

# Adaptive Observer Based Longitudinal Control of Vehicles

Rhee, Hyoung-Chan\*

**Abstract** In this paper, an observer-based adaptive controller is proposed to control the longitudinal motion of vehicles. The standard gradient method will be used to estimate the vehicle parameters such as mass, time constant, etc. The nonlinear model between the driving force and the vehicle acceleration will be chosen to design the state observer for the vehicle velocity and acceleration. It will be shown that the proposed observer is exponentially stable, and that the adaptive controller proposed in this paper is stable by the Lyapunov function candidate. It will be proved that the errors of the relative distance, velocity and acceleration converge to zero asymptotically fast, and that the overall system is also asymptotically stable. The simulation results are presented to investigate the effectiveness of the proposed method.

**요약** 본 논문에서는 주행 차량의 직진운동 제어를 위하여 관측자를 이용한 적응제어기를 제안한다. 차체중량, 시정수 등의 차량 파라미터들을 추정하기 위해 표준형 적응식을 이용한다. 차량의 구동력 입력에서 가속도까지의 비선형 모델을 이용하여 차량주행 속도 및 가속도 관측자를 설계한다. 제안한 관측자의 지수함수적인 안정도 및 관측자에 의거하여 설계한 적응 제어기의 안정도를 리아프노브 함수 후보에 의해 입증한다. 전체 시스템의 안정도 및 차차간 상대거리/속도/가속도 오차들의 점근적인 수렴성도 수학적으로 입증하며, 제안한 방법의 타당성 및 효율성을 시뮬레이션을 통해 검증한다.

**Keywords** : Adaptive observer, Longitudinal vehicles, Gradient method, Lyapunov function candidate, parameter adaptation law, Nonlinear systems, Feedback, linearization control

## 1. Introduction

The longitudinal motion control of vehicles is an important portion of AVHS(Advanced Vehicle Highway Systems). This is a base technique for applying to an adaptive cruise control, collision avoidance control, etc. The core technologies to implement a longitudinal controller are a sensor fusion, decision making, and system control, etc. In this paper, the longitudinal control problem is focused on a system control design. The various control laws have been proposed to apply the longitudinal control of a platoon of vehicles[1]~[11]. In the reference paper[1], it was shown that an adaptive controller for decentralized nonlinear system has an effectiveness for a longitudinal control of vehicles. In the reference paper[2]~[10], the various control laws, sliding mode

control, feedback linearization control, etc., were proposed to control the longitudinal motion of vehicles by using known system parameters and dynamics. In the reference paper[11], it was shown that the measurement error of the intervehicle spacing is well compensated using a nonlinear observer, and that the longitudinal motion of vehicles can be controlled also well.

The contribution of this paper is to establish the sliding mode observer-based adaptive feedback linearization controller to control the longitudinal motion of the vehicles in the platoon. The vehicle mass, one of the system parameters, may be varied by the number of passengers and loads. And time delay of the powertrain dynamics is also variable based on the vehicle speed. It will be shown that the time delay property and the parameter uncertainty of the vehicles in the platoon can be covered by using a standard parameter adaptation law without any modifications. A sliding mode observer will be

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designed to estimate the vehicle speed and acceleration required to the feedback linearization control law. It will be shown that the observer-based adaptive control law guarantees the asymptotic stability and the good tracking performance of the closed loop system.

This paper was constructed as follows: the vehicle model to be controlled and the problem to be solved are formulated in section 2; the design problem of the observer-based adaptive controller and the stability of the closed loop system are investigated in section 3; the computer simulations results are represented to show the validity of the proposed observer-based adaptive control law in section 4; finally, it will be concluded in section 5.

## 2. The Control Model and Problem Formulation

In this paper, the simplified vehicle model of  $i$ th vehicle in the platoon is used to design the longitudinal control law. The simple model has been used for system level studies in longitudinal control of a platoon of vehicles[1],[2]. As the same in the reference papers[1],[2], we do not use complex engine models constructed with ambient temperature, engine temperature, altitude, condition of spark plugs, transmission, etc. To simplify the road and environmental conditions are assumed as follows:

**Assumption:** the road surface is horizontal, there is no wind gust, and all the vehicles travel in the same direction at all times.

