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OPTIMAL DESIGN OF THE MULTIPLAYER DAMPING MATERIALS USING EQUIVALENT MODELING

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ABSTRACT−The viscoelastic layer material is widely used to control the noise and vibration characteristics of the panel structure. This paper describes the design technology of the effective vibration damping treatment using the concept of the equivalent parameter of viscoelastic layer materials. Applying the equivalent parameter concepts based on theories of shell, it is possible to simulate the finite element analysis of damping layer panel treatments on the vibration characteristics of the structure. And it is achieved the reduced computational cost and the optimal design of topological distribution for the reduction of vibration effect.

KEY WORDS : Damping treatment, Equivalent parameter, Vibration characteristics, Panel structure, Topology distribution

NOMENCLATURE

ρ : density
E : Young's modulus
t : thickness
ζ : critical damping ratio
v : Poisson's ratio
$[D]$: elastic constant matrix
$[T]$: transformation matrix
Γ : design space
Ω : design domain
$[K]$: stiffness matrix
$\{F\}$: force vector
$\{x\}$: design variable vector
$[M]$: mass matrix

1. INTRODUCTION

In order to reduce vibration and structure borne noise, the viscoelastic damping materials are used. In automotive structure, panels such as floor, dash, roof, door or around the engine and trunk compartment are treated with viscoelastic damping layers. The effects of damping treatments on the vibration behavior of panels are strongly dependent but have been neglected in structural analysis. Generally, a damping system may consist of a single layer of damping material glued on the panel or may be adhesive two metallic layers. This damping system introduces the damping effect and stiffens the panel and increases the mass of the system. Additionally, the material parameters of the viscoelastic material are slightly frequency and heavily temperature dependent with the complexity of the system and its effects. The damping system with panel structure generally represents the bending deformation. Therefore, to simulate the damping system needs the finite element model representing the damping treatment on the panel structure. Ross-Ungar-Kerwin equations (Kerwin *et al.*, 1959) and single element modeling based on variational asymptotical theory (Agnes, 1995), transfer matrix procedure (Nashif *et al.*, 1985) have been developed to describe the layer damping treatment. These methods have been limited to flat panels. And, By some researchers (Seo *et al.*, 1994; Hwang *et al.*, 1996), the study of multi-layer structure characteristics conducted, but this is not carry out the vibration behavior and topology optimization of the panel structure with damping materials.

In this paper, a simple procedure by the equivalent parameters for simulating the vibrations response of panels with damping treatments is presented. The equivalent parameters for damping treatment are derived using the bending theory of thin plates. For the cost reduction and efficient damping treatment, the topological distribution is studied on the base of the frequency

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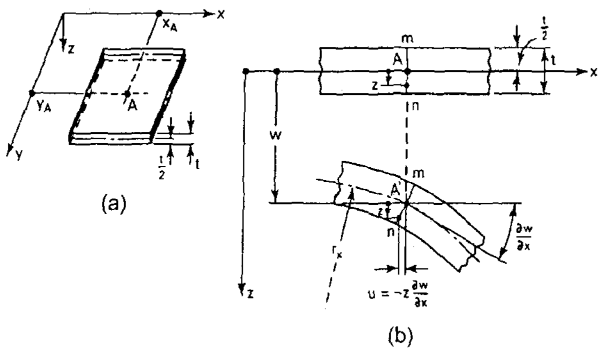

Figure 1. Deformation of a plate in bending.

response analysis.

2. THEORETICAL CONSIDERATIONS

2.1. Bending Deformation

The theory of plates and shells (Ugural *et al.*, 1995) assumes that sections normal to the middle plane remain plane during deformation and the normal stresses in z direction are small (Figure 1), hence strains in that direction can be neglected. With these two assumptions it is easy to see that the total state of deformation can be described as follows. From the assumption that the normal strain ε_z owing to vertical loading may be neglected, the local displacements in the direction of the x, y and z axes are,

$$
\begin{aligned}
u&=-z\frac{\partial w}{\partial x}+f_2(x,y) \quad \text{with } \gamma_{xz}=\partial w/\partial x+\partial u/\partial z=0\\
v&=-z\frac{\partial w}{\partial y}+f_3(x,y) \quad \text{with } \gamma_{yz}=\partial w/\partial y+\partial v/\partial z=0\\
w&=f_1(x,y) \quad \text{with } \varepsilon_z=\partial w/\partial z=0
\end{aligned}
\tag{1}
$$

