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균일단면 선박의 유탄성 수평응답에 대한 해석해

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Exact Solution on the Anti-symmetric Responses of Ships having Uniform Sectional Properties with Hydro-elasticity

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Abstract

Exact solution on the anti-symmetric response of ships having uniform sectional properties in waves is derived. Boundary value problem consisted of Timoshenko beam equation and free-free end condition is solved analytically. The responses are assumed as linear and wave loads are calculated by using strip method. Horizontal bending moment, shear force and torsional moment are calculated. The developed analysis model is used for the benchmark test of the numerical codes in this problem. Also the application on the preliminary design of barge-like ships and VLFS (Very Large Floating Structure) is expected

※Keywords: Anti-symmetric Response(수평응답), Hydro-elasticity(유탄성), Exact Solution (해석해), Timoshenko Beam(티모쉔코 보), Horizontal Bending Moment(수평 굽힘 모멘트), Torsional Moment(비틂 모멘트), Warping(워핑)

1. INTRODUCTION

The vertical responses of ships considering hydro-elasticity have been studied by the authors (박인규 등 2004). The result showed that the exact solution is fairly useful for the validation and early estimation of the ship responses. Under the same concept with those previous works, it is quite natural to extend the scope of the work to horizontal responses of the uniform ship.

In this paper, exact solution on the hydro-elastic response of ships having uniform sectional properties in waves is derived. Timoshenko

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beam equation is used as the governing equation. Free-free end conditions at the bow and stern are used as the boundary conditions. The responses are assumed as linear to get the analytic solutions. Horizontal bending moment, shear force and torsional moment are calculated. Total hydrodynamic forces are calculated using strip method (korobkin 2003).

Ship springing is known as the steady resonance response of the ships to incident waves. There are a lot of previous works on this topic. But only a limited number of them are dealing with the anti-symmetric responses. Bishop/Price(1979) dealt with the anti-symmetric responses. Linear theory with strip methods is studied by Hoffman/van Hooff(1976). Troesch (1984) studied springing experimentally and theoretically. Jensen/Pedersen (1979, 1981) suggested quadratic strip theory, but only for vertical bending moment. Besides, in domestic, 이호영 등(2001, 2003) analyzed the ship springing using 3-D pulsating and translating source method. Jung et al.(2003) and 정종진/박인규(2004) developed a numerical model using quadratic strip theory.

2. BVP FORMULATION

The anti-symmetric responses of the advancing ship are described within the Timoshenko beam theory as follows. They are able to account for the shear deformation as well as St. venant and warping torsion of the ship-like hull girders. Coordinate system is shown in Fig. 1.

Timoshenko Beam Equation

$$\frac{\partial u_y}{\partial x} = \theta_z(x,t) + \gamma_z(x,t) \quad (1)$$

$$EI(x)\left[1+\eta_z(x)\frac{\partial}{\partial t}\right]\frac{\partial \theta_z}{\partial x} = M_z(x,t) \quad (2)$$

$$\frac{\partial M_z}{\partial x} = I_z(x)\frac{\partial^2 \theta_z}{\partial t^2} - V_y(x,t) \quad (3)$$

$$\frac{\partial V_y}{\partial x} + F_y(x,t) = \mu(x)\frac{\partial^2 v_C}{\partial t^2} \quad (4)$$

$$k_y(x)A(x)G\left[1+\alpha_y(x)\frac{\partial}{\partial t}\right]\gamma_z(x,t) = V_y(x,t) \quad (5)$$

$$\frac{\partial \theta_x}{\partial x} = P(x,t) \quad (6)$$

$$\frac{\partial P}{\partial x} = \frac{R(x,t)}{EI_W(x)} \quad (7)$$

$$\frac{\partial R}{\partial x} = GI_t(x)\left[1+\kappa(x)\frac{\partial}{\partial t}\right]P(x,t) - M_x(x,t) \quad (8)$$

$$\frac{\partial M_x}{\partial x} + z_{SC}(x)\frac{\partial V_y}{\partial x} = I_C(x)\frac{\partial^2 \theta_x}{\partial t^2} - K(x,t) \quad (9)$$

$$u_y(x,t) = v_C(x,t) + z_{SC}(x)\sin[\theta_x(x,t)] \quad (10)$$

$$v_C(x,t) = u(x,t) + z_C(x)\sin[\theta_x(x,t)] \quad (11)$$

In (1) - (11), the notations are;