The above assumption was considered in the reference paper[2]. Fig. 1 shows the platoon configuration for a

platoon of vehicles.

The intervehicle spacing  $\Delta_i$  between  $i$ th and  $i-1$ th vehicles can be measured by sensors located in  $i$ th vehicle. The may be expressed as follows:

$$\Delta_i = e_i + L_{si}, \quad \dot{\Delta}_i = \dot{e}_i, \quad \ddot{\Delta}_i = \ddot{e}_i \quad (1)$$

$$e_i = x_{i-1} - (x_i + d_i) - L_{si} \quad (2)$$

where  $x_i$  is the position of  $i$ th vehicle's rear bumper,  $d_i$  is the length of  $i$ th vehicle,  $e_i$  is the error of the intervehicle space  $L_{si}$  and is the constant intervehicle spacing for the safe traveling in the platoon. The simplified model of the  $i$ th vehicle in the platoon is shown in Fig. 2.

In Fig. 2., the term  $K_{di}\dot{x}_i^2$  denotes the force due to air resistance, where  $K_{di}$  is the constant with respect to the specific air mass, drag coefficient, cross sectional area of  $i$ th vehicle, etc. The constant  $d_{mi}$  is the mechanical drag of  $i$ th vehicle;  $m_i$  denotes the  $i$ th vehicle's mass;  $u_i$  denotes the throttle/brake input;  $F_i$  denotes the force produced by  $i$ th vehicle's engine/brake. The longitudinal dynamics of  $i$ th vehicle may be expressed as follows the reference paper[2]:

$$m_i\ddot{x}_i = F_i - K_{di}\dot{x}_i^2 - d_{mi} \quad (3)$$

$$F_i = -\frac{F_i}{\mu_{1i}(\dot{x}_i)} + \frac{u_i}{\mu_{1i}(\dot{x}_i)} \quad (4)$$

where  $\mu_{1i}(\dot{x}_i)$  denotes the  $i$ th vehicle's engine/brake time-constant when the  $i$ th vehicle is traveling with a speed equal to  $\dot{x}_i$ .

From equ.(3) it is obtained the following expression,

$$F_i = m_i\ddot{x}_i + K_{di}\dot{x}_i^2 + d_{mi} \quad (5)$$

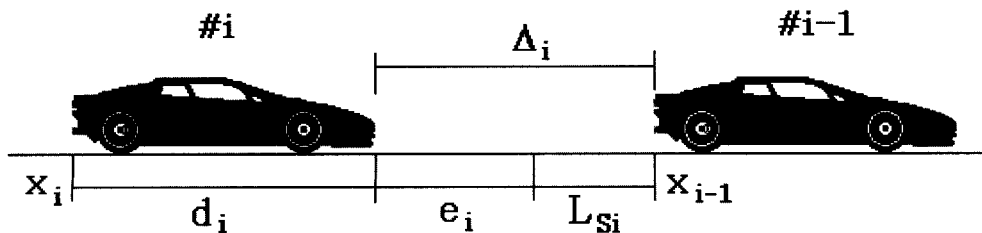


Fig. 1. Platoon of vehicles

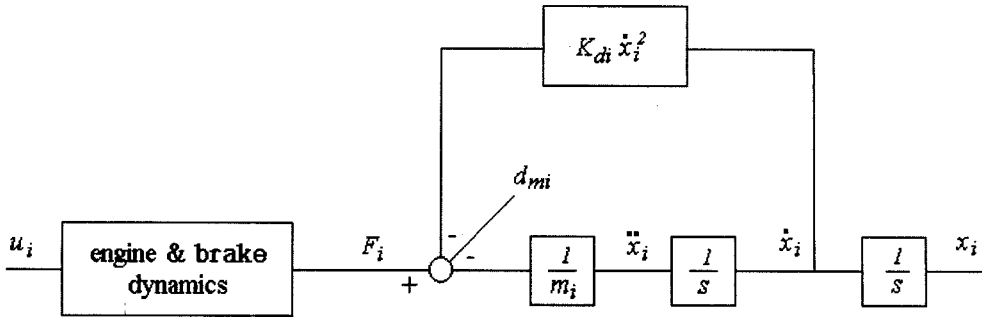


Fig. 2. Simplified model of the *i*th vehicle in the platoon

Substituting equ.(5) into equ.(4), it can be obtained as follows:

$$F_i = -\frac{1}{\mu_{1i}(\dot{x}_i)} m_i \ddot{x}_i + K_{di} \dot{x}_i^2 + d_{mi} + \frac{u_i}{\mu_{1i}(\dot{x}_i)} \quad (6)$$

