRAC theory can be used to control the given unknown plant. If the plant evolves nonlinear time-invariant characteristics, it may be impossible to control it using the control theory developed in chapter 2 because some required assumptions are not satisfied. Intuitively, it may be assumed that the output error would be generated by the nonlinear time-invariant characteristics of the plant.

Assumption 3.1

An arbitrary nonlinear time-invariant plant is assumed to be composed of a linear time-invariant subsystem and a nonlinear time-invariant subsystem. Then the nonlinear time-invariant characteristics of the plant are dependent only upon the nonlinear time-invariant subsystem.

Even for nonlinear systems whose mathematical models cannot be separated into linear and nonlinear terms explicitly, they might be supposed to satisfy assumption 3.1 because they are composed of linear first order terms and nonlinear higher order terms when they are expanded into Taylor series at a fixed time.

Under this assumption, the input/output relation for the plant can be described as in Fig. 2.

Thus the output of the plant y_p is described as

$$y_p = y_{pL} + y_{pN}. \qquad (12)$$

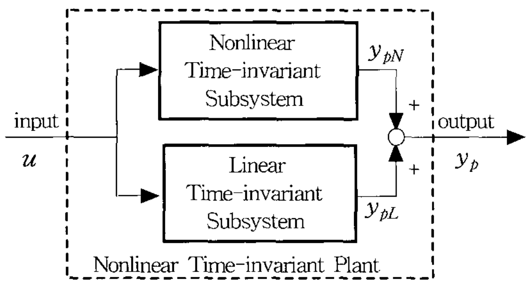


Fig. 2 Input/output relation for the plant under the assumption 3.1

Assumption 3.2

Although the plant which satisfies assumption 3.1 has nonlinear time-invariant characteristics, if it is not known, the standard MRAC theory can still be applied under the assumption that it is linear time-invariant.

When the standard MRAC is applied to the plant described by the (12), the structure of the control system is expressed as Fig. 3.

Then the output error e_1 in Fig. 3 is expressed as

$$e_1 = y_p - y_m = y_{pL} - y_m + y_{pN} \qquad (13)$$

If the nonlinear time-invariant subsystem does not appear in the plant, y_{pN} is naturally equal to zero. In this case the plant is linear time- invariant and the output error e_1 is given as

$$e_1 = y_p - y_m = y_{pL} - y_m \qquad (14)$$

and the steady state output error will converge to zero, that is $\lim_{t\to\infty} e_1 = 0$, by the control action of the standard MRAC. Where the nonlinear time-invariant characteristics are contained in the plant, the steady state error will not converge to zero. In this case the steady state error can be considered as the output of the nonlinear time-invariant subsystem and it can also be considered as the output error of the overall control system in steady state. That is,

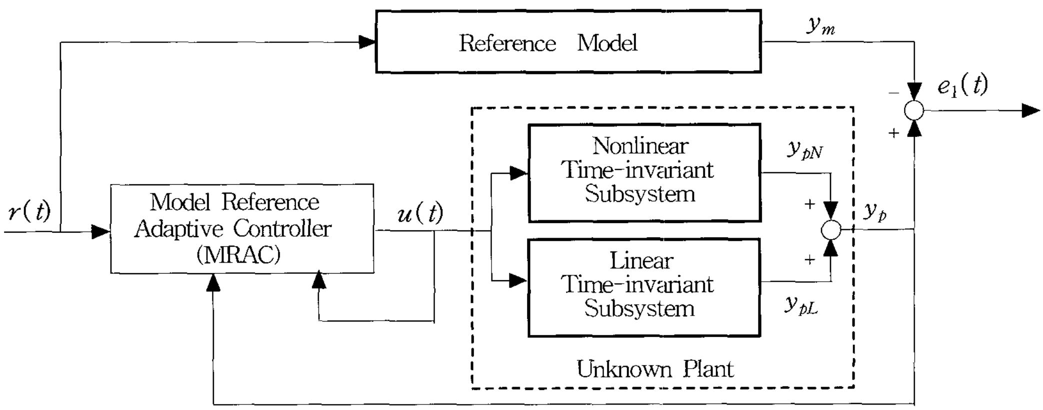


Fig. 3 The standard MRAC control system under the assumption 3.2

$$\lim_{t\to\infty} e_1(t) = \lim_{t\to\infty}[(y_{pL} - y_m) + y_{pN}] = \lim_{t\to\infty}(y_{pL} - y_m) + \lim_{t\to\infty} y_{pN} = \lim_{t\to\infty} y_{pN} \quad (15)$$

If a method exists, which makes the steady state output error $\lim_{t\to\infty} y_{pN}$ converge to zero, the nonlinear time-invariant plant analyzed as Fig. 2 could be controlled completely within the standard MRAC structure.

