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EMERGENCY BRAKING CONTROL OF A PLATOON USING STRING STABLE CONTROLLER

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ABSTRACT–In this paper, a safe control strategy is considered in the situation when a platoon of vehicles need to decelerate rapidly. When the vehicles are spaced closely, it is known that the reference information should be transmitted to the whole platoon to minimize the undesirable effects of small leader disturbances. However, the vehicle control should also depend on the preceding vehicle position to maintain the desired distance. Tracking the preceding vehicle position can lead to amplification of the control input along the following vehicles, therefore the vehicles in the rearward generally exert larger maximum control input than the vehicles in the front. The theoretical bounds for the i^{th} vehicle control input are calculated using a linear vehicle and controller model. In the simple illustrative example, the designed controller maintains string stability, and the control inputs of the following vehicles stay within bounds.

KEY WORDS : String-stable controller, String-stability, Emergency braking, Platoon, Automatic vehicle control, Actuator saturation

1. INTRODUCTION

The safety of an emergency braking situation is the most critical factor in an automated highway system (AHS). In the architecture of an automated highway system, California PATH (Partners for Advanced Transit and Highways) has been using the concept of platoons in which traffic is organized in groups of closely spaced vehicles. In this paper, a safe control strategy is considered in the situation when the platoon needs to decelerate rapidly. The strategy should be designed such that rear end collisions in a platoon can be avoided, taking into consideration the different braking capability of each vehicle. When a platoon of vehicles is braking, a vehicle may skid its wheel and lose control if the controller commands deceleration greater than the vehicle's maximum limit. Consequently, rear end collisions can occur. In general, the maximum deceleration limit of a vehicle is determined by its maximum friction coefficient between the road and tire as follows(Gillespie, 1976).

$$|a|_{\max} \leq \mu_{\max} g \,, \tag{1}$$

where $|a|_{\max}$ denotes the maximum deceleration limit, $\mu_{\max}$ denotes the maximum friction coefficient between the road and tire, and g denotes the gravity constant.

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Modern Antilock Brake Systems (ABS) are designed to avoid wheel-locking and the resulting unstable condition, even if the driver's braking input is greater than the vehicle's maximum limit. However, if the maximum deceleration limit of each vehicle's ABS is significantly different, the vehicles in the platoon will not decelerate at the same rate with each other, and collisions may occur.

Previous studies concerning safety in the AHS were done studying the key factors affecting accident severity in a collision, and the required uniform headways were analyzed (Grimm and Fenton, 1988). The safe maneuvering region of platoons performing basic maneuvers such as join, split and change lane were studied by (Li *et al.*, 1997). Concerning the results of emergency braking under constant headway policy, the safety and throughput were analyzed in a large scale system (Godbole and Lygeros, 2000). The benefits of using leader vehicle information to reduce error propagation were shown by (Swaroop and Hedrick, 1996) and (Seiler *et al.*, 2002) in automated vehicle control.

In this paper, a deceleration strategy to avoid the rear end collisions in a platoon is presented using a string stable controller. In Section 2, several assumptions are given to simplify the problem of our interest. Section 3 and Section 4 describe the important features of the string stable controller and the maximum possible deceleration of the platoon is calculated. Numerical examples of calculating the deceleration limit are given in Section 5.

2. CONTROL STRATEGIES FOR THE PLATOON

In this section, several assumptions are made in order to simplify the platoon models in the AHS. Under those assumptions, control strategies that can enhance the safety in the defined emergency situation are considered.

We make the following assumptions:

(1) The dynamics of the vehicles are identical except for the maximum acceleration and deceleration limits.
(2) All the following vehicles use the same control law.
(3) The desired inter-vehicle spaces are the same for each following vehicle.
(4) Vehicles are equipped with sensors to measure the relative distance and velocity from the preceding vehicle, and a communication method to transmit the information to other vehicles.
(5) The maximum acceleration and deceleration limit of each vehicle can be measured.

The fifth assumption means that we have a proper estimation technique such that the maximum friction coefficients between the road and tire can be estimated and transmitted to the leader vehicle or higher level controller. Maximum friction coefficient estimation techniques have been addressed by many researchers. Depending on the types of measurements used to estimate the friction coefficient, some methods are categorized as cause-based, i.e. using optical sensors (Breuer *et al.*, 1992), and some methods are categorized as effect-based, i.e. measuring the slip between the road and tire (Müller, 2001).

