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EVALUATION OF VEHICLE DYNAMIC CONTROL FOR ROLOVER PREVENTION

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ABSTRACT–Evaluation of active safety control systems usually relies heavily on field testing and is time-consuming and costly. Advances in computer simulations make it possible to perform exhaustive design trials and evaluations before field testing, and promise to dramatically reduce development cost and cycle time. In this paper, a comprehensive simulation-based evaluation procedure is proposed, which combines standard evaluation maneuvers, worst-case techniques, and a driver model for closed-loop path following evaluations. A vehicle dynamic controller (VDC) for a popular Sport Utility Vehicle is evaluated using the proposed procedure. Simulation results show that the proposed procedure can be used to assess the performance of the VDC under various conditions and provides valuable information for the re-design of the VDC.

KEY WORDS : VDC, Rollover, Stability control, Active safety

1. INTRODUCTION

The growing popularity of Sport Utility Vehicles (SUV) over the last decade together with their higher rollover tendency necessitates a closer look at regulations aimed at reducing rollover fatalities. National Highway Traffic Safety Administration (NHTSA) reported 10,647 (19.5%) rollover deaths in cars, vans and trucks in 2001 (NHTSA, 2002), up from 10,013 (18.3%) in 1997 (NHTSA, 1998). The increase in rollover deaths seems to be closely related to the increased population of SUVs and light trucks. The rollover propensity of SUVs thus has caught the attention of consumers, car companies, major suppliers, and government agencies in recent years.

Effort by the National Highway Traffic Safety Administration (NHTSA) on safety standard for rollover resistance goes back to the 1970's (NHTSA, 1973). Tremendous work has been done to establish standard procedures and regulations for vehicle roll performance assessment and minimum performance requirements. Susceptibility to rollover is a complex dynamic phenomenon involving many factors such as vehicle parameters, road conditions, and driver characteristics. The complexity and dynamic nature of this problem make it difficult to define a stability testing procedure for vehicle testing. The problem becomes more complicated when the vehicle is equipped with advanced control devices, such as the vehicle stability control systems (Zanten, 1995; Matsumoto, 1992) or anti-rollover systems (Wielenga, 1999; Hac, 2002).

To assess the on-road, un-tripped rollover propensities of vehicles, two types of metrics have been proposed in the literature. The NHTSA dynamic testing program proposes to use a set of rollover evaluation maneuvers (J-turn and three Fishhook maneuvers) to examine the frequency of vehicle Two-Wheel-Lift (TWL) (Garrick, 2002). The Consumer Union's double lane change driving test also falls into this "dynamic testing" category. On the other hand, the "static" type of rollover metrics are usually based on simple measurement of vehicle parameters related to its roll behavior. For example, Static Stability Factor (SSF), Tilt Table Angle, Tilt Table Ratio, Critical Sliding Velocity, and Side Pull Ratio all fall into this category.

One of the major findings from the NHTSA Phase IV study (Garrick, 2002) is that all path-following (driver in loop) maneuvers they tested (ISO3888, Consumer Union Short Course, Pseudo Double Lane Change) failed either in repeatability or discriminatory capability. This experience provides a valuable lesson for car companies that are designing and evaluating advanced control systems with human driver interactions: repeatable and discriminatory human-in-loop tests are very hard to conduct because of human variations. It is desirable to seek a more reliable alternative.

A driver model was developed in (Ungoren, 2004) specifically targeting VDC evaluations. This model was developed using the adaptive predictive control (APC)

Corresponding author. e-mail: hpeng@umich.edu framework. Three key features are included in the APC framework: use of preview information, internal model identification and weight adjustment to simulate different driving style. The driver uses predicted vehicle information in a future window to determine the optimal steering action. A tunable parameter is defined to assign relative importance of lateral displacement and yaw angle error in the cost function to be optimized. The model is tuned to fit three representative drivers (average, aggressive and smooth) obtained from driving simulator data taken from 22 human drivers. Including this driver model in the evaluation process enables us to evaluate the performance of VDC using virtual drivers with different characteristics.

The main contribution of this paper is the development of a three-stage approach for computer evaluations of a vehicle dynamic control (VDC) system. The procedure iterates through three stages: standard open-loop test matrix, worst-case evaluation, and human-in-loop simulations. The open-loop test matrix can include regulatory (from NHTSA) as well as company accustomed test maneuvers. The worst-case maneuvers are computed based on the Iterative Dynamic Programming technique. This method does not require system gradient, and achieves fast convergence through the use of coarse-grid-search and approximating the identified cost function through curve fitting. The human-in-loop simulations use three virtual drivers based on the model described above (Ungoren, 2004). The basic idea of this evaluation procedure is illustrated by studying the rollover performance of a VDC system.