Assumption 3.3

The nonlinear time-invariant subsystem in Fig. 2 is considered as the error generator which generates the output error of the MRAC system in steady state, in the case where the unknown plant is assumed to be linear time-invariant.

Based on the assumption 3.3, (13) can be expressed as

$$e_1 = y_p - y_m = (y_{pL} - y_m) + y_{pN} = e_{1A} + e_{1N} \quad (16)$$

where $e_{1A} = y_{pL} - y_m$ is the output error of the standard MRAC when the plant is time- invariant, $e_{1N} = y_{pN}$ is the output of the error generator. Then the nonlinear time-invariant unknown plant can be substituted into Fig. 3 as shown in Fig. 4.

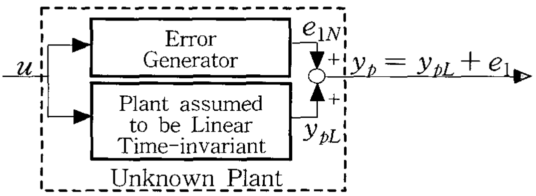


Fig. 4 A description of the unknown plant under the MRAC

The control aim is to find a method which generate an additional control input such that the output of the error generator converges to zero in steady state, that is $\lim_{t\to\infty} e_{1N}(t) = 0$.

3.2 Fuzzy Identification for the Error Generator and Design of a Fuzzy Controller

If the behavior of the error generator is analyzed, it could be achieved to obtain an additional control input so that the output of error generator asymptotically converges to zero in steady state. The error generator of the unknown plant is considered as a fuzzy model and is identified using a fuzzy identification method. A fuzzy controller is designed such that the output of the identified fuzzy error generator goes to zero and resultantly the additional control input added to the control input of the standard MRAC is obtained.

3.2.1 Fuzzy identification of the error generator

Takaki and Sugeno's fuzzy model[8] is adopted as a fuzzy model and it is identified according to the identification steps in Fig. 5 suggested by Sugeno and Kang[9].

Takaki and Sugeno's fuzzy model is composed of fuzzy IF-THEN rules which represent locally linear input/output relations of the error generator.

The ith rule is expressed as follows.

Rule i :

IF $e_1(k)$ *is* F_1^i and $\cdots$ and $e_1(k-m+1)$ *is* F_m^i

$$THEN\, e_1^i(k+1) = a_1^i e_1(k) + a_2^i e_1(k-1) + \cdots + a_m^i e_1(k-m+1) + b^i u_f^i(k) \quad (17)$$

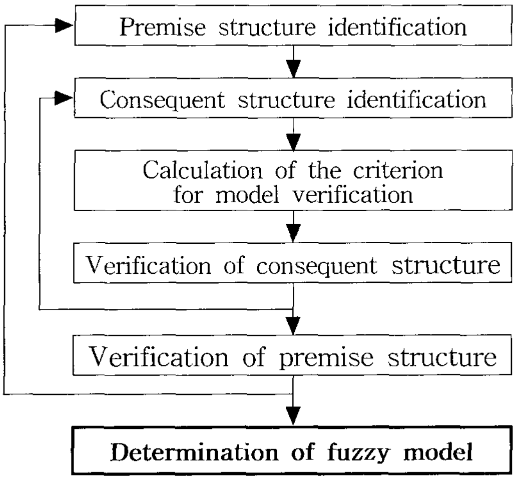


Fig. 5 The identification procedure of a fuzzy model

where $i=1, 2, \cdots, l$, $e_1(k-j+1)(j=1, 2, \cdots, m)$ are state variables of the fuzzy model, a_j^i and b^i are consequent parameters, and F_j^i are fuzzy sets of which membership functions are represented as continuous piecewise-polynomial functions.

If the consequent part of (17) is expressed in vector-matrix notation, it can be written as follows.

Rule i :
IF $e_1(k)$ *is* F_1^i and ⋯ and $e_1(k-m+1)$ *is* F_m^i

$$THEN\ \boldsymbol{e}_1^i(k+1) = \boldsymbol{F}_i\, \boldsymbol{e}_1(k) + \boldsymbol{B}_i\, u_f^i(k) \qquad (18)$$

where $\boldsymbol{e}_1(k)=[e_1(k), e_1(k-1), \cdots, e_1(k-m+1)]^T$ is the state vector of the fuzzy model and $\boldsymbol{e}_1^i(k+1)= [e_1^i(k+1), e_1^i(k), e_1^i(k-1), \cdots, e_1^i(k-m)]^T$ is the output from the ith rule. When a pair of $\{ \boldsymbol{e}_1(k),\ u_f^i(k) \}$ is given, the final output of the fuzzy system is inferred as follows.

$$\boldsymbol{e}_1^i(k+1)=\frac{\sum_{i=1}^{l}\xi_i(k)\ [\boldsymbol{F}_i\boldsymbol{e}_1(k)+\boldsymbol{B}_i u_f^i(k)]}{\sum_{i=1}^{l}\xi_i(k)} \qquad (19)$$

where $\xi_i(k)= \prod_{j=1}^{m} F_j^i(e_1(k-j+1))$ and $F_j^i(e_1(k-j+1))$ is the grade of membership of $e_1(k-j+1)$ in F_j^i.