We are interested in an emergency situation where a large deceleration of a platoon is needed to avoid a collision. In such a situation, it is important that the controller does not command a larger deceleration than the vehicle's deceleration limit. Otherwise, the vehicle will lock its wheels and lose control. In the AHS, where all the vehicles are forming a platoon with tight spacing, a single vehicle out of control can endanger the whole platoon. Therefore, it is important to know the maximum deceleration limit of each vehicle and regulate the control input properly to avoid a dangerous situation.

The maximum deceleration limit of a vehicle is determined by the maximum friction coefficient between the road and tire. In (Evert, 1989), it is shown that the standard deviation of the maximum friction coefficient for the same test surface can be as large as 0.05.

The possible emergency braking strategies are as follows. First, all the vehicles can brake with their maximum deceleration limit when an emergency situation occurs. In this case, the difference of maximum deceleration limits between vehicles in the same platoon and the spaces between vehicles will determine whether collisions will occur.

Another coordinated control strategy is the predecessor following. However, it was studied in (Swaroop and Hedrick, 1996) and this strategy can amplify the initial error from the previous vehicle and cause string instability. But we can overcome this phenomenon by transmitting the information of leader vehicle to the following vehicles. Here, the term 'string stable controller' denotes the combination of both predecessor and leader vehicle following.

3. STRING STABLE CONTROLLER

In this section, the proposed string stable controller is designed and its characteristics are analyzed.

Since we are more interested in the performance of the platoon than that of the individual vehicle, a simple linear vehicle model will be used and denoted by $H(s)$ which is the transfer function with two poles at the origin.

Figure 1 shows the concept of the platoon with a finite number of vehicles following the leader vehicle. x_0 denotes the position of the leader vehicle while x_1 and x_i denote the position of the first and the i^{th} following vehicle. δ denotes the desired distance between vehicles.

In an emergency braking situation, the leader vehicle will follow the reference positions, x_r. The Laplace transform of its position will be represented as,

$$X_0(x)=H(s)U_0(s)\,, \tag{2}$$

where the control input of the leader vehicle, $U_0(s)$ is determined by,

$$U_0(s)=K(s)E_0(s)\,, \tag{3}$$

and $E_0(s)$ represents the error defined by

$$E_0(s)=X_r(s)-X_0(s)\,. \tag{4}$$

$K(s)$ represents the linear controller for the leader vehicle.

By plugging Equation (4) into (3), and (2) into (3), we get

$$U_0(s)=\frac{K(s)}{1+H(s)K(s)}X_r(s)\,. \tag{5}$$

Next, the position of the i^{th} vehicle can be expressed as follows.

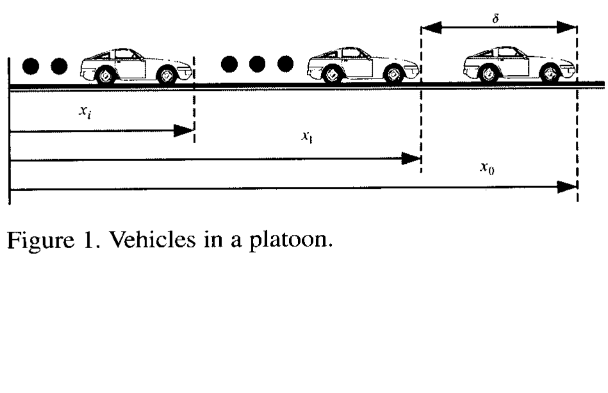

Figure 1. Vehicles in a platoon.

$$X_i(s)=H(s)U_i(s)+\frac{x_i(0)}{s} \quad (i=1,\ 2,\ ...,\ n) \tag{6}$$

where $x_i(0)$ denotes the initial position of the i[th] vehicle and $U_i(s)$ denotes the control input to the i[th] vehicle. The initial position of each i[th] following vehicle, $x_i(0)$, is considered as $-i\delta$ where δ is the desired inter-vehicle spacing.

The control input is decided by the following Equation (7),

$$U_i(s)=K_p(s)E_i(s)+K_r(s)\left(x_r(s)-X_i(s)-\frac{i\delta}{s}\right) \tag{7}$$

where $E_i(s)$ denotes the error between the i-1[th] vehicle and the i[th] vehicle defined as follows,

$$E_i(s)=X_{i-1}(s)-X_i(s)-\frac{\delta}{s}. \tag{8}$$

Notice that the control input (7) is expressed by the summation of two parts. In the first part, $K_p(s)$ represents the transfer function of the preceding vehicle following controller. In the second part, $K_r(s)$ represents the transfer function of the reference trajectory following controller. Since the control input is determined by the summation of the two controllers, the following vehicles will maintain the desired distance, δ, while keeping the desired distance from the leader vehicle $i\delta$. Instead of the leader vehicle position, the reference position is fed back to the following vehicle to eliminate the delay caused by the transfer function of the leading vehicle.

