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All or Nothing: Problem Solving High Achievers in Mathematics[1]

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This article describes the first results gained from a problem solving test that was administered to grade 4 students in the Netherlands. The students involved are all high achievers in mathematics. The analysis of the student responses gives cause for concern. The often-heard belief that teachers do not need to worry about the better students is clearly in need of revision. It turned out that when high achievers in mathematics are challenged to take on non-typical problems, their abilities are more limited than expected. The study revealed that the students wrote down hardly anything on their scrap paper to solve certain problems. Also it was found that they were not very persistent in their looking for a solution. In this paper we illustrate these first findings by discussing the results of one of the test problems.

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INTRODUCTION

In primary schools in the Netherlands, the focus is generally more on weak and average students than on high achievers in mathematics. Often it is thought that the bright students can help themselves and do not need so much assistance. This approach

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[2] From October 2004 though March 2005.

to clever children is also reflected in research of mathematics education. For example, until now no single study was carried out in the Netherlands to investigate how well our best primary school students actually perform.

Therefore we seized the opportunity to comply with the request that came from the University of Leeds[3] to investigate how Dutch high-achieving fourth-graders will do problem solving tasks that have been developed for the World Class Tests. In this paper we report on our experiences with the administration of these tasks and the first results we got from analyzing the data.

METHOD

In collaboration with Peter Pool and John Trelfall, 15 problems were selected and translated into Dutch. These 15 problems were administered in the Netherlands in March of 2004 to a group of high achievers in grade 4. In total, 21 schools took part in the study. The students belonged to the top 20% ability range in mathematics. The teachers themselves selected the children. It was determined that the children should have scored 'good' to 'very good' in a student monitoring test. This means that the children should have, for instance, an A-score in the Dutch CITO-test *Rekenen en Wiskunde M6*. In total 152 children were selected.

The problems were chosen in such a way that their content (in number domain and sub-domains of mathematics) covered more or less the mathematics curriculum the students have dealt with so far. Notwithstanding the foregoing, the nature of these problems was somewhat special for the Dutch students. The tasks are often puzzle-like problems, such as number riddles, that we do not encounter that often in textbook series and tests in the Netherlands.

The 15 problems were put in a test booklet with every problem presented on a separate page. The children were free to use the whole page as scrap paper; and for some problems there was an explicit request to show their calculations.

The test was administered by the teachers according to a set of guidelines. We made observations in three schools during the test and held interviews with some of the children afterwards as well. We also analyzed the student work ourselves.

A SELECTION FROM OUR FINDINGS

[3] Peter Pool and John Trelfall from the Assessment and Evaluation Unit, School of Education, University of Leeds, United Kingdom, contacted us with this question. They are involved in the development of the World Class Tests,

When we looked at the student work in the test booklets, we noticed that quite a lot of the children did not make use of the draft space when solving the problems. Since they did not have any other scrap paper available, this means that they worked on the problems without writing anything down. This is remarkable, especially for problems that involve a lot of data or where you can find a solution by systematically trying out options. In these problems it is often practical to make notes and write down intermediate answers. An example of such a problem is the following.

Find the number
It is smaller than 100.
If you divide it by 7, there is no remainder.
If you divide it by 3, the remainder is 2.
If you divide it by 5, the remainder is 1.

Figure 1. *Find the number* problem

In table 1 it can be seen that this problem was fairly difficult for the Dutch high-achieving fourth-graders that were involved in the study. Only 39 students, a quarter of the total group, found the right number[4]. Furthermore, this table reveals that 93 children, almost two thirds of the total, did not make any use of the scrap paper.

Table 1. The results from high achievers in grade 4

Results for problem 'Find the number'			
	Did not use scrap paper	Did use scrap paper	Total of students
Correct answer	19	20	39
Wrong answer	74	39	113
Total number of students	93	59	152

In this paper we want to focus on this group, the children who did not write down anything for this problem. We will do this in two parts. We will start with the students who reached the right answer without making use of the scrap paper. After that, we will look at the children who did not do so.

[4] The students in the United Kingdom did slightly better on this problem. Of the 184 grades 4–5 students (8.5 years to 9.5 years) who did this problem and who belonged to the 15–20% best students in mathematics, 34.2% came up with the correct answer (Peter Pool and John Trelfall, personal communication). The Dutch score on this problem was 25.7%. However, no significant difference between the British and Dutch students was found on the test as a whole. A more detailed report on the findings is in preparation.

Nothing written down and still the right answer

Although this problem takes a lot of calculation, the difficulty does not lie so much in the knowledge of the tables of multiplication that is required. The bright students in grade 4 often know their tables. What it comes down to in this problem, is that you have to be able to take into account a number of different number requirements at once. That is not easy. On the other hand, since we are dealing with high achievers in mathematics, it is not really surprising that roughly half the children found the right answer mentally.

This is clearly the case with Jasper. He clarified in the interview how he made his calculation. Because he often ran into trouble putting his thoughts into words, his clarification may be a bit cryptic, but his reasoning is entirely correct.

From interviewing Jasper:

From the fact that the number has a remainder of 1 when it is divided by 5, Jasper knows that the number has to end in 6 or 1. He starts with 6: "It has to be something with a 6 and then I tried the table of 7 and then I got 56." But that is not the end of Jasper's explanation, because when you divide by 3, the remainder has to be 2. So he continues with: "In the table of 3 you have 60, and therefore 54; then you have 56, with remainder 2."