Let us assume in this paper that

$\sum_{i=1}^{l} \xi_i(k) > 0$ and $\xi_i(k) \geq 0$ for $i=1,2,\cdots,l$

for all k. Each linear consequent equation represented by linear discrete notation $\boldsymbol{F}_i\, \boldsymbol{e}_1(k)$ is called 'subsystem of the error generator'.

In conclusion, the fuzzy system given as (19) is the fuzzy representation of the nonlinear time-invariant characteristic which is evolved by the error generator. This fuzzy model is important in two aspects. First, it is used as the base model of the fuzzy control system which generates the input to regulate the output of the error generator. Second, it is used to prove the global asymptotic stability for the fuzzy control system.

3.2.2 Design of a fuzzy controller to stabilize the output of the error generator

The fuzzy system identified as (19) presents the nonlinear time-invariant characteristic which is evolved by the error generator. Thus if a fuzzy controller is designed and a regulation input is obtained so that the output $\boldsymbol{e}_1(k+1)$ of (19) converges to zero in steady state, the overall control system could be controlled in a stable fashion. To do this, let consider a fuzzy control system described as Fig. 6.

Since the design purpose of the fuzzy controller is to make the output of the error generator converge to zero in steady state, an external input to the error generator is assumed to be zero. That is, the regulation problem of a free fuzzy system is considered. If an ith control rule is assumed to act only on the same i th rule of the fuzzy system, the following control rules are used in this paper.

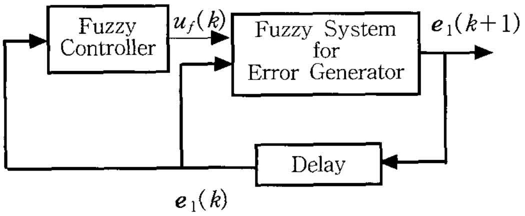


Fig. 6 A fuzzy control system to regulate the error generator

Control rule i :
$IF e_1(k)$ *is Any* and $\cdots$ and $e_1(k-m+1)$ *is Any*

$$THEN\ u_f^i(k) = -K_i e_1(k)\ i = 1, 2, \cdots, l. \qquad (20)$$

where K_i is a proportional feedback gain for the control rule i and *'Any'* is a fuzzy set whose membership function $Any\{e_1(\cdot)\}$ is 1.0 for all $e_1(\cdot)$. This type of proportional controller is known as a special case of a fuzzy proportional controller.

In order to compose a fuzzy control system, each control rule given as (20) is combined into the corresponding subsystem given as (18) for the same i. Thus the ith subsystem of the fuzzy control system is expressed as

Control subsystem i :
$IF\ e_1(k)$ *is* F_1^i and $\cdots$ and $e_1(k-m+1)$ *is* F_m^i

$$THEN\ e_1^i(k+1) = (F_i - B_i K_i) e_1(k). \qquad (21)$$

The resultant output of the fuzzy control system can be obtained as follows.

$$e_1(k+1) = \frac{\sum_{i=1}^{l} \xi_i(k)\ T_i\ e_1(k)}{\sum_{i=1}^{l} \xi_i(k)} \qquad (22)$$

where $T_i = F_i - B_i K_i$.

Since the number of control subsystems corresponds to the number of fuzzy subsystems of the error generator, that is $i = 1, 2, \cdots, l$, the stability analysis of the fuzzy control system is quite simple. This type of rule has the characteristic that the control input $u_f^i(k)$ is applied directly to the corresponding subsystem regardless of premise parameter condition. Therefore, the resultant fuzzy control input $u_f(k)$ must be calculated using the same method that each subsystem controlled by the control $u_f^i(k)$ participates in the resultant output of the fuzzy control system with the weight $\xi_i(k)$. That is,

$$u_f(k) = \frac{\sum_{i=1}^{l} \xi_i(k)\, u_f^i(k)}{\sum_{i=1}^{l} \xi_i(k)} = -\frac{\sum_{i=1}^{l} \xi_i(k)\, K_i e_1(k)}{\sum_{i=1}^{l} \xi_i(k)} \qquad (23)$$

In conclusion, the design problem of the fuzzy controller is to decide the feedback gains K_i to stabilize the given fuzzy system, or in other words, to decide the fuzzy control input $u_f(k)$ in order to make the output of the error generator converge to zero in steady state.