Consider the closed loop error dynamics of this string stable controller. Combining Equations (2), (3) and (4), we get Equation (9)

$$E_0(s)=\frac{1}{1+H(s)K(s)}X_r(s) \tag{9}$$

Similarly, by combining (6), (7) and (8), we get the following relations of the error propagation.

$$E_1(s)=\frac{H(s)\{K(s)-K_r(s)\}}{1+H(s)\{K_p(s)+K_r(s)\}}E_0(s) \equiv T_1(s)E_0(s) \tag{10}$$

$$E_1(s)=\frac{H(s)K_p(s)}{1+H(s)\{K_p(s)+K_r(s)\}}E_{i-1}(s) \equiv T(s)E_{i-1}(s) \quad (i=2,\ 3,\ ...,\ n) \tag{11}$$

From the above relations, it is shown that if $|T(j\omega)|$ is larger than 1 at some frequencies, then error with that frequencies will amplify rearward and the platoon will become "string unstable". However, the string stable controller enables us to make $|T(j\omega)|$ below 1 at all frequencies. If we do not have the reference position feedback term, then $K_r(s)=0$ and $T(s)$ becomes,

$$T_0(s)=\frac{H(s)K_p(s)}{1+H(s)K_p(s)}, \tag{12}$$

where $H(s)$ has two poles at the origin. Because of the pole at the origin, the bode magnitude plot of $T_0(s)$ will start from $T_0(0)=1$ and may increase above 1 at some frequencies. Thus, we get $\|T_0(s)\|_\infty \geq 1$. It is clear that with an input having a frequency component ω where $T_0(j\omega) \geq 1$, the error will not attenuate rearward in the platoon. One example of $T_0(s)$ and $T(s)$ is given in Figure 2. However, if we feed back the reference trajectory and make the $K_r(s)$ nonzero, $|T(j\omega)|$ will start from the value less than 1 and we can design a controller to make $|T(j\omega)|$ smaller than 1 at all frequencies (Seiler *et al.*, 2002).

4. PEAK DECELERATION AMPLIFICATION

As explained in Section 2, we want to design a control strategy that does not command input exceeding the actuator limits. In this section, the relations between the reference position trajectory and the following vehicle's control input are discussed. This relation will provide a way of regulating the following vehicle's control input by manipulating the reference position trajectory which is determined in the upper level platoon maneuver controller.

By manipulating Equation (7) as follows, it is shown that the control input of the i[th] vehicle can be expressed by the summation of errors.

$$\begin{aligned} U_i(s)&=K_p(s)E_i(s)+K_r(s)\{X_r(s)-X_0(s)+X_0(s) \\ &\quad -X_1(s)+X_1(s)...+X_{i-1}(s)-X_i(s)-\frac{i\delta}{s}\} \\ &=K_p(s)E_i(s)+K_r(s)\sum_{j=0}^{i}E_j(s) \end{aligned} \tag{13}$$

Using Equations (9), (10) and (11), we can transform Equation (13) to

$$\begin{aligned} U_i(s)&=\left\{K_p(s)T_1(s)T^{i-1}(s)+K_r(s)\left\{1+T_1(s)\sum_{j=0}^{i-1}T(s)^j\right\}\right\} \frac{X_r(s)}{1+H(s)K(s)} \\ &=\left\{K_p(s)T_1(s)T^{i-1}(s)+K_r(s)\left\{1+T_1(s)\sum_{j=0}^{i-1}T(s)^j\right\}\right\} \\ &\quad \times\frac{U_r(s)}{s^2\{1+H(s)K(s)\}} \\ &\equiv F_i(s)U_r(s) \qquad (i=1,\ 2,\ ...,\ n) \end{aligned} \tag{14}$$

where $U_r(s)$ represents the acceleration of a reference trajectory equivalent to $s^2X(s)$.