Another example is Jacco. He also did not write down anything to support his solution for this problem. During the interview it became apparent that he approached the problem in a systematic manner.

From interviewing Jacco:

Jacco knows that the number has to be in the table of 7. He first tries 9×7, and then 8×7. The number 56 turns out to be right.

Nothing written down and the right answer not found.

A group that gives more cause for concern is that of the 74 children who did not find an answer and who wrote down nothing. These students did not even start the problem. An example of this group is Benny. His overall score for the test is slightly above average. In the interview we found that he has difficulty ordering his thoughts and finds it difficult as well to keep the problem he is working on in mind. Doing the calculations is a problem for him, despite his high score in the student monitoring system.

From interviewing Benny:

Benny starts by apologizing: "I could not get a couple of problems in a row. I thought it was 35, but if you divide that by five, you don't get a remainder of 1. I don't know."

After a bit of help, he writes down all the numbers in the table of 7. This does not go quickly. Having to find out whether a number results in a remainder of 2 when divided by 3 also goes rather slowly. He has to think a long time about 14 divided by 3. He feels ill at ease when writing things down. Every time, he tries to do it mentally and loses his train of thought.

In view of these problems it is almost hopeless to do the whole problem mentally, yet this is apparently the way Benny usually works.

Another example is Frank. He has 91 as the answer and has not written down anything else. The interview revealed that he has no problems with calculating mentally and that his approach is good. He works in a very systematic manner. His problem lies rather in keeping up this systematic inquiry.

From interviewing Frank:

Frank indicates that it should be a number that ends in 6 or 1 which is in the table of 7. He tries 21, that does not fit. Then he says that $13 \times 7 = 91$ and that is it if it is correct. He does not check any further.

SUMMARY OF OUR FINDINGS

We learned from the interviews that the difficulties with this problem are not related to understanding the question. The children understood the intention of the problem. Although the calculation itself did not always go as smoothly as you would expect in this group of students, that also was not the core of their problem. The weakness of these good students, we feel, lies in a different area, namely in not trying and not being able to keep up a particular approach. Moreover, this attitude is strengthened by the children apparently not being used to writing something down to support their thought process.

The trend to not write anything down

We noticed in the interviews that the children often thought for a very long time before starting to write. We had noticed this before during a number of try-outs. It looks as if the children do not start writing until they know, or almost know, the answer. It was also noticeable that some children wanted to use an eraser. We pointed out to them that we would be much more able to see how they were thinking if they did not erase anything. Some children also found it very difficult to cross things out. They want their work to stay tidy. For instance, one girl puts a wavy line under notes that on second thought turned out to be wrong. A boy writes down his notes on his ruler rather than on the paper. Comments made by the children also revealed that they find it irritating if they fail to realise straightaway that they are on the wrong track. It was also quite remarkable that one of the teachers had handed out separate scrap paper before we arrived, so that the children did not have to write down their notes in the test booklet.

The trend to not write anything down raises many questions, but after our experiences in this study we do have some idea of how to explain this behavior. For example, children (and teachers) might think that it is better not to use the paper, because they think that solving the problems mentally is a higher level of mathematics. The trend might also have to do with the fact that bright students hardly use (scrap) paper when they do their assignments in regular mathematics classes.

A very different aspect that might play a part is that the children do not write anything down because they feel that you should not 'make a mess' in a test. They learned from their teachers that it important to work tidily.

The trend to not even start

The fact that the majority of the children who could not solve this problem did not try anything on paper either is also significant. This was the case with 74 of the 113 children who did not find the right number. Except that the trend to not even start could result from the above-mentioned refusal to write down anything in solving mathematics problems, another possible explanation could be that the children have not learned to use notes and organizing data as a support for the solution process. In the case of this problem one could think of listing the numbers that qualify according to a certain rule and then strike out the numbers that do not conform. Trying things out might break through the all or nothing atmosphere that now often prevails.

The trend to not to persist

Another alarming experience is that quite a lot child did not persevere enough when attempting the problem. They often gave up after trying a few numbers. It is remarkable that this behavior also often occurs with weak students. Although the latter group makes more calculation mistakes and will not so easily reach, for instance, the conclusion that it should be a number that ends in 6 or 1. The fact that good students also find it difficult to persevere, may be caused by the fact that they do not often come across problems that require mathematical inquiry. They usually do not have to think long about the problems they normally encounter. As was mentioned at the beginning of this paper, problem solving has a marginal place in the present Dutch mathematics curriculum. We believe that this should change and hope that the start that is given for this in the TAL learning-teaching trajectory for calculation with whole numbers in primary school[5], will have an effect in teaching practice.

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[5] This learning-teaching trajectory has been described in van den Heuvel-Panhuizen, M. (Ed.) (2001).

AN INVITATION TO CONCLUDE

As a follow-up to this paper we would like to invite teachers to try the problem discussed here in their classroom and observe how their students approach it. We would like to ask them to especially pay attention to the use of scrap paper and the issue of persevering. For those who have any doubts regarding the usefulness of this problem, we would like to point out the possibilities for 'productive practice' contained in the problem. And for teachers who foresee that they will not be able to set this difficult problem for the whole group, we would like to point out the option of differentiation. Instead of setting three conditions that the number under 100 has to match, it is also an option to look for numbers that only meet one condition. This way the whole group can participate, while the good students can learn something extra at the same time.

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