3.3 A New Control Structure Named Model Reference Adaptive Fuzzy Control (MRAFC) System

3.3.1 The structure of MRAFC system

Fig. 7 shows the overall control system that the fuzzy control system is combined with the MRAC system in order to control the given nonlinear time-invariant plant.

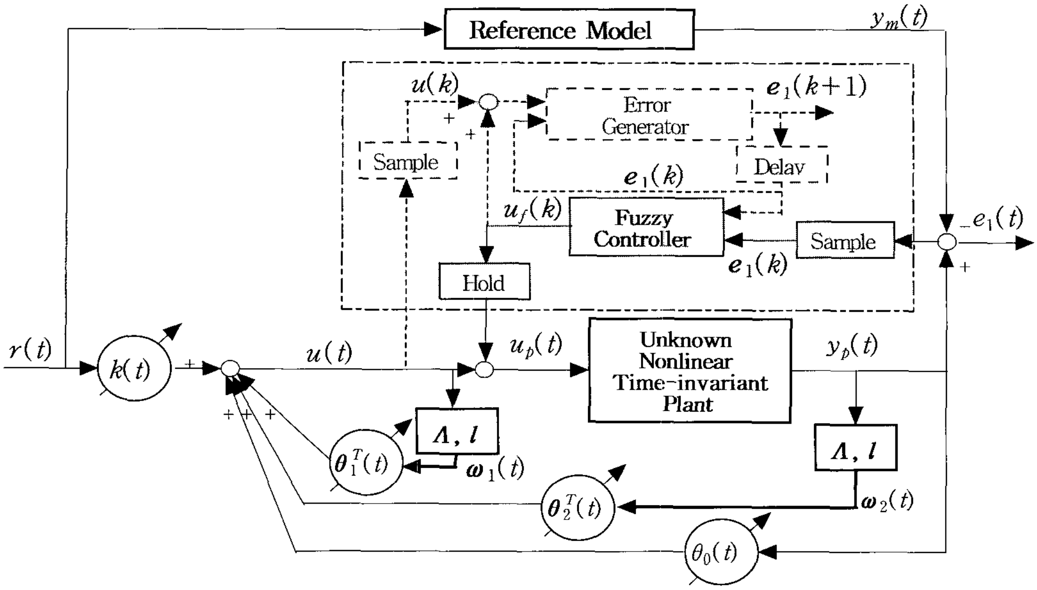


Fig. 7 The structure of the Model Reference Adaptive Fuzzy Control (MRAFC)system

From Fig. 7 the total control input $u_p(t)$ can be obtained by adding the additional control input $u_f(t)$ from the fuzzy controller to the control input $u(t)$ from the MRAC, that is,

$$u_p(t) = u(t) + u_f(t) \tag{24}$$

$$u_f(t) = u_f(k) \cdot \Delta T$$

where ΔT is the sampling period for the fuzzy control system.

It is more useful and exact to use the actual output error $e_1(t)$ than to use the error generator output. The signals expressed by the dashed line '----' in Fig. 7 are not used actually for control action but used only for developing the fuzzy control system discussed in the previous section. The fuzzy model for the error generator is used as a basic mathematical model when the stability for the fuzzy control system is analyzed. If the nonlinear time-invariant characteristic of the given plant is modeled exactly as a fuzzy error generator and if the stability of the fuzzy control system is proved, the nonlinear time-invariant plant could be controlled with global stability.

3.3.2 The stability analysis of the MRAFC system

The fuzzy control system in the MRAC structure regulates the error generator which is represented as a fuzzy model and is assumed to generate the nonlinear time-invariant characteristic of the plant. Thus it is very important to prove the global asymptotic stability of the fuzzy control system.

Theorem 3.1

The equilibrium of a fuzzy free system

of (19) when $u_f^i(k)=0$ is globally asymptotically stable if there exists a common positive definite matrix $\boldsymbol{P}$ for all the subsystems such that

$$\boldsymbol{F}_i^T \boldsymbol{P} \boldsymbol{F}_i - \boldsymbol{P} < 0 \quad \text{for} \quad i=1,2,\cdots,l. \tag{25}$$

Theorem 3.2

The equilibrium of a fuzzy control system expressed as (22) is globally asymptotically stable if there exists a common positive definite matrix $\boldsymbol{P}$ for all subsystems such that

$$\boldsymbol{T}_i^T \boldsymbol{P} \boldsymbol{T}_i - \boldsymbol{P} < 0 \tag{26}$$

where $i=1,2,\cdots,l$.

Proof. The proof follows directly from the theorem 3.1 if the feedback matrices $\boldsymbol{K}_i\ (i=1,2,\cdots,l)$ are selected such that all the resultant matrices $\boldsymbol{T}_i$ of the fuzzy control system satisfy the condition (26) for a common positive matrix $\boldsymbol{P}$. Therefore, as long as $\boldsymbol{T}_i$ satisfy the condition (26), the fuzzy control system (22) to regulate the error generator is always globally asymptotically stable.