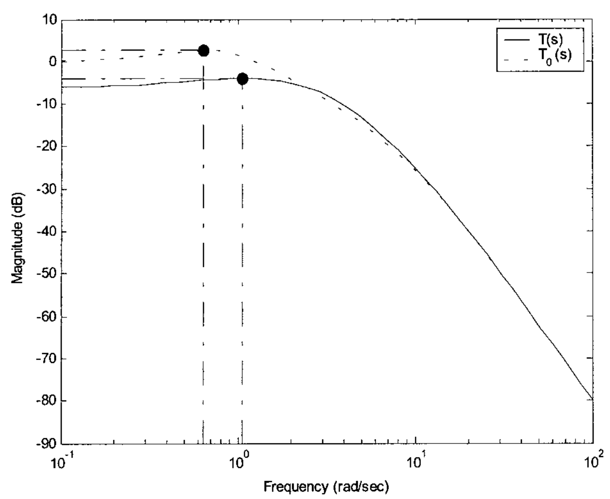

Figure 2. Transfer function of $T(s)$, and $T_0(s)$ given $H(s)$ and $K_p(s)$ as in Equation (17) and (19). $\|T(s)\|_\infty$ =0.62 and $\|T_0(s)\|_\infty$ =1.37.

Equation (14) allows us to define the relationship between $U_i(s)$ and $U_{i-1}(s)$ as follows,

$$\frac{U_i(s)}{U_{i-1}(s)}=\frac{K_p(s)T_1(s)T^{i-1}+K_r(s)\left\{1+T_1(s)\sum_{j=0}^{i-1}T^j(s)\right\}}{K_p(s)T_1(s)T^{i-2}+K_r(s)\left\{1+T_1(s)\sum_{j=0}^{i-2}T^j(s)\right\}}$$

$$=\frac{K_p(s)T_1(s)T^{i-1}+K_r(s)\left\{1+T_1(s)\sum_{j=0}^{i-1}T^j(s)\right\}}{K_p(s)T_1(s)T^{i-1}(s)\left\{\frac{1+H(s)K_p(s)}{H(s)K_p(s)}\right\}+K_r(s)\left\{1+T_1(s)\sum_{j=0}^{i-1}T^j(s)\right\}}$$

$(i=2, 3, ..., n)$. (15)

Comparing the numerator and denominator of Equation (15), notice that the difference is the multiplicative term $(1+H(s)K_p(s))/H(s)K_p(s)$. In Equation (12), it is shown that the inverse of this term is $T_0(s)$, and that this term is larger than or equal to 1 at some frequencies. Thus Equation (15) can be greater than or equal to 1, and this means that more control input can be commanded to the vehicles in the rear of the platoon.

Let $u_i(t)$, $u_r(t)$ and $f_i(t)$ be the impulse response of $U_i(t)$, $U_r(t)$ and $F_i(t)$. Then, in the time domain, Equation (14) can be converted into the following inequality (Desoer, 1975).

$$\|u_i(t)\|_\infty \le \|f_i(t)\|_1 \|u_r(t)\|_\infty , \quad (16)$$

where $\|u_i(t)\|_\infty$ denotes the $\sup_{t\ge 0}|u_i(t)|$ and $\|f_i(t)\|_1$ denotes $\int_0^\infty |f_i(t)|dt$.

From the above inequality (16), it is shown that the possible control input of the i[th] vehicle, $\|u_i(t)\|_\infty$, can be calculated given $\|f(t)\|_1$ and the maximum deceleration of the reference position trajectory, $\|u_r(t)\|_\infty$. Therefore, if we know the maximum friction coefficient of the vehicle, we can regulate the deceleration of the reference trajectory as follows.

$$\|u_r(t)\|_\infty = \lim_i \{\|u_r(t)\|_\infty^i\} \quad (17)$$

where,

$$\|u_r(t)\|_\infty^i \le \frac{\|u_r(t)\|_\infty}{\|f_i(t)\|_1} \le \frac{\mu g}{\|f_i(t)\|_1} . \quad (18)$$

Note that Equation (1) is used to derive the last part of the above inequality.

5. NUMERICAL EXAMPLE

In this section, we show an illustrative example and analyze the behavior of control input and the following system response.

Let all the vehicles in a platoon be modeled with the same transfer functions as follows

$$H(s)=\frac{1}{s^2(0.1s+1)} . \quad (19)$$

The leader and the following vehicle controllers are designed with the transfer functions shown in Equation (5.2) and (5.3).

$$K(s)=\frac{2s+1}{0.1s+1} \quad (20)$$

$$K_r(s)=K_p(s)=\frac{s+0.5}{0.1s+1} \quad (21)$$

The trajectory of the reference position, $x_r(t)$, is generated by integrating the acceleration shown in Figure 3(a). The generated reference velocity is plotted in Figure 3(b) assuming that the initial velocity is 10 m/s.