2. VEHICLE MODEL

The vehicle studied in this paper is a popular mid-size SUV. The vehicle model was developed by TRW Automotive using CARSIM (2003) and the full-car performance has been verified against test data. The nonlinear mathematical model has 14 degrees-of-freedom (6 DOF for the sprung mass, 2DOF for each of the axles, and 1 DOF for each of the wheels) and is quite suitable for simulating vehicle response under significant (± 10 degrees) roll motions. The vehicle simulation model also includes a Vehicle Dynamic Control (VDC) algorithm designed by the TRW Automotive. This version of VDC was modified based an early production-intent design and was meant to be used solely for this research. It has most of the fundamental VDC functions but is not identical to any TRW products. The basic concept of this VDC is to enhance vehicle's yaw rate response (yaw rate following) while maintaining small vehicle side slip angle. The vehicle yaw motion is controlled by applying differential braking force either to reduce the difference between the interpreted driver's desired yaw rate and the actual yaw rate, or to reduce vehicle side slip angle. The driver's steering, throttle and braking commands are disturbance inputs that influence vehicle motions. In the meantime, they also generate reference signal to be followed by the VDC. The evaluation process needs to ensure that the VSC works well under a wide array of disturbance inputs.

The vehicle roll motion was not explicitly considered in the original VDC design. However, since the vehicle side slip motion is regulated, the vehicle with VDC usually has improved tripped and un-tripped rollover resistance, because the likelihood of building up a large lateral speed is greatly reduced. Due to the yaw rate following functionality, it is possible the vehicle with VDC will have slightly higher roll angle when the driver demands a large yaw rate on high friction roads. Due to the relative matureness of VDC technology, it is natural to extend it for rollover prevention purposes, hopefully without compromising its original design goals.

A VDC designed for its original (yaw and side slip only) goals is put through the proposed evaluation process, to be explained in the next section. If the vehicle performance was found to be unsatisfactory, an add-on anti-roll control (ARC) can be designed so that differential braking will be applied when excessive vehicle roll motion occurs. Because we do not have access to the source code of the original VDC, the add-on ARC was not designed in coordination with the yaw-lateral design part. It was only tuned based on a couple of selected maneuvers. Therefore, the revised VDC does not represent a polished design but is adequate to demonstrate the overall evaluation procedure. In this paper, we focus on the evaluation of the roll performance of VDC. In particular, the vehicle outputs we watched closely include roll angle and tire normal forces. When a rollover (roll angle >10 degrees and growing) does not occur, we examine tire normal forces. Zero tire normal force on any tire indicates the occurrence of wheel-lift-off, which is assumed to be an unacceptable roll event.

3. EVALUAITON PROCEDURE

Figure 1 shows the flowchart of the proposed iterative VDC evaluation procedure. When a VDC is designed for a particular vehicle, it will first be tested with open loop maneuvers in a standard test matrix. For example, since NHTSA suggested J-Turn and Fishhook maneuvers for dynamic rollover test, it is natural VDC designers will include the NHTSA identified maneuvers in their standard test matrix. In this paper, to simplify the results, our standard test matrix only includes two J-turns and two Fishhook maneuvers. A real test matrix can be a lot more comprehensive, depending on the preference of the company engineers. If the VDC design performs satisfactorily, it can be allowed to continue for the worst-case evaluation in Stage 2.

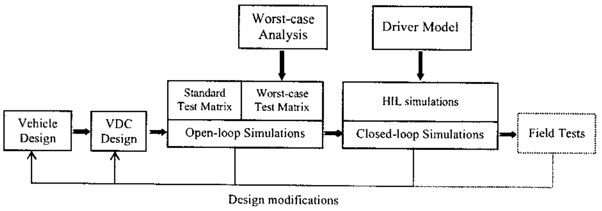

Figure 1. Iterative vehicle and VDC evaluation process.

Worst-case maneuvers (Ungoren, 2001) are computed to identify the worst possible scenario for the vehicle. Since the driver generated steering and braking inputs are modeled as disturbance signals, the worst-case maneuvers are obtained by solving an optimization problem, to identify disturbances that maximize vehicle roll motion in a pre-defined input range (steering angle and brake force). This evaluation phase aims to ensure the VDC works even when atypical inputs were generated by the driver (e.g., panic or evasive maneuvers). In general, this optimization problem is a concave problem for a vehicle described in a numerical format. Therefore, we have to solve the optimization problem numerically. The mathematical core is based on the Iterative Dynamic Programming (IDP) technique (Luus, 2000). The derivative free method with convergence characteristics within the searched grid points provides an excellent balance between computation time and convergence.

The IDP technique used in this paper consists of two phases: a dynamic programming phase and a strategy phase. In the dynamic programming phase a standard dynamic programming problem is solved over a subdomain of input and/or state space with sparse grid points. Sparse grids have to be used because of the high dimensionality (14DOF). In the strategy phase, the solutions at the sparse grid points are approximated by smooth curves, and a local optimum point is estimated. The center, direction and the size of the new search-domain for the next dynamic programming phase are then selected by using the new local optimum point and the most recent old optimum point (see Figure 2, where the red points denote estimated optimum points). The size of the search-domain reduces with the number of iterations, until the convergence rate decreases to a certain level. The domain size can then be increased to avoid trapping at local optimum. Alternating between strategy phases and dynamic programming phases usually leads to improved solutions (Pierre, 1986).