Theorem 3.3

The equilibrium state of the standard MRAC system is globally asymptotically stable along the trajectories (9) and (11), if the fuzzy control system of the MRAFC structure satisfies the theorem 3.2 under the assumptions 3.1 and 3.3.

Proof. If the assumptions 3.1 and 3.3 are satisfied, we can write the output error $e_1(t)$ as

$$e_1(t) = e_{1A}(t) + e_{1N}(t). \tag{27}$$

Then the output error in steady state can be expressed as

$$\begin{aligned}\lim_{t\to\infty} e_1(t) &= \lim_{t\to\infty} [e_{1A}(t) + e_{1N}(t)] \\ &= \lim_{t\to\infty} e_{1A}(t) + \lim_{t\to\infty} e_{1N}(t)\end{aligned} \tag{28}$$

It was assumed that the error generator is modeled as a fuzzy model using input/output data in steady state and it also generates the steady state error which is caused by the nonlinear time-invariant characteristic. Thus, if the fuzzy control system is asymptotically stable, then

$$\lim_{t\to\infty} e_1(t) = 0 \quad \text{and} \quad \lim_{t\to\infty} e_{1N}(t) = 0. \tag{29}$$

Therefore, the following result can be obtained from (28).

$$\lim_{t\to\infty} e_{1A}(t) = \lim_{t\to\infty} (y_{pL} - y_m) = 0 \tag{30}$$

This means the bounded condition

$$\lim_{t\to\infty} \int_0^t |e_{1A}(\tau)|\, d\tau < \infty \tag{31}$$

must be satisfied because the unboundedness of the limit (31) contradicts (30). From (30) and (31), the output error $e_{1A}(t)$ of the MRAC for the linear time-invariant subsystem belongs to $L^1 \cap L^\infty$ and hence it is uniformly bounded[2]. Since $W_m(s)$ in (10) is a stable matrix and hence $e_{1A}(t)$ and $\boldsymbol{\phi}^T(t)\boldsymbol{\omega}(t)$ grow at the same rate[1], $\boldsymbol{\phi}^T(t)\boldsymbol{\omega}(t)$ is also uniformly bounded. These mean that all the signals in the standard MRAC are bounded as long as y_m is bounded. Therefore, the adaptive law given as (11) holds true and the equilibrium state of the MRAC system along (9) and (11) is globally asymptotically stable.

In fact, $e_1(t)$ is used instead of $e_{1A}(t)$ which cannot actually be separated from $e_1(t)$ in Fig. 7. Nevertheless, all the internal signals of the MRAC are also bounded as long as $e_1(t)$ is bounded and converges asymptotically to zero.

In conclusion, if the error generator is identified as a fuzzy model with confidence and the feedback matrices $\boldsymbol{K}_i$ are decided so that the fuzzy control system is globally asymptotically stable, the overall MRAFC system is globally asymptotically stable and it can control the given unstable nonlinear time-invariant plant.

4. Simulations

In this chapter, two simple plant models are adopted in order to test and to verify the control performance and the efficiency of the suggested MRAFC structure.

4.1 Simulation 1

A plant to be controlled is expressed as the following unstable nonlinear differential equation with a bounded disturbance.

plant : $\ddot{y}_p = \dot{y}_p + 0.5y_p^2 + \dot{u} + u + v$

$y_p(0) = 1,\ \dot{y}_p(0) = 0$

disturbance : $v = 0.5\sin t + e_1\cos 2t + 0.5e_1^2\cos t$

In order to acquire input/output data from the MRAC system for the identification, the standard MRAC must be applied to the given plant under the assumption that the given plant is linear time-invariant. A reference model was conveniently chosen to satisfy SPR condition since the relative degree of the given plant is $n^* = 1$. To analyze the MRAC system, two kinds of reference signals was used. The unit step function was used to analyze the transient response and the sinusoidal function with two distinct frequencies was used to analyze the tracking performance.

reference model : $\dot{y}_m = -y_m + r,\ \ y_m(0) = 0$

unit step reference input : $r = u_s(t)$

sinusoidal reference input : $r = \cos t + 5\cos 5t$

Input/output data were acquired for the MRAC system when the given sinusoidal input as the reference input was used.

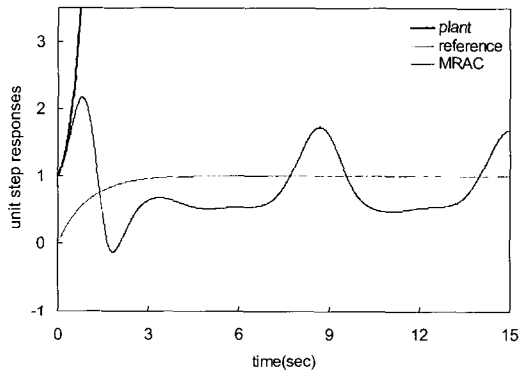


Fig. 8 presents the unit step responses of the given plant, the reference model and the MRAC system.