It is clear that f_2 and f_3 represent, respectively, the values of u and v corresponding to z=0. Because of an in-plane strain; $f_2=f_3=0$. The stress components σ_x, σ_y and $\tau_{xy}=\tau_{yx}$ are related to the strains by Hooke's law, which for a thin plate becomes,

$$
\begin{aligned}
\sigma_x&=-\frac{Ez}{1-v^2}\left(\frac{\partial^2 w}{\partial x^2}+v\frac{\partial^2 w}{\partial y^2}\right)\\
\sigma_y&=-\frac{Ez}{1-v^2}\left(\frac{\partial^2 w}{\partial y^2}+v\frac{\partial^2 w}{\partial x^2}\right)\\
\tau_{xy}&=-\frac{Ez}{1-v}\frac{\partial^2 w}{\partial x\partial y}
\end{aligned}
\tag{2}
$$

Here, the damping layers will be assumed to be isotropic.

The stresses distributed over the side surfaces of the plate, while producing no net force, do result in bending and twisting moments. These moment resultants per unit length are denoted M_x, M_y and M_{xy}. The bending and twisting moments per unit length are,

$$
\begin{aligned}
M_x&=\int_{-t/2}^{t/2} z\sigma_x dz\\
M_y&=\int_{-t/2}^{t/2} z\sigma_y dz\\
M_{xy}&=\int_{-t/2}^{t/2} z\tau_{xy} dz
\end{aligned}
\tag{3}
$$

Introducing into Equation (3) the stresses given by Equation (2), and taking into account the fact that $w=w(x, y)$, we obtain

$$
\begin{aligned}
M_x&=-D\left(\frac{\partial^2 w}{\partial x^2}+v\frac{\partial^2 w}{\partial y^2}\right)\\
M_y&=-D\left(\frac{\partial^2 w}{\partial y^2}+v\frac{\partial^2 w}{\partial x^2}\right)\\
M_{xy}&=-D(1-v)\frac{\partial^2 w}{\partial x\partial y}
\end{aligned}
\tag{4}
$$

where $D=\dfrac{Et^3}{12(1-v^2)}$

The coefficient D is the flexural rigidity of the panel.

On the panel structure with damping materials, it is easy to see that the damping material can be dealt with the bending and membrane deformations, while the panel structure has the bending deformation. And the total deformations can be represented by a superposition of bending deformations and pure in-plane deformations. In the case that the bending deformation is the main behavior in the damping materials, its flexural rigidity may be proportional to the cubic thickness. The shear and bending deformations can be included for the layers. By tuning the bending stiffness and damping factor of layer material with the same boundary condition of panel structural characteristics, the equivalent structural characteristics can be made (Hur *et al.*, 1994). By choosing the equivalent thickness t_{eq} and the equivalent Poisson's ratio v_{eq}, the equivalent flexural rigidity, D_{eq}, can be expressed by Equation (5).

$$
D_{eq}=\frac{E_{eq}t_{eq}^3}{12(1-v_{eq}^2)}
\tag{5}
$$

The coefficient E_{eq} is the equivalent Young's modulus of bending deformation. For the equivalent thickness t_{eq}, the thickness of damping layer or panel structure is chosen.

Under the identical bending moment, the radius of curvature is the same for the deformation behavior.

$$
\frac{1}{r_x}\sum_{i=1}^{n} E_iI_i \cong \frac{1}{r_x}E_{eq}I_{eq}=M_b
$$

$$
E_{eq}t_{eq}^3=\sum_{i=1}^{n} E_it_i^3
\tag{6}
$$

Equation (5) can be rewritten by,

$$D_{eq}=\frac{\sum_{i=1}^{n} E_i t_i^3}{12(1-\nu_{eq}^2)} \tag{7}$$

Since the dissipation can be integrated over all layers, the equivalent critical damping ratio is yielding the following expression.

$$\zeta_{eq}=\frac{1}{E_{eq}t_{eq}^3}\sum_{i=1}^{n}\zeta_i E_i t_i^3 \tag{8}$$

where the critical damping ratio of layers, ζ_i, can be derived from the panel modal test or numerical solutions. The structural damping is calculated as $C_s=2\zeta$.