Differentiating equ.(3) with respect to time and substituting into equ.(6) it is obtained as follows:

$$\frac{d^3 x_i}{dt^3} = -2 \frac{K_{di}}{m_i} \dot{x}_i \ddot{x}_i - \frac{1}{\mu_{1i}(\dot{x}_i)} \left[ \ddot{x}_i + \frac{K_{di}}{m_i} \dot{x}_i^2 + \frac{d_{mi}}{m_i} \right] + \frac{u_i}{m_i \mu_{1i}(\dot{x}_i)} \quad (7)$$

The accurate values of the *i*th vehicle's velocity and acceleration denoted as  $\dot{x}_i$  and  $\ddot{x}_i$ , are required to obtain an exact linearization of vehicle dynamics. The vehicle mass  $m_i$  may be varied at each traveling because of the increasing/decreasing the number of passengers, etc.

In this paper, the state observer with switching action will be designed to estimate  $\dot{x}_i$  and  $\ddot{x}_i$ . The feedback linearization control law will be designed by using the observed states. It will be shown that the standard parameter adaptation law for the vehicle mass  $m_i$  and time-constant  $\mu_i(\dot{x}_i)$  guarantees the stability of the adaptive control system. It is the objective of the paper that the proposed observer-based adaptive control law guarantees the convergence of the errors of intervehicle distance, velocity and acceleration between *i*th and *i*-1th vehicles.

### 3. The Observer and Adaptive Controller Design

#### 3.1 THE STATE OBSERVER

In this section, the switching mode nonlinear observer is designed to estimate  $\dot{x}_i$  and  $\ddot{x}_i$ . The vehicle speed  $\dot{x}_i$  may be measured directly by the speed meter. However, in order to guarantee the robustness for the measurement noise, the estimates of  $\dot{x}_i$  will be used to design the adaptive longitudinal controller. The measured value of will be used in the switching mode nonlinear observer. To simplify the structure of the observer, the model equ.(3) is used to design the observer. The equ.(3) can be rewritten using the parameterization method as follows:

$$\ddot{x}_i = \frac{1}{m_i} F_i - \frac{K_{di}}{m_i} \dot{x}_i^2 - \frac{d_{mi}}{m_i} = p_{1i}^T Y_{1i} \quad (8)$$

where  $p_{1i}^T = \left[ \frac{1}{m_i} - \frac{K_{di}}{m_i} \dot{x}_i^2 - \frac{d_{mi}}{m_i} \right]$  and  $Y_{1i}^T = [F_i \ \dot{x}_i^2 \ 1]$  are denoted the parameter vector and the measurement vector, respectively. For the observer model equ.(8), the proposed observer is designed as follows:

$$\begin{aligned} \ddot{\hat{x}}_i &= -k_{ol}(\hat{x}_i - \dot{x}_i) + p_{1i}^{-T} Y_{1i} - \xi_{1i} S(\dot{\hat{x}}_i), \\ S(\dot{\hat{x}}_i) &= \begin{cases} 1, & \dot{\hat{x}}_i > 0 \\ -1, & \dot{\hat{x}}_i < 0 \end{cases}, \end{aligned} \quad (9)$$

$\xi_{1i} \geq \left\| p_{1i}^{-T} - p_{1i}^T \right\|_{\infty} \|Y_{1i}\|_{\infty}$ , where  $k_{ol}(>0)$  denotes the observer gain;  $p_{1i}^{-T}$  denotes the primary vector of the parameter vector  $p_{1i}^T$ ;  $\|\cdot\|_{\infty}$  denotes  $L_{\infty}$  norm;  $\tilde{x}_i = \hat{x}_i - x_i$  denotes the state observation error. The upper bound of the deviation between  $p_{1i}^T$  and  $p_{1i}^{-T}$  can be obtained primarily. Subtracting equ.(8) from equ.(9), the state error model can be written as follows:

$$\ddot{\tilde{x}}_i = -k_{ol}\dot{\tilde{x}}_i + \overline{p}_{1i}^{-T} Y_{1i} - p_{1i}^T Y_{1i} - \xi_{1i} S(\dot{\tilde{x}}_i) \quad (10)$$