Fig. 9 shows the time responses of the given plant, the reference model and the MRAC system for the sinusoidal input which is persistently excited. According to the Fig. 8 and Fig. 9, the outputs of the standard MRAC system cannot follow those of the reference model but they are bounded within a certain limit. Since the plant output is bounded by the MRAC in spite of the unstable plant, it is possible to acquire input/output data from the plant in the MRAC system so as to identify an error generator.

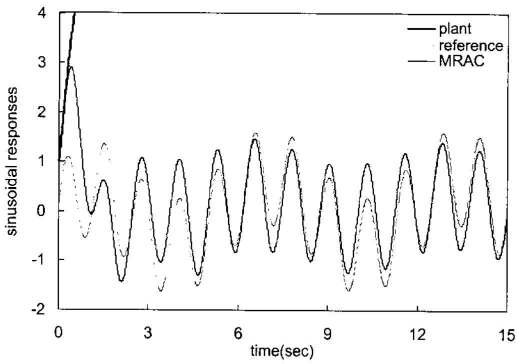
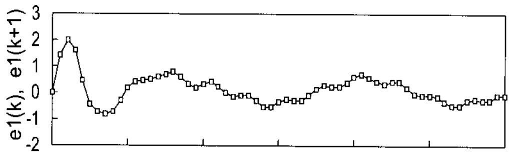
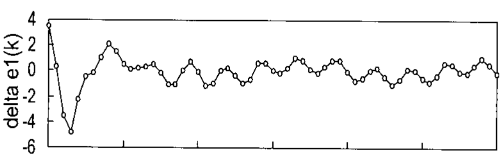


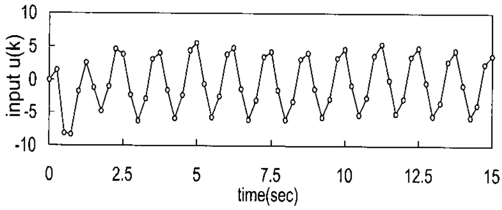
Fig. 9 Output comparison for the persistently excited sinusoidal input



(a) $e_1(k)$ and $e_1(k+1)$



(b) $\Delta e_1(k)$

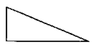


(c) $u(k)$

Fig. 10 Input/output data used for the identification of an error generator

Fig. 10 shows the data that are used for the identification of the error generator as a fuzzy model when the sinusoidal reference input is used. The sampling period $\Delta T = 0.1$ second was used.

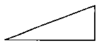
Using the data $e_1(k)$, $\Delta e_1(k)$ $(= [e_1(k) - e_1(k-1)]/\Delta T)$, $u(k)$ and $e_1(k+1)$ acquired from the MRAC, Takaki and Sugeno's fuzzy model for the error generator was identified according to the procedure presented in Fig. 5. Although five fuzzy models dependent upon the partitions of the input spaces for the $e_1(k)$ and $\Delta e_1(k)$ were identified, the following simple model was chosen as the resultant identification model for the error generator, which has the least performance index defined as the root mean square of the output errors.

Rule 1 : *IF* $e_1(k)$ is 

-1.0 2.5

THEN $e_1^1(k+1) = 0.9988\, e_1(k) + 0.0096\, \Delta e_1(k) + 0.009\, u_f^1(k)$

$= 1.0948\, e_1(k) - 0.096\, e_1(k-1) + 0.009\, u_f^1(k)$

Rule 2 : *IF* $e_1(k)$ is 

-1.0 2.5

THEN $e_1^2(k+1) = 0.997\, e_1(k) + 0.0109\, \Delta e_1(k) - 0.013\, u_f^2(k)$

$= 1.106\, e_1(k) - 0.109\, e_1(k-1) - 0.013\, u_f^2(k)$

The feedback matrices for fuzzy control rules were selected such that the damping ratios are nearly 0.7 to regulate the consequent equations of the above fuzzy subsystems, that is, $\boldsymbol{K}_1 = [21.0 \quad 22.9]$ and $\boldsymbol{K}_2 = [-12.0 \quad -15.64]$. Therefore, the resultant fuzzy control input can be calculated by using (23). The additional control input from the fuzzy controller which is added to the control input from the MRAC, can be calculated by (24).