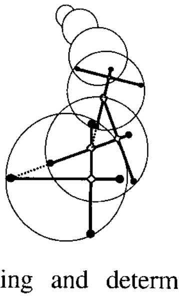

Figure 2. Curve-fitting and determination of search domain (2-dimension example).

The next stage of the evaluation procedure is the (virtual) driver-in-loop simulations. The VDC is tested with steering profiles generated by a driver model, tuned to represent three different driver characteristics. This stage uses "closed-loop simulations" because the road path to be followed is specified, and the driver model closes the loop by using the sensed path and yaw motion errors. This stage more closely resembles on-road driving compared with the open-loop simulations in the first two stages. This stage complements the first two stages by including human-VDC interactions, which is unfortunately lacking in the NHTSA proposed dynamic tests.

After each set of simulations, the designer could decide to go back to the drawing board and come back with a better design, or continue with the next stage. Once all simulation results are satisfactorily, the VDC can be sent to the final stage--field testing with the prototype controllers implemented on an actual vehicle. The field test stage is beyond the scope of this paper

4. EVALUAITON RESULTS

A simulation exercise of the proposed procedure is presented in this section. Two iterations are reported. In the first iteration, the original VDC design is tested. Based on the finding from this iteration, an anti-rollover patch is designed to improve the performance of the original VDC design. The new design is then tested in the second iteration.

4.1. Iteration 1, Stage 1: Standard Test Matrix

For this stage we use four open loop maneuvers: J-Turn, J-Turn with braking, Fishhook#1, and Fishhook#2. These maneuvers are recommended by NHTSA for dynamic rollover testing (Garrick, 2002).

The vehicle is evaluated at three different initial vehicle speeds (80, 100, and 120 kph). Hand-wheel steering angle range and maximum braking force for the simulations are set to [-80, 80] degrees and 25 lbf respectively. These levels are somewhat smaller than the NHTSA recommendations because we want to test at higher vehicle speeds. Figure 3 shows the steering angle, braking force and roll angle response both with and without VDC. The roll angle is reduced slightly by VDC, mainly because of the reduction in vehicle speed. Figure 4 shows the steering angle and roll angle of the Fishhook #1 maneuver. The vehicle without VDC rolls over, while the vehicle with VDC has a maximum roll angle of 5.83 degrees.

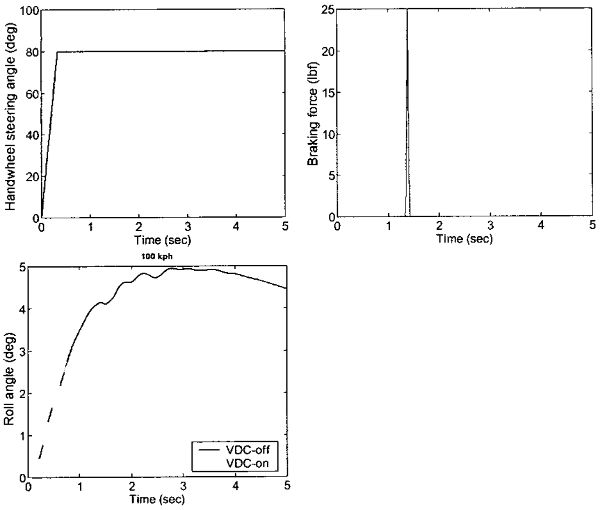

Figure 3. J-Turn with braking at 100 kph-inputs and roll response.

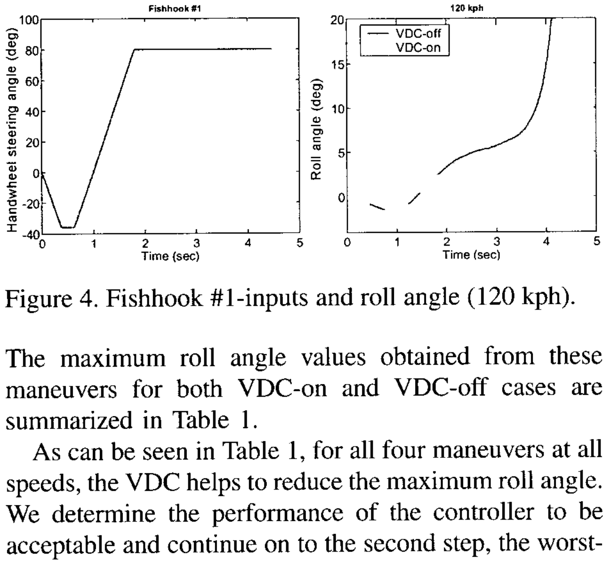

Figure 4. Fishhook #1-inputs and roll angle (120 kph).

