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Operational Availability Under A Continuous Review Inventory Model for Logistics Support

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Abstract. Relationships between inventory policy and operational availability of military equipment maintained under a logistics support system are analyzed. A continuous review inventory model with a stochastic demand typically used in a military logistics support is considered and some numerical studies are provided.

Key Words : *inventory policy, operational availability, logistics support, shortage time*

1. INTRODUCTION

Two principal purposes of military logistics support lie in minimizing total cost and keeping equipment availability under an optimal condition without loss of military forces. Maintaining appropriate spare parts inventories is one of the most important military affairs related to the readiness of military forces. However, the demand of parts is uncertain and to cope with the uncertainty, basic inventory policies are used to maintain appropriate stocks against shortages and unnecessarily excessive inventories. Excessive stocks result in additional inventory carrying cost. On the contrary, shortages of spare parts may lead to greater losses since shortage may bring about failures in providing parts required to repair broken equipments. In that case, readiness of military forces may be critically damaged.

When we consider the inventory model, we usually mean one of the models; the continuous review model or the periodic review model. The continuous review model orders a fixed quantity Q units as soon as the stock level reduces to the reorder point. The

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operating doctrine for the periodic review model requires that an order of Q units be placed at each periodic review time if the stock level is under the reorder point, but that no orders be placed if not under the reorder point. The continuous review model is studied by many researchers, sometimes called as the fixed quantity policy with safety stocks (Newberry(1960)). The exact formulas of this model with Poisson demands and fixed procurement lead time have been presented in Hadley and Whitin(1963). When the procurement lead time is distributed by the exponential probability density function, we can refer to Galliber et al. (1959). The numerical approximation of this model using service level instead of shortage cost has been devised in Clark and Rowe (1960). Liu (1990) has studied an inventory model in which depletions are due to random demands and random failures, but in his model the lead time is assumed to be zero. Kalpakam and Sapna (1994) consider an inventory model with exponential lead times in which the demands occurring during the stock-out periods are assumed to be lost.

Inventory control methods in logistics support system pursue to have the correct amount for the lowest total cost. The economic inventory principle involves the optimization between the inventory holding cost and procurement cost. The economic order quantity (EOQ) model is most common and it is generally applicable in instances where there are relatively large quantities of common spares and repair parts. The EOQ model is a deterministic model in which the rate of demand is assumed known with certainty and constant and shortages are not allowed. On the other hand, the demand occurs stochastically under the logistics support system for supplying spare parts of operated equipment, which usually results in lead time or logistic delay after the order of spare parts. If the spare parts fall short during the lead time, the operational readiness or effectiveness of equipments may be impaired significantly. The effectiveness of equipment can be expressed as a measure of how well the system performs the intended function. A representative measure is availability. Availability indicates the possibility of operational readiness when any mission is demanded at a stated time.

This paper analyzes the effect of the inventory policies for the spare parts on operational availability. Operational availability is defined as the following formula:

$$Operational\ Availavility = \frac{MTBF}{MTBF + MTTR + MLDT} \cdot \tag{1}$$

In the above formula, MTBF stands for Mean Time Between Failures, MTTR for Mean Time To Repair, and MLDT for Mean Logistics Delay Time.

We consider a continuous review inventory model with a fixed reorder quantity in which a lot of size Q is ordered when the inventory level drops to a reorder point, ROP. We assume that the lifetime of a related part follows an exponential lifetime distribution and the lead time between placement and receipt of an order is a constant. We also assume that, if spare parts are available, time to repair or replace a failed part of equipment is negligible. In this case, the demand process for spare parts is a Poisson process since the time between failures follows an exponential distribution.

2. CONTINUOUS REVIEW INVENTORY MODEL FOR LOGISTICS SUPPORT

We consider a fixed reorder quantity model under the circumstances of continuously reviewed inventory policy. The terms and notations used for inventory management for logistics support are defined as follows:

Safety level S/L: minimum stock level against any shortage from uncertain demand and supply delay.

Storage objective S/O: maximum on-hand inventory level.

Lead time L: lead time between placement and receipt of an order

Reorder point ROP: reordering point at which an order is placed to replenish the inventory

Order quantity Q: fixed order quantity at reorder points

We consider an inventory policy in which an order of a fixed quantity Q is placed whenever the storage level reaches a reorder point ROP. Typical behavior of the inventory policy can be described as the following Figure 1.