The simulation results of the designed controller are plotted in Figure 4 and 5.

It is shown that the errors attenuate for vehicles in the rear of the platoon. This is because $T(j\omega) \le 1$ for all ω, as shown in the Figure 2.

In Figure 5, it is shown that the absolute values of the peak accelerations are increasing. However, this increment is decreasing geometrically due to the factor $T^{i-1}(s)$ multiplied to $(1+H(s)K_p(s))/H(s)K_p(s)$ in Equation (15).

Also in Figure 5, the bounds for the possible control inputs, $\|f_i(t)\|_1\|u_r(t)\|_\infty$, are plotted below the acceleration curves. From Figure 3(a), we know $\|u_r(t)\|_\infty$ equals to 1, and the term $\|f_i(t)\|_1$ is calculated by integrating the

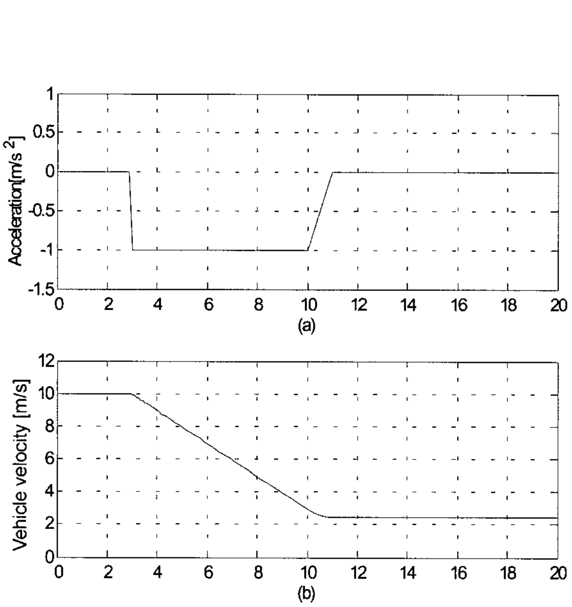

Figure 3. (a) Reference trajectory acceleration. (b) Reference trajectory velocity with initial velocity 10 m/s.

absolute value of the impulse responses of $F_i(s)$ numerically. In this result, it is shown that for a unit step-like reference acceleration shown in Figure 3(a), the controllers of the following vehicles command desired acceleration around -1.6 ~ -1.7 m/s^2 to the actuators. If this value is within the saturation limit of the actuator, the vehicles will show good performance. Otherwise, we have commanded too much deceleration on the reference trajectory, and may regulate the reference trajectory more.

Assume that three vehicles have a maximum deceleration limit of 1.2, 1.3 and 1.1 m/s^2, respectively. Then, according to Equation (17) and (18), the maximum deceleration of the reference trajectory will be determin-

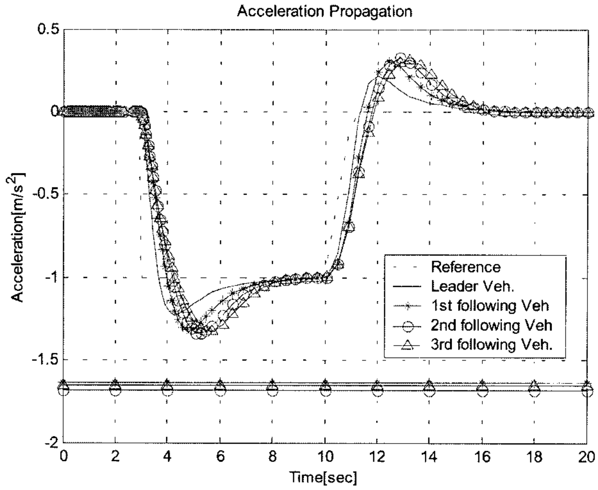

Figure 5. Acceleration of the following vehicle $K_p(s)=K_r(s)$.

ed as a minimum of 0.73, 0.77 and 0.66 m/s^2, which is 0.66 m/s^2.

In another simulation, $K_r(s)$ is multiplied by 1.5 and $K_p(s)$ is multiplied by 0.5 in order to make the controller more sensitive to the reference position than the preceding vehicle position. In Figure 6, the errors of each vehicle are shown, and they show similar results with previous ones attenuating errors rearward. The following vehicles are braking faster than before, and the absolute values of the peaks are less than before. Therefore, it is shown in Figure 7 that the absolute values of the bounds have become smaller.