Defining the Lyapunov function candidate as  $V_{1i} = \frac{1}{2}\dot{\tilde{x}}_i^2$ , its 1st derivative is written as follows:

$$\dot{V}_{1i} = \dot{\tilde{x}}_i \ddot{\tilde{x}}_i = -k_{ol}\dot{\tilde{x}}_i + \overline{p}_{1i}^{-T} Y_{1i} \dot{\tilde{x}}_i - p_{1i}^T Y_{1i} \dot{\tilde{x}}_i - \xi_{1i} S(\dot{\tilde{x}}_i) \dot{\tilde{x}}_i \quad (11)$$

$$\leq -k_{ol}\dot{\tilde{x}}_i^2 + \|\overline{p}_{1i}^{-T} - p_{1i}^T\|_\infty \|Y_{1i}\|_\infty |\dot{\tilde{x}}_i| - \xi_{1i} |\dot{\tilde{x}}_i|.$$

where  $\xi_{1i}$  will be used to guarantee the boundedness of the error equation. Using the relationship  $\xi_{1i} \geq \|\overline{p}_{1i}^{-T} - p_{1i}^T\|_\infty \|Y_{1i}\|_\infty$  in equ.(9), the above equ.(11) can be written as follows:

$$\begin{aligned} \dot{V}_{1i} &\leq -k_{ol}\dot{\tilde{x}}_i^2 = -2k_{ol}V_{1i} \\ \dot{V}_{1i} + 2k_{ol}V_{1i} &\leq 0, \\ V_{1i} &\leq V_{1i}(0)e^{-2k_{ol}t} \end{aligned} \quad (12)$$

It can be shown that  $V_{1i}$  is exponentially stable as the reference paper[12], and thus  $V_{1i}$ ,  $\dot{V}_{1i}$ ,  $\dot{\tilde{x}}_i$  and  $\ddot{\tilde{x}}_i$  converge to zero exponentially fast, i.e., the observation errors can be satisfied as  $\dot{V}_{1i} \rightarrow 0$ ,  $V_{1i} \rightarrow 0$  and  $\dot{\tilde{x}}_i \rightarrow 0$ ,  $\ddot{\tilde{x}}_i \rightarrow 0$ ,  $\forall t \geq 0$ .

### 3.2 The adaptive controller

In this section, the adaptive feedback linearization controller is designed using the state estimates  $\dot{\tilde{x}}_i$  and  $\dot{\tilde{x}}_i$ . Using the parameterization technique, the control model equ.(7) can be rewritten as follows:

$$\begin{aligned} \frac{d}{dt}x_i &= -2\frac{K_{di}}{m_i}\dot{\tilde{x}}_i\ddot{\tilde{x}}_i - \frac{1}{\mu_{1i}(\dot{\tilde{x}}_i)}\left[\ddot{\tilde{x}}_i + \frac{K_{di}}{m_i}\dot{\tilde{x}}_i^2 + \frac{d_{mi}}{m_i}\right] + \frac{u_i}{m_i\mu_{1i}(\dot{\tilde{x}}_i)} \\ &= \Theta_{1i}^T \Psi_{1i} + \theta_{5i}u_i \end{aligned} \quad (13)$$

where

$$\Theta_{1i}^T = -2\frac{K_{di}}{m_i} - \frac{1}{\mu_{1i}(\dot{\tilde{x}}_i)} - \frac{K_{di}}{m_i\mu_{1i}(\dot{\tilde{x}}_i)} - \frac{d_{mi}}{m_i\mu_{1i}(\dot{\tilde{x}}_i)} \text{ and}$$

$$\theta_{5i} = \frac{1}{m_i\mu_{1i}(\dot{\tilde{x}}_i)} \text{ denote the parameters, respectively;}$$

$$\Psi_{1i}^T = [\dot{\tilde{x}}_i \ddot{\tilde{x}}_i \dot{\tilde{x}}_i^2 \ 1] \text{ denotes the measurement}$$