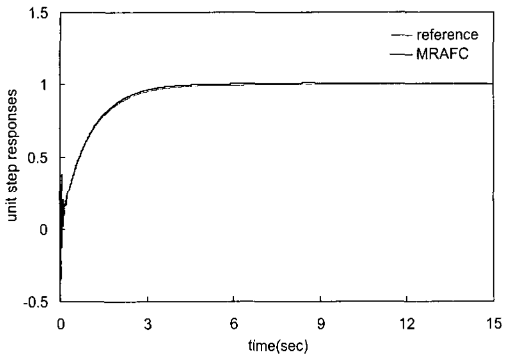


Fig. 11 Output comparison for the unit step reference input

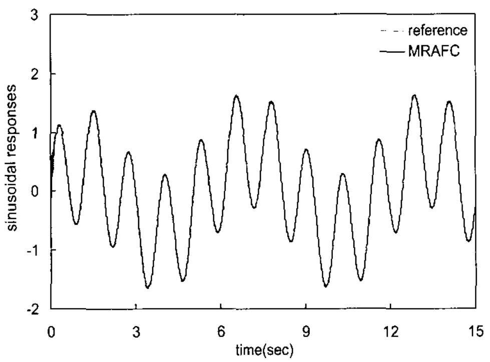


Fig. 12 Output comparison for the persistently excited sinusoidal input

Fig. 11 and Fig. 12 show the transient responses of the suggested MRAFC system which are compared with those of the reference model for the step input and the persistently excited sinusoidal input, respectively. As can be seen, they follow the outputs of the reference model very well and thus exhibit good tracking and steady state behavior even if the given plant has a highly nonlinear characteristic.

It is necessary to check whether the overall control system is stable or not. This is performed through demonstrating the asymptotic stability of the fuzzy control system. For the following matrices $\boldsymbol{T}_1$ and $\boldsymbol{T}_2$ of the fuzzy control subsystems, if $\boldsymbol{T}_1=\begin{bmatrix}0.906 & -0.302\\ 1 & 0\end{bmatrix}$, $\boldsymbol{T}_2=\begin{bmatrix}0.950 & -0.312\\ 1 & 0\end{bmatrix}$ a positive definite matrix $\boldsymbol{P}=\begin{bmatrix}2.5 & -1\\ -1 & 1\end{bmatrix}$ is selected, then the condition (26) is always satisfied for $i=1,2$. Therefore, the fuzzy control system is asymptotically stable and thus the MRAFC system is globally asymptotically stable.

In conclusion, the proposed method can be applied to control unstable nonlinear plants with global asymptotic stability, as long as some assumptions in section 3.1 are satisfied.

4.2 Simulation 2

A simulation was accomplished on an unstable nonlinear time-varying plant with the following mathematical model. The same reference model was adopted and the same disturbance was used as in simulation 1.

Plant : $\ddot{y}_p = \dot{y}_p + (2.0+\cos t)y_p^2 + \dot{u} + u + v$

$$y_p(0) = 1,\ \dot{y}_p(0) = 0$$

Fig. 13 and Fig. 14 demonstrate the time responses of the given plant and the MRAC compared with that of the reference model for the unit step input and sinusoidal input, respectively. As can be seen, the plant outputs diverge to infinity because it is unstable. The outputs of the MRAC do not follow those of the reference model but remain within finite limits. Therefore, it is possible to apply the suggested method to control the given plant.

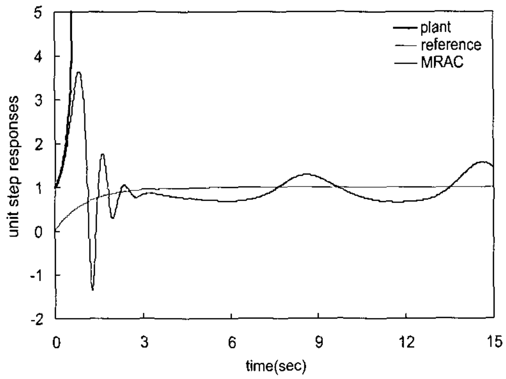


Fig. 13 Output comparison for the unit step reference input

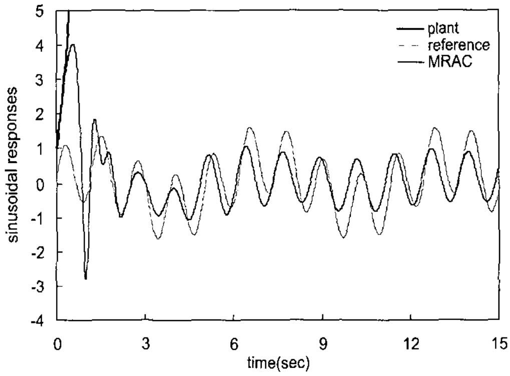
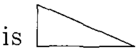


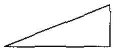
Fig. 14 Output comparison for the persistently excited sinusoidal input

The following fuzzy model is an error model which is assumed to evolve the nonlinear bounded time-varying characteristic of the given plant. It was identified using the data obtained from the output error of Fig. 14.