$u_y(x,t)$, Horizontal displacement of the shear center of the ship section

$v_C(x,t)$, Horizontal displacement of the mass center of the ship section

$u(x,t)$, Horizontal displacement of the point O' of the ship section

x, Distance of the ship section from the stern

t, Time

$\theta_z(x,t)$, Horizontal bending slope

$\gamma_z(x,t)$, Vertical shear deformation per unit ship length

E, Modulus of elasticity for the material of the ship

$I(x)$, Cross sectional second moment

$\eta_z(x)$, Horizontal structural damping coefficient per unit ship length

$M_z(x,t)$, Horizontal bending moment of the ship girder

$M_x(x,t)$, Torsional moment of the ship girder

$V_y(x,t)$, Transverse shear force per unit ship length

$I_z(x)$, Inertia moment of the cross section in horizontal bending

$I_t(x)$, Saint Venant' s uniform torsion constant

$I_w(x)$, Bredt torsion constant

$z_C(x)$, Distance between the mass center and the point O'

$z_S(x)$, Distance between the shear center and the point O'

$z_{SC}(x)$, Distance between the mass center and the shear center

$\mu(x)$, Mass of the ship per unit length

$F_y(x,t)$, Horizontal total hydrodynamic force per unit ship length

$K(x,t)$, Hydrodynamic torsional moment with respect to the mass center

$A(x)$, Cross sectional steel area of the ship

$k_y(x)$, Horizontal shear coefficient per unit ship length

G , Modulus of elasticity in shear for steel

$\alpha_y(x)$, Horizontal shear structural damping coefficient per unit ship length

$P(x,t),\quad R(x,t)$, Auxiliary functions

The solution of equation (1) - (11) satisfies the free-free beam boundary conditions

$$V_y = 0,\ M_z = 0,\ M_x = 0,\ R = 0,\ at\ x = 0\ and\ x = L \tag{12}$$

where x=0 corresponds to the ship stern and x=L to the ship bow.

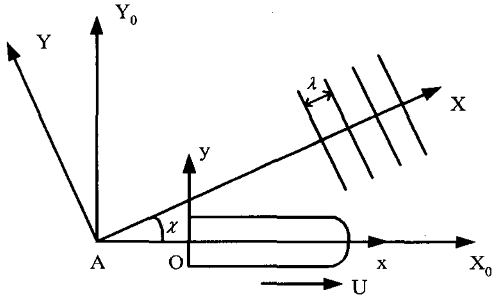

Fig. 1 Local and global coordinate systems

The solution of the boundary value problem is sought using standard method. The partial differential equation is converted into ordinary differential equation. If the warping is not taken into account, equations (6)-(8) must be replaced by the equation

$$GI_t(x)\left[1 + \kappa(x)\frac{\partial}{\partial t}\right]P(x,t) = M_x(x,t) \tag{13}$$

Within the linear strip theory, the horizontal total hydrodynamic force per unit ship length $F_y(x,t)$ is given as;

$$F_y(x,t) = -\Re\left[\frac{D}{Dt}\left(m^c_{yy}(x,\omega_e)\frac{D\overline{Y}_1}{Dt}\right) + \frac{D}{Dt}\left(m^c_{y\phi}(x,\omega_e)\frac{D\overline{\Phi}_1}{Dt}\right)\right] + \Re\left[\widetilde{F}^{FK}_y(x)e^{-i\omega_e t}\right] \tag{14}$$

$$\widetilde{F}^{FK}_y(x) = -\rho g i A k_s U_2(x) e^{ik_c x} \tag{15}$$

and the hydrodynamic torsional moment as,

$$K(x,t) = -\Re\left[\frac{D}{Dt}\left(m^c_{y\varphi}\frac{D\overline{Y}_1}{Dt} + m^c_{\varphi\varphi}\frac{D\overline{\Phi}_1}{Dt}\right)\right] + z_c(x)\Re\left[\frac{D}{Dt}\left(m^c_{yy}\frac{D\overline{Y}_1}{Dt} + m^c_{y\varphi}\frac{D\overline{\Phi}_1}{Dt}\right)\right] + \Re\left[\widetilde{K}^{FK}(x)e^{-i\omega_e t}\right] \tag{16}$$