vector. Using the parameterized control model equ.(13), the adaptive feedback linearization controller is designed as follows:

$$u_i = \frac{1}{\hat{\theta}_{5i}}\{v_i - \hat{\Theta}_{1i}^T \hat{\Psi}_{1i}\} \quad (14)$$

where  $\hat{\theta}_{5i}$  and  $\hat{\Theta}_{1i}$  denote the parameter estimates;  $\hat{\Psi}_{1i}$  denotes the observed vector of the measurement. If the exact linearization is satisfied, the error equation with the exogeneous input  $v_i$  may be written as follows:

$$\begin{aligned} \frac{d}{dt}\ddot{\tilde{x}}_i &= v_i \\ &= \frac{d}{dt}\ddot{x}_{di} + k_{1i}(\ddot{x}_{di} - \ddot{\tilde{x}}_i) + k_{2i}(\dot{x}_{di} - \dot{\tilde{x}}_i) + k_{3i}(x_{di} - x_i) \end{aligned} \quad (15)$$

Adding  $k_{1i}\ddot{\tilde{x}}_i + k_{2i}\dot{\tilde{x}}_i - k_{1i}\ddot{x}_{di} - k_{2i}\dot{x}_{di}$  in the above equ.(15), it can be obtained as follows:

$$\begin{aligned} \frac{d}{dt}(\ddot{x}_{di} - \ddot{\tilde{x}}_i) + k_{1i}(\ddot{x}_{di} - \ddot{\tilde{x}}_i) + k_{2i}(\dot{x}_{di} - \dot{\tilde{x}}_i) \\ + k_{3i}(x_{di} - x_i) &= k_{1i}\ddot{\tilde{x}}_i + k_{2i}\dot{\tilde{x}}_i \end{aligned} \quad (16)$$

where  $x_{di}$  is desired state. Using  $\dot{\tilde{x}}_i \rightarrow 0$ ,  $\ddot{\tilde{x}}_i \rightarrow 0$ , the results in equ.(12), the above error equation is asymptotically stable, i.e.,  $\frac{d}{dt}(\ddot{x}_{di} - \ddot{\tilde{x}}_i) \rightarrow 0$ ,  $(\ddot{x}_{di} - \ddot{\tilde{x}}_i) \rightarrow 0$ ,  $(\dot{x}_{di} - \dot{\tilde{x}}_i) \rightarrow 0$  and  $(x_{di} - x_i) \rightarrow 0$ ,  $\forall t \geq 0$ .

Substituting equ.(14) into equ.(13) to investigate the exact linearizing ability of equ.(14), it is obtained as follows:

$$\begin{aligned} \frac{d}{dt}\ddot{\tilde{x}}_i &= \Theta_{1i}^T \Psi_{1i} + \theta_{5i} \frac{1}{\hat{\theta}_{5i}} \{v_i - \hat{\Theta}_{1i}^T \hat{\Psi}_{1i}\} \\ &= \Theta_{1i}^T \Psi_{1i} + \frac{\theta_{5i} - \hat{\theta}_{5i} + \hat{\theta}_{5i}}{\hat{\theta}_{5i}} \{v_i - \hat{\Theta}_{1i}^T \hat{\Psi}_{1i}\} \\ &= \Theta_{1i}^T \Psi_{1i} - \hat{\Theta}_{1i}^T \hat{\Psi}_{1i} + \hat{\theta}_{5i}u_i + v_i \end{aligned} \quad (17)$$

Adding  $\Theta_{1i}^T \hat{\Psi}_{1i} - \hat{\Theta}_{1i}^T \hat{\Psi}_{1i}$  to the right hand side in equ.(17), it is obtained as follows:

$$\begin{aligned} \frac{d}{dt}\ddot{\tilde{x}}_i &= \Theta_{1i}^T \Psi_{1i} - \hat{\Theta}_{1i}^T \hat{\Psi}_{1i} + \Theta_{1i}^T \hat{\Psi}_{1i} - \hat{\Theta}_{1i}^T \hat{\Psi}_{1i} + \hat{\theta}_{5i}u_i + v_i \\ &= \left\{ \Theta_{1i}^T - \hat{\Theta}_{1i}^T \right\} \hat{\Psi}_{1i} - \hat{\Theta}_{1i}^T \left\{ \hat{\Psi}_{1i} - \Psi_{1i} \right\} + \hat{\theta}_{5i}u_i + v_i \end{aligned}$$

$$= \tilde{\Theta}_{1i}^T - \hat{\Psi}_{1i} + \tilde{\Theta}_{1i}^T \hat{\Psi}_{1i} + \tilde{\theta}_{5i} u_i + v_i \quad (18)$$