The maximum roll angle values obtained from these maneuvers for both VDC-on and VDC-off cases are summarized in Table 1.

As can be seen in Table 1, for all four maneuvers at all speeds, the VDC helps to reduce the maximum roll angle. We determine the performance of the controller to be acceptable and continue on to the second step, the worst-case analysis.

Table 1. Vehicle speed vs. maximum roll angle.

Speed (km/hr)			80	100	120
J-Turn	VDC-off	max roll angle (deg)	3.04	4.85	rollover
	VDC-on		2.99	4.57	5.97
J-Turn with braking	VDC-off		3.18	4.95	rollover
	VDC-on		3.05	4.59	5.89
Fishhook#1	VDC-off		2.95	4.70	rollover
	VDC-on		2.95	4.53	5.83
Fishhook#2	VDC-off		2.92	4.60	rollover
	VDC-on		2.92	4.55	5.84

4.2. Iteration 1, Stage 2: Worst-Case Analysis

In this stage, the VDC is turned on all the time and the worst-case performance for the vehicle plus VDC is examined carefully. The purpose of a worst-case study is to ensure that the active safety system works well in the field, when panicked drivers, under a wide variety of conditions, could produce unpredictable steering and braking patterns. The worst-case analysis is thus valuable for all active safety products of ground vehicles. The worst-case analysis searches for the worst-case inputs (steering angle and braking force) within the same range identified in stage 1 (hand-wheel steering angle ∈[-80, 80] degrees and brake force ∈[0,25] lbf). The initial speed can be set at the same three levels as stage one ∈(-80,100 and 120 kph). However, we decide to only focus on the highest speed (120 kph) because higher forward speed always produces higher worst-case roll angle. The worst-case study is extremely time-consuming and we need to focus the limited resource on the most important case. It should be noted that the IDP technique used in this paper tries to perform search through a range that shrinks and grows, and thus is able to escape from some local optimum. However, due to the extremely large state space (14 DOF, 28 states), we start from many different initial conditions for the search. The results are multiple worst-case scenarios, rather than a single global optimum.

More than 120 input profiles, including the standard maneuvers in stage 1 tests are used as initial guesses for the worst-case maneuvers search. Figure 5 shows the steering and braking inputs of one of the identified worst-case scenarios on high friction surface, which results in a rollover (Figure 6). For comparison purposes, we use the same steering and braking inputs for the vehicle with the VDC turned off. The vehicle without VDC was found to have a maximum roll angle of 4.27 degrees (Figure 6). This is quite alarming because the maneuver does not pose a real threat for the VDC-off case. But there is a good explanation for this: this scenario resembles an evasive obstacle avoidance steering (lane change) with heavy braking, which is not that uncommon. It seems the VDC was trying to achieve yaw rate following and in the

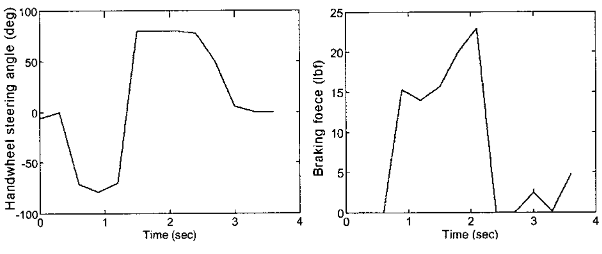

Figure 5. Worst-case hand-wheel steering and braking force for VDC.

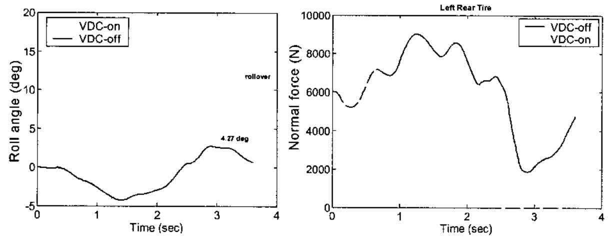

Figure 6. Roll angle and left rear tire normal force under a worst-case scenario.

process helps the vehicle to become more oversteer. A good patch for this problem is to add additional logic to ensure that the goal of yaw-rate following will be overridden when the vehicle is in danger of rollover. At this moment, we can iterate back to the design stage. However, we choose to continue to show a complete comparison of the two designs.

4.3. Iteration 1, Stage 3: Human In Loop

Designing an active safety system without considering how drivers interact with it may result in unsatisfactory performance. How the control authority is shared between the driver and the controller is a question that designers need to consider seriously. Analyzing the interaction among vehicle-controller-environment through human-in-the-loop (HIL) simulations is critical to identify possible problems in the design stage. In this section the driver model proposed in (Ungoren 2004) is used to evaluate the VDC system on a path following test. We use Toyota's peak-to-peak yaw rate measure (Toyota 2001) to investigate the driver and controller interactions. In the Toyota's method, a star rating system is suggested, which uses maximum entry speed to a Moose Test track without hitting the obstacles, together with the peak-to-peak yaw rate to determine a vehicle's performance. The obstacle avoidance, double-lane change test path is shown in Figure 7.