$$\widetilde{K}^{FK}(x) = -\rho g\left[U_3(x) + z_C(x)S(x)\right]\widetilde{\theta}_x(x) + i\rho g A\left[U_1(x) + kz_C(x)U_2(x)\right]\sin\chi\, e^{ik_c x} \tag{17}$$

where, $\frac{D}{Dt} = \frac{\partial}{\partial t} - U\frac{\partial}{\partial x}$.

3. EXACT SOLUTION

By applying the uniform beam assumption, the hydrodynamic force $F_y(x,t)$, given by the equation (14), can be reduced as followings,

$$F_y(x,t) = -\Re\left[m_{yy}^c(\omega_e)\frac{D}{Dt}\left(\frac{D\overline{Y}_1}{Dt}\right) + m_{y\varphi}^c(\omega_e)\frac{D^2\overline{\Phi}_1}{Dt^2} + \rho giAk_sU_2e^{ik_cx}e^{-i\omega_et}\right] \quad (18)$$

$$\overline{\Phi}_1(x,t) = \left[\tilde{\theta}_x(x) - \tilde{\gamma}_1(x)\right]e^{-i\omega_et} \quad (19)$$

$$\frac{D\overline{Y}_1}{Dt} = \left[-i\omega_e\tilde{u}(x) - U\frac{d\tilde{u}}{dx} - \tilde{v}(x)\right]e^{-i\omega_et} \quad (20)$$

$$\tilde{v}(x) = A\omega_0\sin\chi\, E_S\, e^{ik_cx} \quad (21)$$

$$\tilde{\gamma}_1(x) = iAk_sE_S\, e^{ik_cx} \quad (22)$$

$$\frac{D}{Dt}\left(\frac{D\overline{Y}_1}{Dt}\right) = \left\{i\omega_e\left[i\omega_e\tilde{u} + U\frac{d\tilde{u}}{dx} + \tilde{v}\right] + U\left[i\omega_e\frac{d\tilde{u}}{dx} + U\frac{d^2\tilde{u}}{dx^2} + \frac{d\tilde{v}}{dx}\right]\right\}e^{-i\omega_et}$$
$$= \left\{-\omega_e^2\tilde{u} + 2i\omega_eU\frac{d\tilde{u}}{dx} + U^2\frac{d^2\tilde{u}}{dx^2} + i\omega_e\tilde{v} + U\frac{d\tilde{v}}{dx}\right\}e^{-i\omega_et} \quad (23)$$

$$U\frac{d\tilde{v}}{dx} + i\omega_e\tilde{v} = (ik_cU + i\omega_e)A\omega_0\sin\chi\, E_S\, e^{ik_cx} = iA\omega_0^2\sin\chi\, E_S\, e^{ik_cx} \quad (24)$$

$$\frac{D^2\overline{\Phi}_1}{Dt^2} = \left\{-\omega_e^2\left[\tilde{\theta}_x - \tilde{\gamma}_1\right] + 2i\omega_eU\frac{d}{dx}\left[\tilde{\theta}_x - \tilde{\gamma}_1\right] + U^2\frac{d^2}{dx^2}\left[\tilde{\theta}_x - \tilde{\gamma}_1\right]\right\}e^{-i\omega_et} \quad (25)$$

Equation (18) takes the form,

$$F_y(x,t) = -\Re\left[m_{yy}^c(\omega_e)\left\{-\omega_e^2\tilde{u} + 2i\omega_eU\frac{d\tilde{u}}{dx} + U^2\frac{d^2\tilde{u}}{dx^2}\right\} + m_{y\varphi}^c(\omega_e)\left\{-\omega_e^2\tilde{\theta}_x + 2i\omega_eU\frac{d\tilde{\theta}_x}{dx} + U^2\frac{d^2\tilde{\theta}_x}{dx^2}\right\} + Fe^{ik_cx}\right]e^{-i\omega_et} \quad (26)$$

$$F = \rho giAk_sU_2 + m_{yy}^c\omega_0^2\sin\chi\, E_S + m_{y\varphi}^c\omega_0^2k_sE_S = iA\sin\chi\left[\rho gkU_2 + m_{yy}^c\omega_0^2E_S + m_{y\varphi}^c\omega_0^2kE_S\right] \quad (27)$$