Under the same volume, the equivalent density, ρ_{eq}, is given by.

$$\rho_{eq}=\sum_{i=1}^{n}\rho_i t_i/t_{eq} \tag{9}$$

But for the in-plane deformations such as membrane, the in-plane stiffness and loss factor can be predicted over all layers as the followings,

$$E_{eq}t_{eq}=\sum_{i=1}^{n}E_i t_i \tag{10}$$

$$\zeta_{eq}=\frac{1}{E_{eq}t_{eq}}\sum_{i=1}^{n}\zeta_i E_i t_i \tag{11}$$

In defining the equivalent parameters, the behavior of damping layers have to taken into account.

2.2. Topology Optimization

The topology optimization (Bendson *et al.*, 1988; Yang, 1997; Ruben, 2002) is to distribute the material density of element, which represents the strain energy evolution supporting the applied load with the given boundary conditions. On the given domain, each element can be distributed with the following material property relationship. Once the parameter is chosen, the Young's modulus of cell can be directly represented by Equation (12). When $n>1$, the ratio of relative density is forced to 0 or 1 by

$$\rho_i=\kappa_i\rho_0$$

$$E_i=(\kappa_i)^n E_0 \tag{12}$$

where E_i is an elastic modulus of element i, E_0 is a reference elastic modulus, ρ_i is a density of element i, ρ_0 is a reference density of element i, κ_i is a relative density ratio of element i and n is a density index.

From the relationship of stress-strain ($\{\sigma\}=[D]\{\varepsilon\}$), the elastic constant $[D]$ can be given as the relative density ratio. The elastic constants of a plane stress and plane strain problem for an isotropic material are given by,

$$D(\rho_i)=\frac{E_0\kappa_i^n}{1-\nu^2}\begin{bmatrix}1 & \nu & 0\\ \nu & 1 & 0\\ 0 & 0 & (1-\nu)/2\end{bmatrix} \tag{13-a}$$

$$D(\rho_i)=\frac{E_0\kappa_i^n}{(1+\nu)(1-2\nu)}\begin{bmatrix}1-\nu & \nu & 0\\ \nu & 1-\nu & 0\\ 0 & 0 & (1-2\nu)/2\end{bmatrix} \tag{13-b}$$

In order to make analogy to the idea of a cellular body consisting of unit cell with rectangular hole κ_i may be written as,

$$\kappa_i=1-a_ib_i \tag{14}$$

The matrix of elasticity constant of shell with isotropic material and a parabolic variation of transverse shear strain through the thickness can be written as,

$$D(\rho_i)=\frac{E_0(1-a_ib_i)^n}{1-\nu^2}\begin{bmatrix}1 & \nu & 0 & 0 & 0 & 0\\ \nu & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & (1-\nu)/2 & 0 & 0\\ 0 & 0 & 0 & 0 & 5(1-\nu)/2 & 0\\ 0 & 0 & 0 & 0 & 0 & 5(1-\nu)12\end{bmatrix} \tag{15}$$

where the matrix of elasticity constant in the local directions is transformed to one in the global directions given by,

$$D'(\rho_i)=[T_i]^T[D(\rho_i)][T_i]$$

In order to design the lightweight structure with high structural rigidity, the objective must be defined as mean compliance and the constraint as mass.

$$\textit{Minimize} \int_{\Gamma} F_i u_i d\Gamma$$

$$\textit{Subject to} \int_{\Omega}\rho_i(\kappa)d\Omega \le V_0,\ at\ 0\le\kappa\le 1 \tag{16}$$

where F_i is a force vector on element i, u_i is a displacement vector of element i, Γ is a design space, Ω is a design domain and V_0 is a given volume

The variation of structural rigidity with respect to material element density can be calculated using the following relationships. For the static problem,

$$[K]\{u\}=\{F\}$$

$$\{F\}^T\frac{\partial\{u\}}{\partial\rho_i}=-\{u\}^T\frac{\partial[K]}{\partial\rho_i}\{u\}=-\{u_i\}^T\frac{\partial[k_i]}{\partial\rho_i}\{u_i\} \tag{17}$$