The exogeneous input  $v_i$  is designed as follows:

$$v_i = \frac{d}{dt} \ddot{x}_{ri} + \lambda_i (\ddot{x}_{ri} - \ddot{x}_i), \quad (19)$$

$$\frac{d}{dt} \ddot{x}_{ri} = \frac{d}{dt} \ddot{e}_i + a_{1i} \ddot{e}_i + a_{2i} \dot{e}_i + \frac{d}{dt} \ddot{x}_i \quad (20)$$

where  $\lambda_i$  ( $> 0$ ) is a design parameter; is a auxiliary reference input. Substituting equ.(19) into equ.(18), it is obtained as follows:

$$\frac{d}{dt} \ddot{x}_i = \frac{d}{dt} \ddot{x}_{ri} + \lambda_i (\ddot{x}_{ri} - \ddot{x}_i) + \tilde{\Theta}_{2i}^T \hat{\Psi}_{2i} - \tilde{\Theta}_{1i}^T \hat{\Psi}_{1i}, \quad (21)$$

where  $\tilde{\Theta}_{2i}^T \hat{\Psi}_{2i} = \tilde{\Theta}_{1i}^T \hat{\Psi}_{1i} + \tilde{\theta}_{5i} u_i$  depends on the parameter estimation error term;  $\tilde{\Theta}_{2i} = [\tilde{\theta}_{1i} \tilde{\theta}_{2i} \tilde{\theta}_{3i} \tilde{\theta}_{4i} \tilde{\theta}_{5i}]$ ;  $\hat{\Psi}_{1i} = [\hat{\psi}_{1i} \hat{\psi}_{2i} \hat{\psi}_{3i} \hat{\psi}_{4i} \hat{\psi}_{5i}]$ . Defining the auxiliary error as  $x_{ei} = x_{ri} - x_i$ , the above equ.(21) can be rewritten as follows:

$$\frac{d}{dt} \ddot{x}_{ei} = -\lambda_i \ddot{x}_{ei} - \tilde{\Theta}_{2i}^T \hat{\Psi}_{2i} + \tilde{\Theta}_{1i}^T \hat{\Psi}_{1i} + \lambda_i (\ddot{x}_i - \ddot{x}_i). \quad (22)$$

As the result, it can be shown that the auxiliary error  $\ddot{x}_{ei}$  does not grow more rapidly than the exponential fashion. Defining the Lyapunov function candidate as  $V_{2i} = \frac{1}{2} \ddot{x}_{ei}^2 + \frac{1}{2} \tilde{\Theta}_{ei}^T \Gamma^{-1} \tilde{\Theta}_{2i}$ , its 1st derivative can be obtained as follows:

$$\begin{aligned} \dot{V}_{2i} = & -\lambda_i \ddot{x}_{ei} - \tilde{\Theta}_{2i}^T \hat{\Psi}_{2i} \ddot{x}_{ei} + \tilde{\Theta}_{1i}^T \Gamma^{-1} \dot{\tilde{\Theta}}_{2i} \\ & + \tilde{\Theta}_{1i}^T \hat{\Psi}_{1i} \ddot{x}_{ei} + \lambda_i \ddot{x}_i \ddot{x}_{ei} \end{aligned} \quad (23)$$

Using a standard gradient method, the parameter adaptation law is designed as follows:

$$\dot{\tilde{\Theta}}_{2i} = \Gamma \hat{\Psi}_{2i} (\ddot{x}_{ri} - \ddot{x}_i), \quad \Gamma = \Gamma^T > 0 \quad (24)$$