Also applying the uniform beam assumption, the hydrodynamic torsional moment $K(x,t)$,the equation (16), can be reduced as,

$$K(x,t) = -\Re\left[\left(m_{y\varphi}^c - z_cm_{yy}^c\right)\frac{D}{Dt}\left(\frac{D\overline{Y}_1}{Dt}\right) + \left(m_{\varphi\varphi}^c - z_cm_{y\varphi}^c\right)\frac{D^2\overline{\Phi}_1}{Dt^2} + \rho g(U_3 + z_cS)\tilde{\theta}_x^{(1)}e^{-i\omega_et} - i\rho gA(U_1 + kz_cU_2)\sin\chi\, e^{ik_cx}e^{-i\omega_et}\right], \quad (28)$$

$$K(x,t) = -\Re\left[\hat{m}_{y\varphi}^c(\omega_e)\left\{-\omega_e^2\tilde{u} + 2i\omega_eU\frac{d\tilde{u}}{dx} + U^2\frac{d^2\tilde{u}}{dx^2}\right\} + \hat{m}_{\varphi\varphi}^c(\omega_e)\left\{-\omega_e^2\tilde{\theta}_x + 2i\omega_eU\frac{d\tilde{\theta}_x}{dx} + U^2\frac{d^2\tilde{\theta}_x}{dx^2}\right\} + E_1\tilde{\theta}_x + E_2e^{-i\omega_et}\right]e^{-i\omega_et}, \quad (29)$$

$$\hat{m}_{y\varphi}^c = m_{y\varphi}^c - z_cm_{yy}^c \quad (30)$$

$$\hat{m}_{\varphi\varphi}^c = m_{\varphi\varphi}^c - z_cm_{y\varphi}^c \quad (31)$$

$$E_1 = \rho g(U_3 + z_cS) \quad (32)$$

$$E_2 = i\hat{m}_{y\varphi}^cA\omega_0^2E_s\sin\chi + i\hat{m}_{\varphi\varphi}^cA\omega_0^2k_sE_s - i\rho gA(U_1 + kz_cU_2)\sin\chi \quad (33)$$

Therefore, with the constant coefficients, the boundary value problem takes the following form,

$$\frac{d\tilde{u}}{dx} = \tilde{\theta}_z + \tilde{\gamma}_z - z_s\tilde{P} \quad (34)$$

$$\frac{d\tilde{\theta}_z}{dx} = \tilde{M}_z / a_1^c \quad (35)$$

$$\frac{d\tilde{M}_z}{dx} = -\omega_e^2I_z\tilde{\theta}_z - \tilde{V}_y \quad (36)$$

$$\frac{d\tilde{V}_y}{dx} + F_y(\tilde{x}) = -\omega_e^2\mu\left[\tilde{u} + z_c\tilde{\theta}_x\right] \quad (37)$$

$$\tilde{\gamma}_z = \frac{\tilde{V}_y}{a_2^c} \quad (38)$$

$$\frac{d\tilde{\theta}_x}{dx} = \tilde{P}(x) \quad (39)$$

$$\frac{d\widetilde{P}}{dx} = \frac{\widetilde{R}(x)}{EI_W} \quad (40)$$

$$\frac{d\widetilde{R}}{dx} = a_3^c\widetilde{P} - \widetilde{M}_x \quad (41)$$

$$\frac{d\widetilde{M}_x}{dx} + z_{SC}\frac{d\widetilde{V}_y}{dx} = -\omega_e^2 I_C\widetilde{\theta}_x - \widetilde{K}(x) \quad (42)$$

$$a_1^c = EI[1 - i\omega_e\eta_z] \quad (43)$$

$$a_2^c = k_y AG[1 - i\omega_e\alpha_y] \quad (44)$$

$$a_3^c = GI_t(x)[1 - i\omega_e\kappa] \quad (45)$$

$$F_y(x,t) = \Re\left[\widetilde{F}_y(x)e^{-i\omega_e t}\right] \quad (46)$$

$$K(x,t) = \Re\left[\widetilde{K}(x)e^{-i\omega_e t}\right] \quad (47)$$

The solution of the system of ordinary differential equations (34)-(42) has to satisfy the free-free beam boundary conditions.