For the eigenvalue problem,

$$[K]\{\phi\}-\lambda[M]\{\phi\}=0$$

$$\begin{aligned}\frac{\partial\lambda_i}{\partial\rho_j}&=\frac{\{\phi_i\}^T\left(\frac{\partial[K]}{\partial\rho_j}-\lambda_i\frac{\partial[M]}{\partial\rho_j}\right)\{\phi_i\}}{\{\phi_i\}^T[M]\{\phi_i\}}\\&=\{\phi_i\}^T\frac{\partial[K]}{\partial\rho_j}\{\phi_i\}-\lambda_i\{\phi_i\}^T\frac{\partial[M]}{\partial\rho_j}\{\phi_i\}\\&=\frac{E'(\rho_j)}{E(\rho_j)}\{\phi_i\}^T[K]\{\phi_i\}-\frac{1}{\rho_j}\lambda_i\{\phi_i\}^T[M]\{\phi_i\}\end{aligned} \tag{18}$$

where $\{\phi_i\}^T[M]\{\phi_i\}=1$

The entries of overall stiffness matrix $[K]$ can be written by.

$$[k_i]=\int_{V_e}[B]^T[D'(\rho_i)][B]dV_e \tag{19}$$

where $[B]$ is a spatial derivative matrix of displacement variables

Figure 2 shows the microstructure of element cell. In two-dimensional case such as shell element, the microstructure is formed inside an empty rectangle in a unit cell, where a, b and θ are regarded as the design variables. In order to develop a complete void, both a and b must be 1, whereas for solid material a and b must be 0.

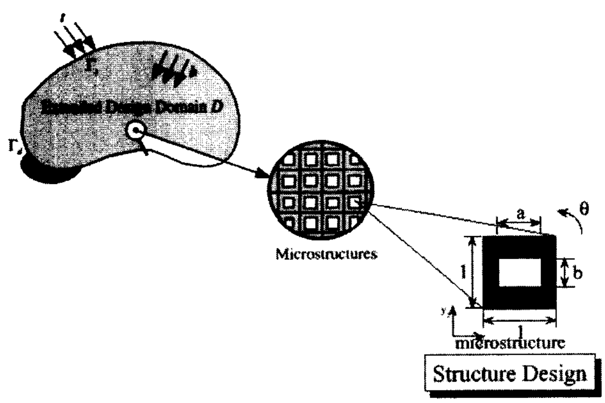

Figure 2. Design domain of microstructure.

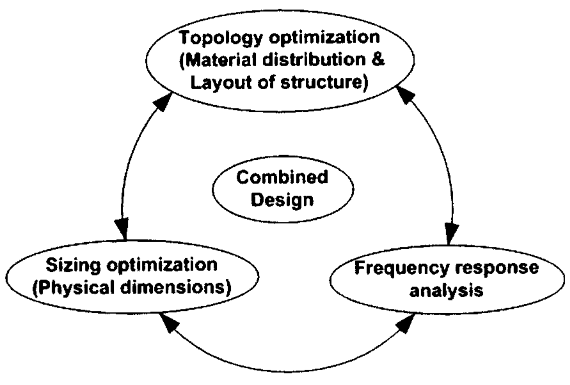

Figure 3. Combined design for layered structure.

2.3. Design Procedure

In the loop of layered structure design as shown in Figure 3, material distributions of layers are solved in three separate steps for reaching the optimum design. First is to define the dynamic response of panel structure. Second is to define the material layout on the design domain. Third is to define the layer thickness under the global topological layout, in order to secure the structural rigidity and lightweight design. Each step is closely interrelated. Through the equivalent method, a single panel structure can be made. The subsequent changes of physical dimensions, geometries and material distributions in the sublevel can help to find the convergence of structure design. Through the design flows shown in Figure 3, the topological distribution of reinforcement beads and damping materials can be applied for the improvement of dynamic stiffness.

3. EXAMPLE

3.1. Design of Equivalent Panel Structure

In order to verify the feasibility of the methodology, two-layer panel structure has been used. The frequency response analysis is made by MSC/NASTRAN. A panel structure has the bead that reinforces the structural rigidity, which is applied to the floor panel of vehicle. A panel structure with clamped edges is loaded at its center with a unit force in the normal direction of panel as shown in Figure 4. Table 1 is the material properties of floor panel, which is composed of steel panel and damping plate attached to it.