Rule 1 : *IF* $\Delta e_1(k)$ is 

-3.5 8.0

$$THEN\ e_1^1(k+1) = 1.002\, e_1(k) + 0.009\, \Delta e_1(k) + 0.01\, u_f^1(k) = 1.092\, e_1(k) - 0.09\, e_1(k-1) + 0.01\, u_f^1(k)$$

Rule 2 : *IF* $\Delta e_1(k)$ is 

-3.5 8.0

$$THEN\ e_1^2(k+1) = 1.001\, e_1(k) + 0.01\, \Delta e_1(k) - 0.002\, u_f^2(k) = 1.101\, e_1(k) - 0.1\, e_1(k-1) - 0.002\, u_f^2(k)$$

In order to regulate the above fuzzy subsystems, the feedback matrices K_1 and K_2 are selected as $\boldsymbol{K}_1 = [19.0 \quad 21.9]$ and $\boldsymbol{K}_2 = [-57.0 \quad -112.0]$. The resultant matrices $\boldsymbol{T}_1$ and $\boldsymbol{T}_2$ of the fuzzy control subsystems are given as

$$\boldsymbol{T}_1 = \begin{bmatrix} 0.902 & -0.309 \\ 1 & 0 \end{bmatrix},\ \boldsymbol{T}_2 = \begin{bmatrix} 0.987 & 0.324 \\ 1 & 0 \end{bmatrix}.$$

For these system matrices if a positive definite matrix $\boldsymbol{P} = \begin{bmatrix} 2.5 & -1 \\ -1 & 1 \end{bmatrix}$ is selected, the asymptotic stability condition (26) is always satisfied for $i = 1, 2$. Therefore, since the fuzzy control system for the error generator is asymptotically stable, the overall MRAFC system is globally asymptotically stable.

Fig. 15 and Fig. 16 present the time responses of the MRAFC system compared with those of the reference model. As expected, the transient response and the tracking performance are enhanced that the MRAFC can follow the reference model. Even though the given system is a nonlinear time-varying plant, the responses of the MRAFC are nearly equal to those of the nonlinear plant in simulation 1.

Therefore, it is concluded that the MRAFC structure can be applied to control unknown unstable nonlinear time-varying plants if only an error model is identified correctly and a fuzzy control system is asymptotically stable.

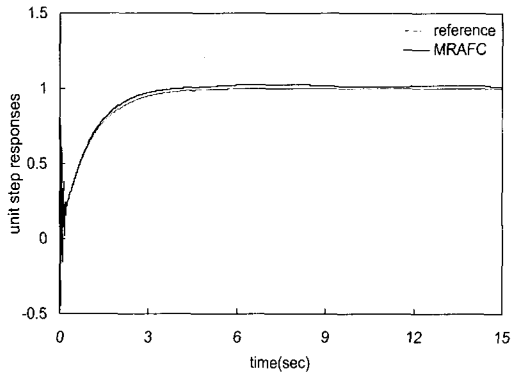


Fig. 15 Output comparison for the unit step reference input

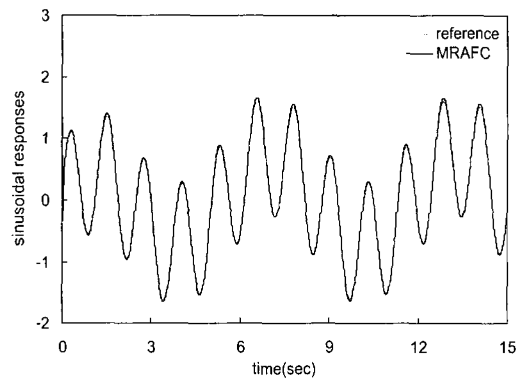


Fig. 16 Output comparison for the persistently excited sinusoidal input

5. Conclusion

A new adaptive control theory was developed for unstable nonlinear time-invariant plants such that a fuzzy control system is combined with the standard model reference adaptive control theory. The fuzzy control system was used to compensate the nonlinear time-invariant characteristic of the given plant which is assumed be the output of an error generator. To achieve this purpose, the fuzzy identification method was adopted and the additional control input was generated such that the output of the identified error generator converges to zero asymptotically.

By means of the simulation results, it was verified that the suggested MRAFC could improve the transient response of the given unstable nonlinear and/or time-invariant plant with global asymptotic stability. That is, the transient and steady state output of the MRAFC system followed that of the reference model quite well.

Although it may not easy to carry out the identification procedure for the fuzzy model of an error generator, nevertheless, if the fuzzy model with the lowest performance index is identified, the given system can be easily controlled by using the well-known linear control theory. Also, since the identification procedure is carried out in off-line, it does not increase the computational burden.

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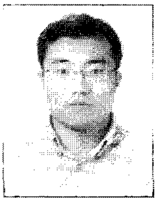

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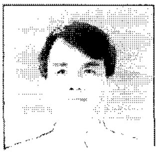

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