$$V_y = 0,\ M_z = 0,\ M_x = 0,\ R = 0,\ at\ x = 0\ and\ x = L \quad (48)$$

Therefore, the boundary value problems are reduced as following form. In order to evaluate the second derivative, $\frac{d^2\widetilde{u}}{dx^2}$, we differentiate (34) and use (35), (37), (38) and (40).

$$\frac{d^2\widetilde{u}}{dx^2} = \frac{1}{a_1^c}\widetilde{M}_z + \frac{1}{a_2^c}\left[-\omega_e^2\mu(\widetilde{u} + z_c\widetilde{\theta}_x) - \widetilde{F}_y\right] - \frac{z_s}{EI_w}\widetilde{R} \quad (49)$$

The function $F_y(x)$ is obtained from (18)

$$F_y(x) = -m_{yy}^c\left[-\omega_e\widetilde{u} + 2i\omega_e U(\widetilde{\theta}_z + \frac{\widetilde{V}_y}{a_2^c} - z_s\widetilde{P})\right] - m_{y\varphi}^c\left[-\omega_e^2\widetilde{\theta}_x + 2i\omega_e U\widetilde{P} + \frac{U^2}{EI_w}\widetilde{R}\right] - Fe^{ik_c x} - m_{yy}^c U^2\frac{d^2\widetilde{u}}{dx^2} \quad (50)$$

By substituting (50) into equation (49) and collecting the terms with the second derivative $\frac{d^2\widetilde{u}}{dx^2}$, we obtain

$$\begin{aligned}
\left(1 - \frac{m_{yy}^c U^2}{a_2^c}\right)\frac{d^2\widetilde{u}}{dx^2} &= \left(-\frac{\omega_e^2\mu}{a_2^c} - \frac{m_{yy}^c\omega_e^2}{a_2^c}\right)\widetilde{u} \\
&+ \frac{2i\omega_e U m_{yy}^c}{a_2^c}\widetilde{\theta}_z + \frac{1}{a_1^c}\widetilde{M}_z + \frac{2i\omega_e U m_{yy}^c}{(a_2^c)^2}\widetilde{V}_y \\
&+ \left(-\frac{\omega_e^2\mu z_c}{a_2^c} - \frac{m_{yy}^c\omega_e^2}{a_2^c}\right)\widetilde{\theta}_x \\
&+ \frac{1}{a_2^c}\left[-2i\omega_e U m_{yy}^c z_s + 2i\omega_e U m_{y\varphi}^c\right]\widetilde{P} \\
&+ \left(m_{y\varphi}^c\frac{z_s}{EI_w a_2^c} - \frac{z_s}{EI_w}\right)\widetilde{R} + \frac{F}{a_2^c}e^{ik_c x}
\end{aligned} \quad (51)$$

It is convenient to introduce the unknown vector-function

$$\vec{Y} = (Y_1, Y_2, Y_3, Y_4, Y_5, Y_6, Y_7, Y_8) \quad (52)$$

$$\begin{aligned}
&Y_1(x) = \widetilde{u}(x),\quad Y_2(x) = \widetilde{\theta}_z(x), \\
&Y_3(x) = \widetilde{M}_z(x), Y_4(x) = \widetilde{V}_y(x), \\
&Y_5(x) = \widetilde{\theta}_x(x),\quad Y_6(x) = \widetilde{P}(x), \\
&Y_7(x) = \widetilde{R}(x),\quad Y_8(x) = \widetilde{M}_x(x)
\end{aligned} \quad (53)$$