When the floor structure is excited under the frequency range from 20 Hz to 500 Hz, Figure 5 shows the mobility of three nodes on the floor panel without the damping treatment. Under the full damping effect of damping material, the dynamic characteristic of floor panel is

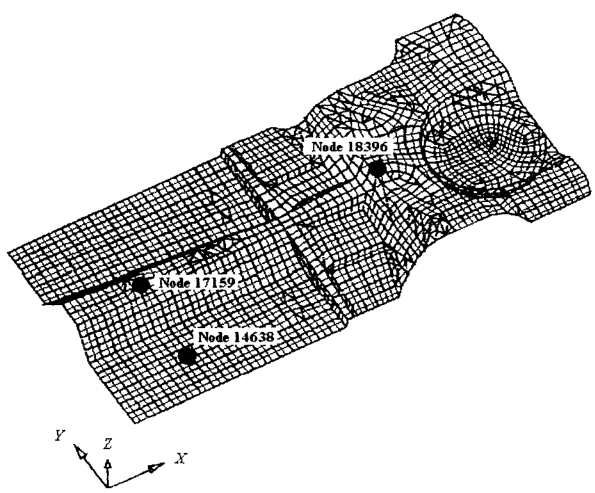

Figure 4. Floor panel structure.

Table 1. Material properties elements numbers of panel structure.

Parameter	Value	
	Floor panel	Damping plate
Thickness (mm)	0.7	3.0
Young's modulus (Mpa)	200000	1030
Density (10^{-9} ton/mm^3)	7.82	1.3
Poisson's ratio	0.29	0.1
Critical damping ratio	0.002	0.03
Elements Numbers	Shell : 3643	

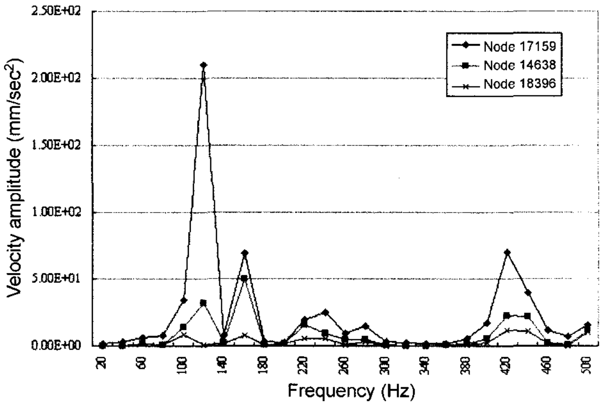

Figure 5. The mobility of panel structure without the damping treatment.

shown in Figure 6.

For the reduction of computational cost, the damping treatment panel structure can be made by the equivalent parameters considering the material properties of damping material and steel panel. Figure 7 shows the mobility of floor structure with the equivalent materials. The material properties of an equivalent panel can be calculated by Equations (5)~(9) and is shown in Table 2. The panel structure with damping treatment can efficiently be replaced with an equivalent panel. Assuming that the total strain results from the superposition of two materials and the strains of each edge are identical ($\varepsilon_{y,d}=\varepsilon_{y,p}$, $\varepsilon_{x,d}=\varepsilon_{x,p}$), the equivalent Poisson's ratio (ν_e) can be given by

Table 2. Material properties of equivalent panel.

Parameter	Value
Thickness (mm)	0.86
Young's modulus (MPa)	151574.7
Density (10^{-9} ton/mm^3)	10.9
Poisson's ratio	0.2
Critical damping ratio	0.017

$$\varepsilon_{y,d}+\varepsilon_{y,p}=-\nu_d\varepsilon_{x,d}-\nu_p\varepsilon_{x,p}$$

$$\varepsilon_{y,e}=-\frac{\nu_d+\nu_p}{2}\varepsilon_{x,e} \quad (20)$$

$$\varepsilon_{y,e}=-\nu_e\varepsilon_{x,e}$$

where ε_p is the strain of panel and ε_d is the strain of damping material.

3.2. Topological Structure of Damping Treatment

For the design of weight reduction of damping treatment, the topology optimization is performed. Let the optimization problem be stated as follows:

$$\textit{Minimize} \quad V$$
$$\textit{Subject to } f_i \le (f)_{ui} \quad (i=1, \ldots, n) \quad (21)$$

where V is a volume of structure, f_i is the ith frequency and $(f)_{ui}$ is the upper bound of ith frequency.

Figure 8 shows the topological distribution of damping treatment on the floor subject to the frequency boundary. The topological panel structure and the mobility are

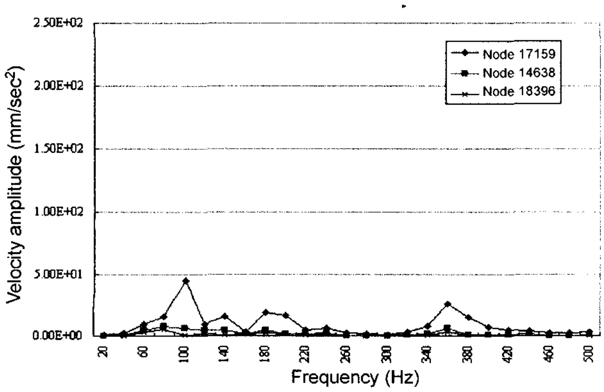

Figure 6. The mobility of panel structure with full damping treatment.