The proposed driver model is tuned to fit three different drivers: average, aggressive, and smooth drivers. These drivers were obtained by tuning the driver model parameters to fit the average, and average plus/minus one standard deviation of hand-wheel steering angle responses from 22 human drivers. The path performance of driver 1 (aggressive: mean+σ), driver 2 (average: mean) and driver 3 (smooth: mean$-\sigma$) over the Moose test track for the VDC-off case are shown in Figure 8. When the vehicle hits any orange cone that delineates the un-allowed region, we declare that the vehicle failed the test (see Figure 9). A driver will be put through the HIL test starting from lower initial speed. If it passed, the entrance speed will be increased by 2kph and repeat the test, until it fails. We found that driver 3 (smooth driver) fails at very low speed. Obviously, to pass this demanding test, a driver needs to respond and steer quickly.

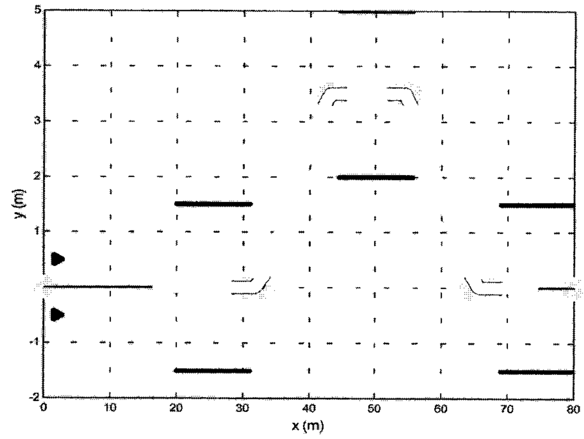

Figure 7. Moose test track used in HIL tests.

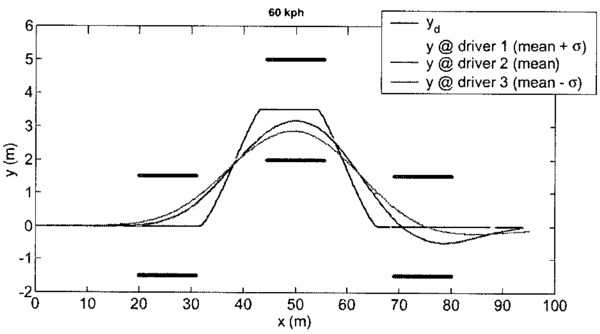

Figure 8. Trajectory of the three drivers over the Moose test track.

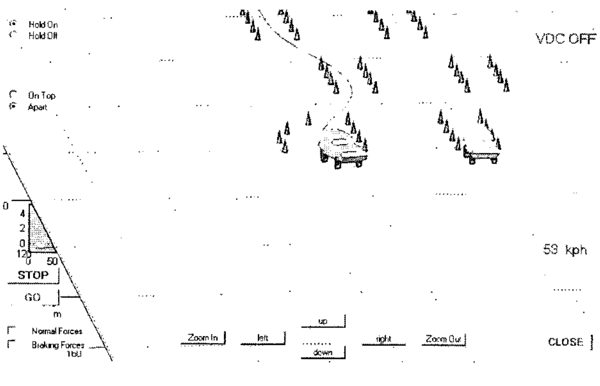

Figure 9. Animation of test results: VDC-off vs. VDC-on.

Figure 10 shows the performance of the three drivers, with and without VDC, on Toyota's suggested star-ranking system. In Figure 10, moving in the direction toward the lower right corner of the figure denotes improvement-either the entry speed is increased, or the peak-to-peak yaw rate is reduced. VDC is able to improve the performance of both driver 1 (aggressive) and driver 2 (aver-

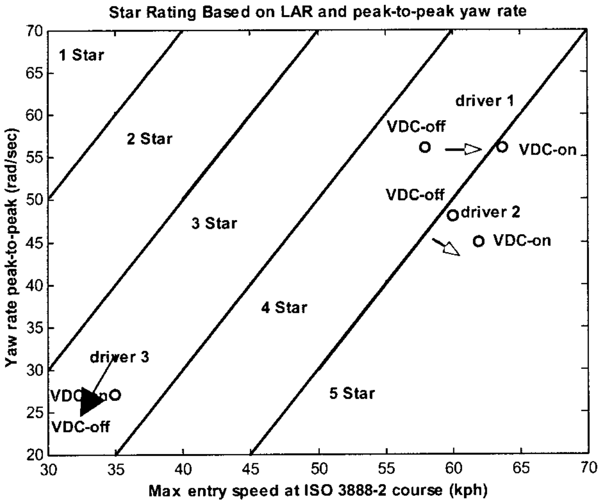

Figure 10. Star rating based on Toyota's proposed dynamic testing procedure.