with the help of which equation (51) can be written in the compact form

$$\frac{d^2\widetilde{u}}{dx^2} = \sum_{n=1}^{7} G_n Y_n(x) + G_0 e^{ik_c x} \quad (54)$$

$$\begin{aligned}
&G_1 = -\omega_e^2\frac{\mu + m_{yy}^c}{a_2^c - U^2 m_{yy}^c},\quad G_2 = \frac{2i\omega_e U m_{yy}^c}{a_2^c - U^2 m_{yy}^c}, \\
&G_3 = \frac{a_2^c / a_1^c}{a_2^c - U^2 m_{yy}^c},\quad G_4 = \frac{G_2}{a_2^c}, \\
&G_5 = -\omega_e^2\frac{\mu z_c + m_{y\varphi}^c}{a_2^c - U^2 m_{yy}^c}, G_6 = 2i\omega_e U\frac{m_{y\varphi}^c - z_c m_{yy}^c}{a_2^c - U^2 m_{yy}^c}, \\
&G_7 = -\frac{1}{EI_w}\frac{a_2^c z_s - U^2 m_{y\varphi}^c}{a_2^c - U^2 m_{yy}^c},\quad G_0 = \frac{F}{a_2^c - U^2 m_{yy}^c}
\end{aligned} \quad (55)$$

By substituting (50) and (54) into the equation (37), we obtain

$$\frac{dY_4}{dx} = -\omega_e^2 \mu\left[\tilde{u} + z_c\tilde{\theta}_x\right] + m_{yy}^c\left[-\omega_e^2\tilde{u} + 2i\omega_e U\left(\tilde{\theta}_z + \frac{\tilde{V}_y}{a_2^c} - z_s\tilde{P}\right)\right] + m_{y\varphi}^c\left[-\omega_e^2\tilde{\theta}_x + 2i\omega_e U\tilde{P} + \frac{U^2}{EI_w}\tilde{R}\right] + Fe^{ik_c x} + m_{yy}^c U^2\left(\sum_{n=1}^{7} G_n Y_n + G_0 e^{ik_c x}\right) = \sum_{n=1}^{8} A_{4n}Y_n + A_{40}e^{ik_c x} \quad (56)$$

$$A_{41} = -\omega_e^2\frac{\mu + m_{yy}^c}{1 - U^2 m_{yy}^c / a_2^c}, \quad (57)$$

$$A_{42} = \frac{2i\omega_e U m_{yy}^c}{1 - U^2 m_{yy}^c / a_2^c}, \quad (58)$$

$$A_{43} = \frac{U^2 m_{yy}^c}{a_1^c}\frac{1}{1 - U^2 m_{yy}^c / a_2^c}, \quad (59)$$

$$A_{44} = \frac{A_{42}}{a_2^c}, \quad (60)$$

$$A_{45} = -\omega_e^2\frac{\mu z_c + m_{y\varphi}^c}{1 - U^2 m_{yy}^c / a_2^c}, \quad (61)$$

$$A_{46} = 2i\omega_e U\frac{m_{y\varphi}^c - z_s m_{yy}^c}{1 - U^2 m_{yy}^c / a_2^c}, \quad (62)$$

$$A_{47} = \frac{U^2}{EI_w}\frac{m_{y\varphi}^c - z_s m_{yy}^c}{1 - U^2 m_{yy}^c / a_2^c}, \quad (63)$$

$$A_{48} = 0, \quad (64)$$

$$A_{40} = \frac{F}{1 - U^2 m_{yy}^c / a_2^c}, \quad (65)$$

Consider equation (42) taking into account (56), (29) and (54)

$$\frac{d\vec{Y}_8}{dx} = \sum_{n=1}^{8} A_{8n}Y_n + A_{80}e^{ik_c x} \quad (66)$$

$$A_{81} = \frac{\omega_e^2}{1 - U^2 m_{yy}^c / a_2^c} \times\left[z_{sc}\left(\mu + m_{yy}^c\right) - \hat{m}_{y\varphi}^c - \frac{U^2 \mu \hat{m}_{y\varphi}^c}{a_2^c}\right], \quad (67)$$