Substituting equ.(24) into equ.(23), it can be obtained as follows:

$$\begin{aligned} \dot{V}_{2i} = & -\lambda_i \ddot{x}_{ei}^2 - \tilde{\Theta}_{2i}^T \hat{\Psi}_{2i} \ddot{x}_{ei} + \tilde{\Theta}_{2i}^T \hat{\Psi}_{2i} (\ddot{x}_{ri} - \ddot{x}_i) \\ & + \tilde{\Theta}_{1i}^T \hat{\Psi}_{1i} \ddot{x}_{ei} + \lambda_i \ddot{x}_i \ddot{x}_{ei} \end{aligned} \quad (25)$$

Adding into the parenthesis at the right hand in the above, it is obtained as follows:

$$V_{2i} = -\lambda_i \ddot{x}_{ei}^2 - \tilde{\Theta}_{2i}^T \hat{\Psi}_{2i} \ddot{x}_{ei} + \tilde{\Theta}_{1i}^T \hat{\Psi}_{1i} \ddot{x}_{ei} + \lambda_i \ddot{x}_i \ddot{x}_{ei} \quad (26)$$

In the above,  $\dot{\tilde{x}}_i$ ,  $\ddot{\tilde{x}}_i$ ,  $\tilde{\Psi}_{1i}$  converge to zero exponentially fast, the following inequality can be satisfied

$$\dot{V}_{2i} \leq -\lambda_i \ddot{x}_{ei} + \varepsilon_i \quad (27)$$

where  $\varepsilon_i$  denotes the residual decreasing exponentially fast. And thus, it can be shown that  $V_{2i}$  is asymptotically stable, i.e.,  $\ddot{x}_{ei}$ ,  $V_{2i}$  converge to zero asymptotically. And it can be also shown that  $\tilde{\Theta}_{2is}$ ,  $\tilde{\Theta}_{2is}$ ,  $\tilde{\Theta}_{2i}$  and the signals in the closed loop system are all bounded[12].

Using the relationships equ.(1), equ.(2), equ.(20) and the result of  $\ddot{x}_{ei} \rightarrow 0$ , the error equation can be obtained as follows:

$$\begin{aligned} \frac{d}{dt} \ddot{x}_{ri} - \frac{d}{dt} \ddot{x}_i = & \dot{\Delta}_i + a_{1i} \dot{\Delta}_i + a_{2i} \Delta_i + \ddot{x}_i + a_{2i} L_{si} - \ddot{x}_i \\ = & \dot{e}_i + a_{1i} \dot{e}_i + a_{2i} e_i + \ddot{x}_i \rightarrow 0 \end{aligned} \quad (28)$$

Therefore it can be shown that  $e_i$ ,  $\dot{e}_i$ ,  $\ddot{e}_i$  and  $\dot{\Delta}_i$ ,  $\Delta_i$  converge to zero asymptotically, and also that  $\Delta_i$  converges to  $L_{si}$ . It is easily shown that the  $i$ th vehicle tracking to  $i-1$ th vehicle can keep the safe intervehicle spacing  $L_{si}$ .

## 4. Simulation Results

In this section, the numerical examples are presented to investigate the effectiveness of the proposed longitudinal control law. The three vehicles in the platoon is used to simulate the controller. It is assumed that all vehicles travel to the same direction. The vehicle parameters represented in table 1. are the same as in the reference paper[2]. The order in which the vehicles followed lead vehicle was as follows: Daihatsu Charade CLS followed by Buick Regal Custom followed by BMW 750iL. The number of passengers in each vehicle and their respective masses were as follows: Daihatsu Charade CLS- 3 passengers each with a mass of 91[Kg]; Buick Regal Custom- 2