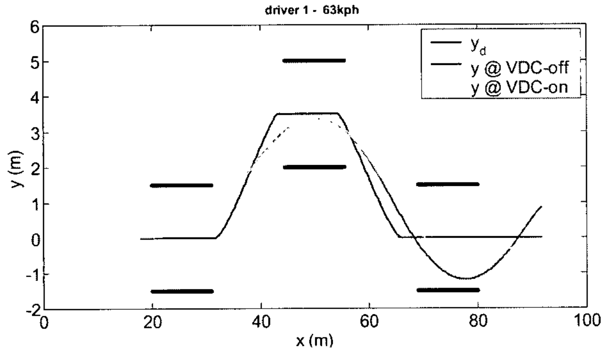

Figure 11. Trajectory profiles for driver 1 on the test track.

age). It could not help driver 3 (smooth) because the driver is too slow and failed at extremely low speeds. We are not too concerned about this because driver 3 does not represent the nimble response of a professional driver. The trajectories of the vehicle with and without VDC at 63 kph are shown in Figure 11. It can be seen that VDC helps driver 1 by reducing the tail swing at the exit of the test track.

Even though the VDC helps to improve vehicle performance in this HIL test, the VDC needs to be re-designed due to its worst-case performance-which suggests that more can be done to improve its resistance to rollover. This conclusion is not surprising since this particular VDC was originally designed only for lateral stability.

4.4. VDC Design Iteration

In this section an add-on anti-rollover "patch" is designed to improve the vehicle's roll stability under worst-case scenarios. The patch plus the original VDC system is then re-evaluated through another iteration. A predictive approach is used to design the anti-rollover patch. Future vehicle roll angle profile is predicted, based on which proper control actions may be taken before the vehicle reaches unsafe situations. Roll angles are predicted using an adaptive discrete-time vehicle model. If the model is accurate, it will be able to predict the occurrence of an unacceptable event. For example, a threshold vehicle roll angle can be chosen corresponding to the occurrence of wheel lift-off. In such a case at time t, a time-to-wheel lift-off (TTW) can be calculated. If the input is held constant and the system is predominantly linear, TTW counts down to 0 (i.e. wheel lift-off) with a slope of -1 with time (Chen, 2001), thus is an excellent threat index.

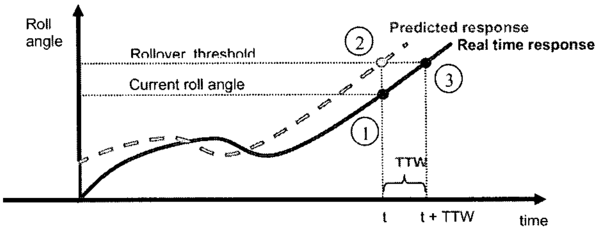

Figure 12. Roll angle prediction and time-to-wheel life-off (TTW).

The anti-rollover controller is designed to operate on the difference between the predicted roll angle and the rollover threshold. The prediction window (t_p) used for the simulations is 0.2 seconds. A 4th order Recursive Auto Regressive with eXtra input (RARX) model is used for roll angle predictions, which can be represented in the discrete time transfer function form as

$$\varphi(t)=\frac{B(z^{-1})}{A(z^{-1})}z^{-d}\delta(t-1)+\frac{1}{A(z^{-1})}e(t) \tag{1}$$

where $A(z^{-1})=1+a_1z^{-1}+\cdots+a_{na}z^{-na}$ and $B(z^{-1})=b_1z^{-d}+\cdots+b_{nb}z^{-nb-d}$. φ is the predicted roll angle, δ is the hand-wheel steering angle, d is the dead time of the system, and $e(t)$ is the exogenous input which is assumed to be white. Parameters used are: d=1, nb=3, na=4, and $A(z^{-1})$ and $B(z^{-1})$ are constructed using the bicycle model. The sampling time of the RARX model is 25 ms, which achives reliable roll angle prediction. The RARX model is trained on-line with steering and roll signals from CARSIM.

Based on the predicted roll angle, the differential braking torque (Bt) may be applied either to the left front ($\varphi<0$) or right front ($\varphi>0$) tire according to the following logic:

If $|\varphi(t+t_p)|>\Phi$,

$$B_t=\left\{K_o+K_p[|\varphi|-\Phi]+K_d\cdot\frac{1}{2}[1+\mathrm{sgn}(\varphi\dot{\varphi})]|\dot{\varphi}|\right\}\frac{1}{0.3s+1} \tag{2}$$

else $B_t=0$

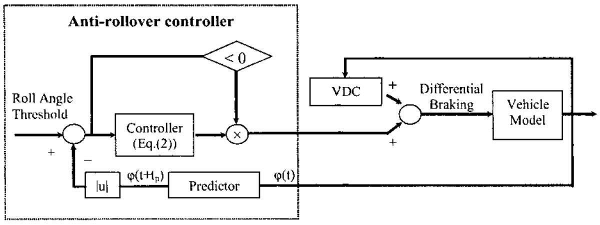

Figure 13. Anti-rollover controller patch with the original VDC.