$$A_{82} = -z_{sc}A_{42} + 2i\omega_e U\hat{m}_{y\varphi}^c + \hat{m}_{y\varphi}^c U^2 G_2, \quad (68)$$

$$A_{83} = -z_{sc}A_{43} + \hat{m}_{y\varphi}^c U^2 G_3, \quad (69)$$

$$A_{84} = -z_{sc}A_{44} + 2i\omega_e U\frac{\hat{m}_{y\varphi}^c}{a_2^c} + \hat{m}_{y\varphi}^c U^2 G_4, \quad (70)$$

$$A_{85} = -\omega_e^2 I_c - z_{sc}A_{45} - \omega_e^2\hat{m}_{\varphi\varphi}^c + E_1 + \hat{m}_{y\varphi}^c U^2 G_5, \quad (71)$$

$$A_{86} = -z_{sc}A_{46} - 2i\omega_e U z_s\hat{m}_{y\varphi}^c + 2i\omega_e U\hat{m}_{\varphi\varphi}^c + \hat{m}_{y\varphi}^c U^2 G_6, \quad (72)$$

$$A_{87} = -z_{sc}A_{47} + \frac{U^2}{EI_w}\hat{m}_{\varphi\varphi}^c + \hat{m}_{y\varphi}^c U^2 G_7, \quad (73)$$

$$A_{88} = -z_{sc}A_{48} = 0, \quad (74)$$

$$A_{80} = -z_{sc}A_{40} + E_2 + \hat{m}_{y\varphi}^c U^2 G_0, \quad (75)$$

Other equations of system (34)-(42) have the forms

$$\frac{d\vec{Y}_j}{dx} = \sum_{n=1}^{8} A_{jn}Y_n, \quad (j = 1,2,3,5,6,7) \quad (76)$$

$$A_{11} = 0, \quad A_{12} = 1, \quad A_{13} = 0, \quad A_{14} = \frac{1}{a_2^c}, \quad A_{15} = 0, \quad A_{16} = -z_s, \quad A_{17} = 0, \quad A_{18} = 0 \quad (77)$$

$$A_{23} = \frac{1}{a_1^c}, \quad A_{2n} = 0, \quad (n \neq 3), \quad (78)$$

$$A_{32} = -\omega_e^2 I_z, A_{34} = -1, A_{3n} = 0, (n \neq 2,4), \quad (79)$$

$$A_{56} = 1, \quad A_{5n} = 0, \quad (n \neq 6), \quad (80)$$

$$A_{67} = \frac{1}{EI_w}, \quad A_{6n} = 0, \quad (n \neq 7), \quad (81)$$

$$A_{76} = a_3^c, A_{78} = -1, A_{7n} = 0, (n \neq 6,8). \quad (82)$$

In matrix form

$$\frac{d\vec{Y}}{dx} = A\vec{Y} + \vec{F}e^{ik_c x} \quad (83)$$

$$\vec{F} = [0, \ 0, \ 0, \ A_{40}, \ 0, \ 0, \ 0, \ A_{80}]^T \tag{84}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 1/a_2^c & 0 & -z_s & 0 & 0 \\ 0 & 0 & 1/a_2^c & 0 & 0 & 0 & 0 & 0 \\ 0 & -\omega_e^2 I_z & 0 & 0 & 0 & 0 & 0 & 0 \\ A_{41} & A_{42} & A_{43} & A_{44} & A_{45} & A_{46} & A_{47} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1/(EI_w) & 0 \\ 0 & 0 & 0 & 0 & 0 & a_3^c & 0 & -1 \\ A_{81} & A_{82} & A_{83} & A_{84} & A_{85} & A_{86} & A_{87} & 0 \end{bmatrix} \tag{85}$$

General solution of the equation (83) is

$$\vec{Y} = \vec{Y}^{(0)} e^{ik_c x} + \sum_{n=1}^{8} C_n \vec{Y}^{(n)} e^{\lambda_n x} \tag{86}$$

where $\vec{Y}^{(0)}$ is the solution of the algebraic equation

$$\mathrm{A}\vec{Y}^{(0)} + \vec{F} = ik_c \vec{Y}^{(0)} \tag{87}$$

λ_n are the eigenvalues of the matrix A

$$\det(\mathrm{A} - \lambda_n \mathrm{E}) = 0 \tag{88}$$

and $\vec{Y}^{(n)}$ the corresponding eigenvectors of the matrix A.