It is a desirable characteristic for the emergency braking controller to have small overshoot in its control input.

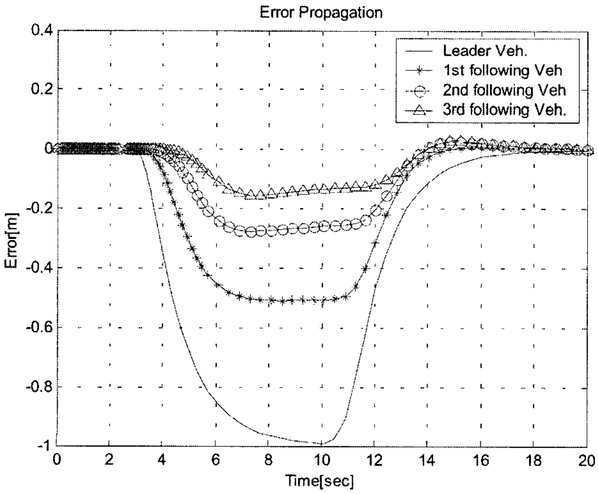

Figure 4. Error of the following vehicle with $K_p(s)=K_r(s)$.

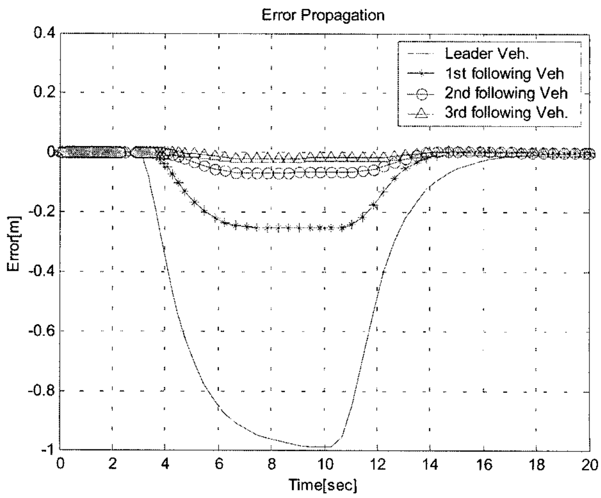

Figure 6. Error of the following vehicle with $3K_p(s)=K_r(s)$.

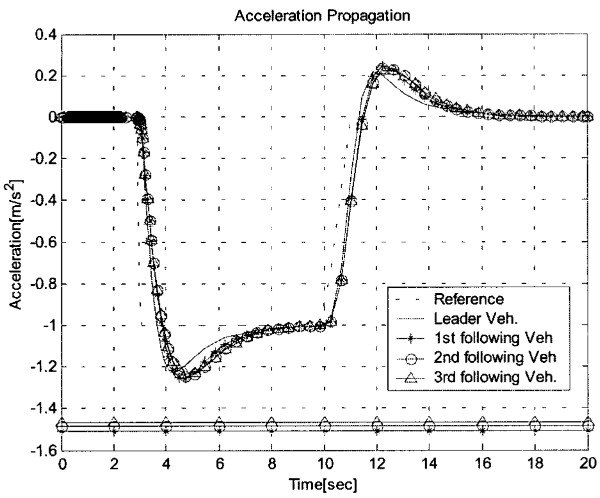

Figure 7. Acceleration of the following vehicle $3K_p(s) = K_r(s)$.

However, having too much portion to controller gain on the reference position side can have undesirable effects, i.e. the platoon will become less sensitive to the disturbance of the previous vehicle. Therefore, a reasonable compromise should be made in determining the portion of the controller gain for the preceding vehicle following and reference trajectory following.

5. CONCLUSION

This paper presents a methodology for using a string stable controller in an emergency braking situation. To avoid brake saturation and the subsequent wheel skidding one should reasonably bound the leader vehicles allowable deceleration.

To regulate the peak deceleration of the control inputs, we used the one norm of the impulse response function, which is acquired from the transfer function of reference trajectory deceleration to the i^{th} vehicle controller input. By dividing the deceleration limits of each following vehicle with this one norm of the impulse response, and then taking the minimum quotient over the string of vehicles, we can get the maximum allowable deceleration for the reference trajectory. Subsequently, actuator saturation will not occur and rear end collisions in the platoon can be avoided.

In the beginning of this paper, we made several assumptions regarding the availability of communication and knowledge of friction coefficients between the road and tire, which are not yet practical in most vehicles. However, this study shows how the development of communication and sensor technology can increase the safety in an advanced transportation system.

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