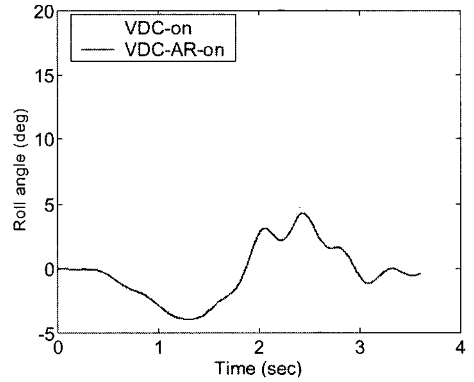

Figure 14. Roll angle profiles: VDC vs. VDC-AR under inputs shown in Figure 5.

where Φ is the roll angle threshold, which is selected to be high enough to prevent degradation of handling performance but low enough to be effective in rollover prevention. A first order lag with 0.3 sec time constant is included to approximate the brake dynamics. The controller gains are chosen to be K_o=300 Nm, K_p=1250 Nm/deg, K_d=4 Nm·sec/deg, and Φ=4 deg. The structure of the anti-rollover patch and the original VDC is shown in Figure 13.

This add-on unit plus the original VDC is assumed to be the new VDC design, which will be put through the evaluation process again in the second iteration. The combined controller will be referred to as VDC-AR (VDC with anti-roll) for the remainder of this paper. In Figure 14 the roll responses under the VDC-on worst-case maneuver (Figure 5) are given for the VDC-on and VDC-AR-on cases. The improved roll performance of VDC-AR is mainly because of its prediction capability.

4.5. Iteration 2, Stage 1: Standard Test Matrix

Simulation results with the VDC-AR controller under the standard test matrix are shown in Table 2. By comparing Table 1 and Table 2, we can see that the VDC-AR system improves vehicle's maximum roll angle under all standard tests.

Table 2. VDC-AR results under the standard test matrix.

Initial speed (kph)			80	100	120
J-Turn	VDC-AR-on	max roll angle (deg)	2.99	4.21	4.26
J-Turn w/b	VDC-AR-on		3.05	4.03	4.49
Fishhook#1	VDC-AR-on		2.95	4.19	4.18
Fishhook#2	VDC-AR-on		2.92	4.20	4.28

4.6. Iteration 2, Stage 2: Worst-Case Simulations

The steering angle and brake force range is the same at [-80, 80] deg and [0, 25] lbf, and the initial speed is again set at 120 kph. Multiple initial guesses are chosen, including the VDC-on worst-case maneuver. Out of all the identified local optimum results under different initial guesses, the one that results in highest roll angle is shown below. The steering and braking time trajectories look quite similar to a standard fishhook maneuver with heavy braking coincides with the roll-reversal point. Figure 16 shows comparison of no-control, VDC-on and VDC-AR-on cases under the "fishhook" worst-case maneuver identified for the VDC-AR-on case. Due to the early activation of differential braking, VDC-AR achieves lowest roll and side slip and in the meantime achieves smallest turning radius. Table 3 shows the maximum roll angle

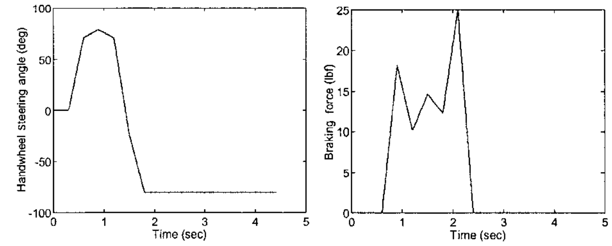

Figure 15. Worst-case hand-wheel steering and braking force for VDC-AR.

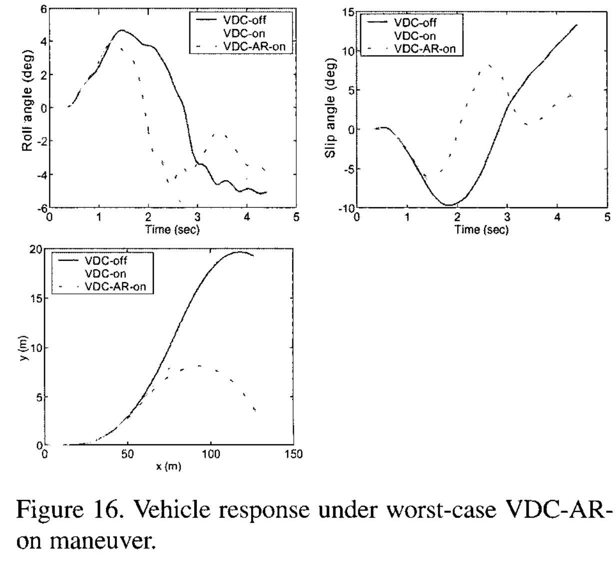

Figure 16. Vehicle response under worst-case VDC-AR-on maneuver.