In representation (86) it is assumed that $\lambda_n \neq \lambda_m$, when $n \neq m$, which is not necessary the case. The eigenvalues λ_n and the eigenvectors $\vec{Y}^{(n)}$ are calculated with the help of standard routines. Calculations have been done with controlled accuracy. Eight complex coefficients C_n in (86) are determined from the boundary conditions (48). The conditions provide eight algebraic equations which are solved by Gauss method. The obtained solution can be considered as exact one because the original problem has been reduced to several standard problems – to find eigenvalues of a matrix, to find eigenvectors of a matrix, to solve a system of algebraic equations – each of them can be solved with a prescribed accuracy.

4. NUMERICAL EXAMPLES

Equation (86) provides exact solutions of the problem under consideration. Derived solutions can be used to calculate the artificial ship which has uniform sectional properties. The example ship is selected as the 281 meters long container ship. Hull and mass data with extensive calculations were provided by the Bishop/ Price(1979). Corresponding uniform artificial ship is established by averaging all the sectional properties along the ship length.

The calculated horizontal bending moment and torsional moment at midship in 135 degrees wave heading are plotted on the Fig. 2 and 3, respectively.

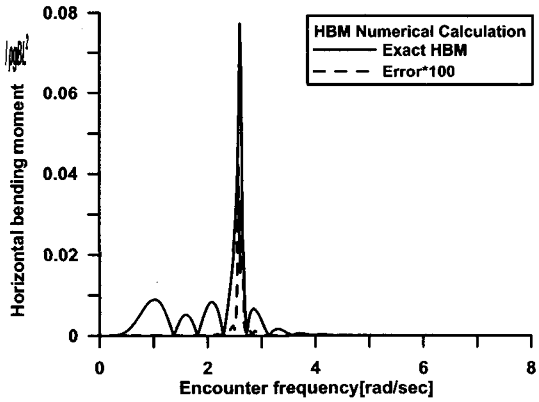

Fig. 2 HBM of uniform ship at 135 Deg. heading

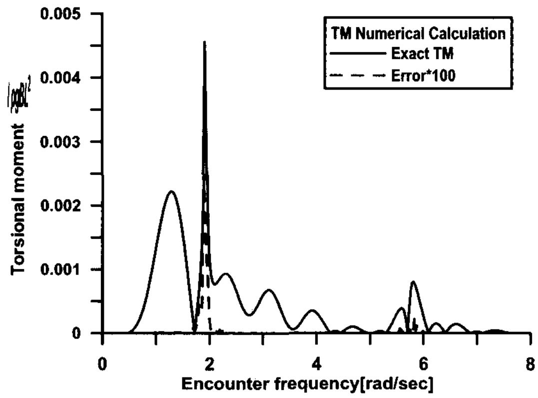

Fig. 3 TM of uniform ship at 135 Deg. heading

Horizontal bending moment is plotted along the encounter frequencies of up to 8 rad/sec in Fig. 2. The abscissa is encountered wave frequency in radian/sec while the ordinate is horizontal bending moment normalized by ρgBL^2 . Springing is observed at encounter frequency around 2.6 rad/sec. The difference between numerical and analytical solution is regarded as the error and presented by dotted line exaggerated by 100 times for reference. We can observe here the degree of the correlation with an existing numerical code, for example, by Jung et al (2003). The difference is observed for small frequencies and near peaks. Therefore, the numerical code used in this example case may be considered more carefully for small encounter frequencies. Torsional moment is also plotted in the Fig. 3. Here, the ordinate is also normalized by ρgBL^2 . Again, good correlation is obtained between the exact solution and a numerical one.

5. CONCLUSIONS

From the previous studies, exact solution on the anti-symmetric hydro-elastic response of uniform ships is derived. The solutions are used as not only preliminary design of barge-like ships but also the benchmark test of the existing numerical codes. Further application on the beam-like body such as VLFS and risers can be expected.

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