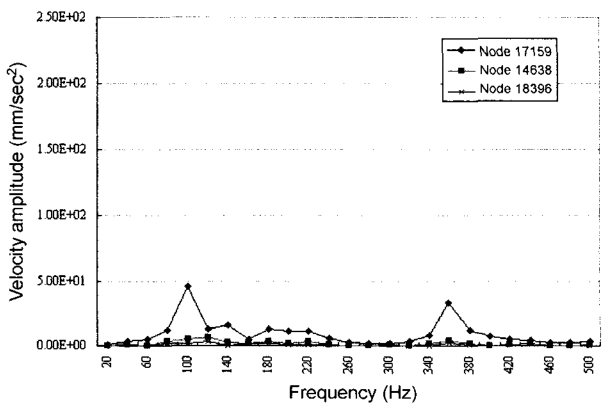

Figure 7. The mobility versus frequency of equivalent panel structure.

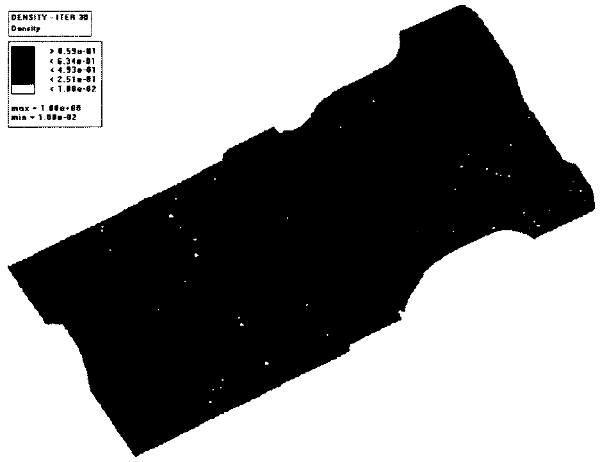

Figure 8. Topological damping treatment on the floor.

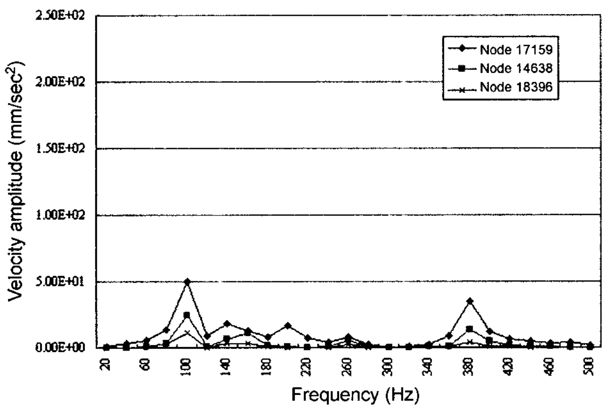

Figure 9. The mobility versus frequency of topological panel structure.

shown in Figure 9. From Figure 6 and Figure 9, the panel structure with topological damping treatment has the same damping effect as the full damping treatment.

4. CONCLUSION

The damping treatment is common in reducing the vibration and noise radiation. The effects of damping treatments on the vibrational behavior of panel structures cannot be neglected in the structural analysis.

This paper presents a modeling method for simulating the vibrations of panels with damping materials, using the equivalent panel modeling and topology optimization of damping material distributions. Through the example of the floor panel, the validity of the equivalent single panel was shown. In comparison with the full damping treatment model, the lightweight rate of damping material distribution is 49% and the mobility is adequately maintained except the local frequency range. If the shapes of reinforcement bead dimensions are changed, the local mobility in the overall ranges can be improved. And the 75% of computational process reduced in the optimal design of topological distribution.

The damping treatment can be taken into account locally and assigning the different equivalent parameters of damping materials can represent the subsection structures of automotive. Using this methodology, the optimization of treatment selection and treatment distribution is easy to perform the common optimization techniques. The change from one treatment configuration to another can be represented by a change of the related modification of the assignments of equivalent material parameters to the different subsections. Also, the geometric dimensions such as thickness, width and height in the damping materials and panel structure can be studied for the improvement of dynamic characteristics.

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