Table 3. Maximum roll angle under worst-case evaluation.

Speed (kph) Optimized			120
VDC-on	VDC-off	max roll angle (deg)	4.27
	VDC-on		rollover
	VDC-AR-on		4.26
VDC-AR-on	VDC-off		5.16
	VDC-on		5.65
	VDC-AR-on		4.72

obtained through simulations with the two worst-case scenarios: for the VDC-on case and the VDC-AR-on case, respectively. As can be seen, the worst-case scenario for the VDC-on case results in a rollover, in contrast, worst-case scenario for the VDC-AR-on case results in a maximum roll angle of 4.72 degrees. Overall, the anti-rollover patch seems to be working really well to without compromising other vehicle performance.

4.7. Iteration 2, Stage 3: Human in Loop

The Toyota peak-to-peak yaw rate based star rating is again used. It is observed that for all three drivers the anti-rollover patch was not activated. The reason is that the predicted roll angle on this test course never exceeds the treshhold value of 4 degrees. In other words, the VDC-AR performs exactly like the original VDC in this test. Since the VDC-AR works satisfactorily in all three simulation tests, it can be forwarded to the field test stage. That, of course, is beyond the scope of this paper.

5. CONCLUSION

In this paper a computer-based evaluation process is outlined. As presented in Figure 1, the procedure consists of three stages: standard test matrix, worst-case test, and human (model)-in-loop Moose test. A Vehicle Dynamic Control (VDC) system was evaluated through this process. Both the standard test matrix and the driver-in-loop simulations show that in general VDC improves vehicle's performance. However, we identified a worst-case scenario with steering input that looks like a lane change maneuver, which results in rollover for the vehice with VDC. This needs to be corrected because the same maneuver does not pose a threat for a vehicle without VDC. An anti-rollover patch using predicted vehicle roll motion and a switch based on time-to-wheel-lift-off is designed to improve the original VDC systems performace. It was shown that for the new controller, VDC-AR successfully passed the evaluation with improved roll performance in all tests. The proposed evaluation procedure provide a comprehensive evaluation for the VDC, and could significantly reduce the field testing time by providing valuable feedbacks through computer simulations.

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REFERENCES

CARSIM User Manual (2003). http://www.carsim.com/

Chen, B. and Peng, H. (2001). Differential-braking-based rollover prevention for sport utility vehicles with human-in-the-loop evaluations. *Vehicle System Dynamics* **36, 4-5,** 359–389.

Garrick, J. F., Garrott, W. K., and O'Harra, B. C. (2002). A comprehensive experimental examination of test maneuvers that may induce on-road. Untripped, Light Vehicle Rollover-Phase IV of NHTSA's Light Vehicle Rollover Research Program, DOT HS 809 513.

Hac, A. (2002). Influence of active chassis systems on vehicle propensity to maneuver-induced rollovers. *SAE Paper No.* 2002-01-0967. SAE Congress and Exposition.

Luus, R. (2000). *Iterative Dynamic Programming*. Chapman & Hall/Crc. Monographs and Surveys in Pure and Applied Mathematics.

Matsumoto, S., Yamaguchi, H., Inoue, H. and Yasuno, Y. (1992). Improvement of vehicle dynamics through braking force distribution control. *SAE Paper No.* 920645, SAE Congress and Exposition.

National Highway Traffic Safety Administration. (1973). *Advance Notice of Proposed Rulemaking (ANPRM) on Resistance to Rollover*, NHTSA 38 FR 959.

National Highway Traffic Safety Administration. (1998). *Traffic Safety Facts 1997.*

National Highway Traffic Safety Administration. (2002). *Traffic Safety Facts 2001.*

Pierre, D. A. (1986). *Optimization Theory with Applications.* Dover Publications. New York.

Toyota (2001). Comments on part 575 consumer information regulations. Rollover resistance-Dynamic rollover testing. *Docket No.* NHTSA-2001-9663.

Ungoren, A. Y. and Peng, H. (2004). An adaptive lateral preview driver model. to appear in *Vehicle System Dynamics*.

Ungoren, A.Y., Peng, H. and Milot, D. (2001). Rollover propensity evaluation of an SUV equipped with a TRW VSC System. *SAE Paper No.* 2001-01-0128, SAE Congress and Exposition.

Wielenga, T. J. (1999). A method for reducing on-road rollovers-anti-rollover braking. *SAE Paper No.* 1999-01-0123, SAE Congress and Exposition.

Zanten, A. V., Erhardt, R. and Pfaff, G. (1995). VDC, The vehicle dynamics control system of bosch. *SAE Paper No.* 950759, SAE Congress and Exposition.