**Table 1.** The vehicle parameters

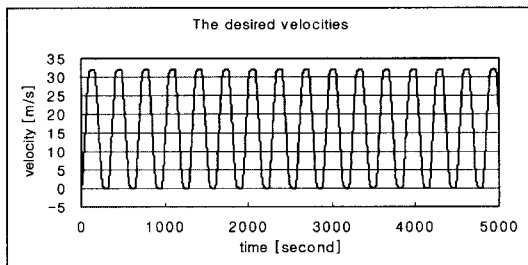
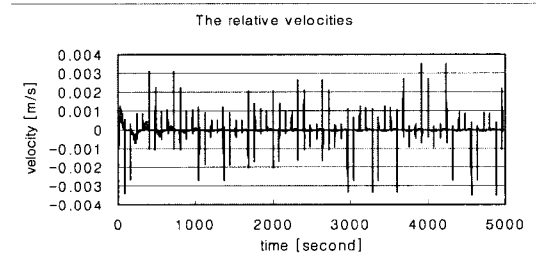
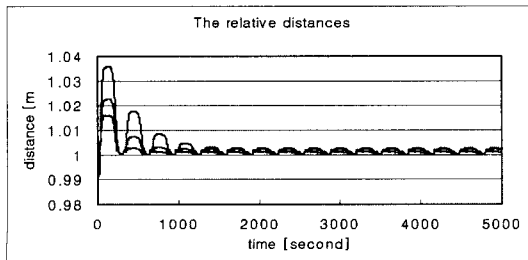
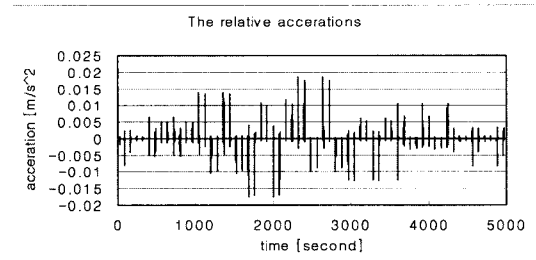
vehicles type	curb mass	cross-sectional area	drag coefficient	time-constant
Daihatsu Charade CLS	916 Kg	1.9 m <sup>2</sup>	$K_d=0.44$ Kg/m	0.2 sec
Buick Reagal Custom	1464 Kg	2.2 m <sup>2</sup>	$K_d=0.49$ Kg/m	0.25 sec
BMW 750iL	1925 Kg	2.25 m <sup>2</sup>	$K_d=0.51$ Kg/m	0.2 sec

passengers each with a mass of 64[Kg]; BMW 750iL- 4 passengers with the following masses, 45, 45, 91, 59[Kg].

The speed profile of the lead vehicle is represented as in the Fig. 3. For  $i=1,2,3$ , the control gains were chosen as follows:  $a_{1i}=10$ ,  $a_{2i}=5$ ,  $\lambda_i=5$ . The safe intervehicle space was selected as  $L_{si}=1$ [m]. The diagonal elements of the parameter adaptation gain matrix  $\Gamma_i$  were all selected as 0.00002. The relative position(i.e., intervehicle spacing) error does not increase more than 4[cm] as shown in Fig. 4. The relative speed errors and the relative acceleration errors of the intervehicles are bounded as shown in Fig. 5 and Fig. 6. The parameter estimates norms of each vehicle are also bounded as shown in Fig. 7.

## 5. Conclusions

In this paper, the sliding mode observer-based adaptive feedback linearization controller was proposed to control the longitudinal motion of the vehicles in the platoon. The vehicle mass, one of the system parameters, may be varied by the number of passengers and loads. And time delay of the powertrain dynamics is also variable based on the vehicle speed. It was shown that the time-varying property and the uncertainty of the vehicle parameters can be covered by using a standard parameter adaptation law without any modifications. A sliding mode observer was designed to estimate the vehicle speed and acceleration required to the feedback linearization control law. It was shown that the

**Fig. 3.** The velocity trajectory followed by the lead car**Fig. 5.** The relative velocity errors**Fig. 4.** The relative position**Fig. 6.** The relative acceleration errors

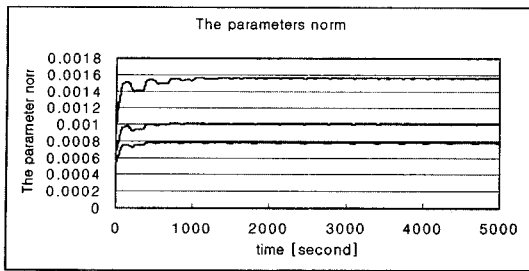


Fig. 7. The parameter estimates norms of each vehicle

observer-based adaptive control law guarantees the asymptotic stability and the good tracking performance of the closed loop system. It was also shown that the proposed control method was effective for the longitudinally traveled vehicles with the unknown parameters and the unknown states.

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