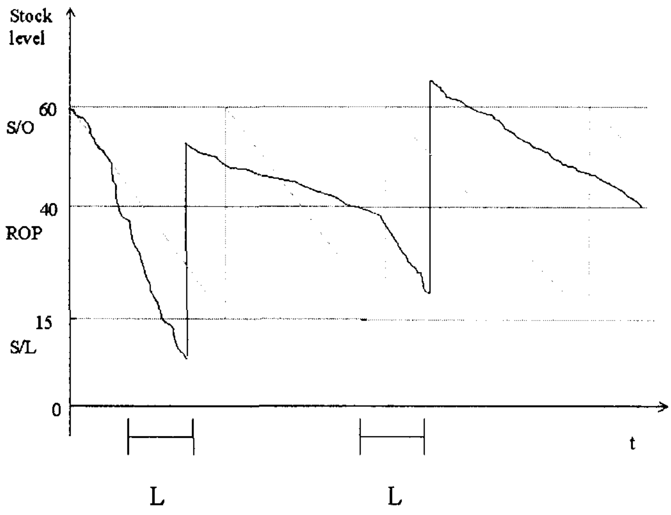

Figure 1. Behave of the inventory policy

3. AVERAGE SHORTAGE TIME AND OPERATIONAL AVAILABILITY

In this section, expected shortage time is derived and the corresponding operational availability is discussed under the inventory policy mentioned in the previous section. We consider a situation where there are N equipments to be maintained and we will consider the inventory policy and operational availability of a specified part that is installed in the equipment. We assume that each equipment has exactly one that part. If the failure rate of a part is λ, the failure rate of N parts is $\lambda_N = \lambda \times N$. The total number of failures N_L of the parts during the lead time L has a Poisson distribution with mean is $\lambda_N \times L$, i.e.,

$$N_L \sim Poisson(\lambda_N \cdot L). \tag{2}$$

Inventory level m at the reorder point ROP is

$$m = \lambda_N \times ROP. \tag{3}$$

We know that, under the inventory policy, m-th failure time T_m follows a Gamma distribution with parameters m and $1/\lambda_N$. In this case, period of zero stock during the lead time L is

$$Z = \begin{cases} L - T_m, & T_m < L \\ 0, & T_m \geq L \end{cases} \tag{4}$$

and the expected period of zero stock is given by

$$\begin{aligned} E(Z) &= \int_0^L (L-t)\, f(t)dt \\ &= L \cdot \Pr(N_L \geq m) - \frac{m}{\lambda_N} \cdot \Pr(N_L \geq m+1). \end{aligned} \tag{5}$$

In addition, the expected period of shortage during which failed part cannot be repaired or replaced because there is no spare part remained is obtained by

$$E(S) = L \cdot \Pr(N_L \geq m+1) - \frac{m+1}{\lambda_N} \cdot \Pr(N_L \geq m+2). \tag{6}$$

For the case of single equipment ($N = 1$), the expected shortage period is

$$E(S^*) = L \cdot \Pr(N^* \geq m+1) - \frac{m+1}{\lambda} \cdot \Pr(N^* \geq m+2) \tag{7}$$

where N^* has a Poisson distribution with parameter is $\lambda \times L$. Since the average cycle length of the inventory policy is Q/λ and the repair or replace time is assumed to be negligible, the operational availability of single equipment is given by

$$Operational\ Availavility = \frac{Q/\lambda - E(S^*)}{Q/\lambda}. \tag{8}$$

4. NUMERICAL EXAMPLES

The following tables show examples usually applied to logistics supply. The convert days in Table 1 are the value obtained from the quantity divided by one day-demand.

Table 1. An example of convert days for logistics support

Classification	Order quantity Q	Safety level (S/L)	Storage objective (S/O)	Order & shipping time (OST)	Reorder point (ROP)
convert days	45	15	60	25	40

Let's consider a situation where a military unit possesses 18 equipments and all the 18 equipment should be in operational readiness for the military purpose. The following part is a critical one for the operation of the equipment and we will study the effect of inventory policy on operational availability for this part.

Part number	Part name	Lifetime distribution	*MTBF* (hour)	Failure rate (/hour)
MS1234-12	Lamp	Exponential	507.5	0.0018994

The failure rate of the above part is λ= 0.0018994 /hour. Suppose that the equipment containing the item operates 240 hour/year. Then the average demand per day is λ_d = 0.0018994 × 240 / 365 = 0.001249/day and the demand rate for 18 equipments is λ_N = 0.001249 × 18 = 0.022482/day. Since ROP = 40 days, inventory level at the reorder point is $m = \lambda_N \times ROP = 0.022482 \times 40 = 0.89928 \cong 1$, i.e., the quantity of residual stock at reorder point is one. Similarly, order quantity Q is obtained by

$$Q = \lambda_N \times (O/L) = 0.022482 \times 45 = 1.0117.$$

Suppose that the lead time L is 25 days. Then the average demand during L is

$$\lambda_N \times L = 0.022482 \times 25 = 0.56205$$

and the total number of failures N_L has a Poisson distribution with mean 0.56205. Therefore, we have

$$E(Z) = 5.875335 \text{ days} = 141.01 \text{ hours}$$

and

$$E(S) = 1.001652 \text{ days} = 24.04 \text{ hours}.$$

The above results indicate that more than one equipment can not be repaired and operated for average 24.04 hours during the lead time L. Next, the operational availability of 18 equipments will be discussed. We assume that repair times of the 18 equipments never overlap because the repair time is assumed to be sufficiently small.

For the part, the average number of failures during the average cycle length of the inventory policy is 1.01169 and we have

$$MTTR = 0.34 \text{ hours} = 0.34 \times 365/240 \text{ days} = 0.517083 \text{ days}$$

and the average operational availability is obtained by

$$A = (45\text{-}1.001652\text{-}\ 0.517083\times1.01169)/45 = 0.9661.$$

Using the above results, we can see the effects the adjusted convert day of the part may have on availability. When a system or subsystem of the equipment is composed of k different parts and they constitute a series structure, system or subsystem availability can be calculated by

$$A_S = A_1 \cdot A_2 \cdot \cdots \cdot A_k \tag{9}$$

where $A_1 \cdot A_2 \cdot \ldots \cdot A_k$ are availabilities of the k parts.

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