Robust Fuzzy Control of a Class of Nonlinear Descriptor Systems with Time-Varying Delay

Yan Wang, Zeng-Qi Sun*, and Fu-Chun Sun

Abstract: A robust fuzzy controller is designed to stabilize a class of solvable nonlinear descriptor systems with time-varying delay. First, a new modeling and control method for nonlinear descriptor systems is presented with a fuzzy descriptor model. A sufficient condition for the existence of the fuzzy controller is given in terms of a series of LMIs. Then, a less conservative fuzzy controller design approach is obtained based on the fuzzy rules and weights. This method includes the interactions of the different subsystems into one matrix. The effectiveness of the presented approach and the design procedure of the fuzzy controller are illustrated by way of an example.

Keywords: Multithreading, SMT, ILP, TLP, in-order issue and completion, grouping.

1. INTRODUCTION

Recently, fuzzy logic control technique is generally accepted as a powerful tool for both collecting human knowledge and dealing with uncertainties in the control process. With the development of the fuzzy identification technique, some fuzzy-model-based fuzzy control system design methods [5, 15, 16, etc.] have appeared in the fuzzy control field. A typical model is the Takagi-Sugeno (T-S) fuzzy model, which is a nonlinear model described by a set of if-then rules providing local linear representation of the underlying system. Therefore, the linear control theory can be used to stabilize the local subsystems. Based on the T-S fuzzy model, some results concerning fuzzy control for nonlinear systems have been reported throughout the literature [1-4, etc.]. For examples, Cao and Frank have investigated the fuzzy control problem of nonlinear time-delay systems [2]; Ma and Sun have considered output tracking and regulation of nonlinear systems based on the T-S fuzzy model [13]; and, Chen *et al.* have designed a mixed H_2/H_∞ fuzzy output feedback controller for nonlinear dynamic systems [4].

Conversely, the research of descriptor systems has been an active area because of the extensive applications in many engineering systems, such as constrained robot systems [9], circuit systems [14] and chemical processes [8]. Many contributions have been made to the study of descriptor systems [6, 7, 11]. Due to the difficulties of constructing Lyapunov function and the complexity of the existence and uniqueness of the solution, there still remain some difficulties in controlling nonlinear descriptor systems. The fuzzy descriptor system was first stated by Tadanari *et al.* in [17] and [18]. It has been proven that the T-S fuzzy model is a universal approximator of any smooth nonlinear systems having a first order that is differentiable [19]. Therefore, it is meaningful to consider applying the fuzzy descriptor model to approximate the nonlinear descriptor system, and designing the fuzzy controller. In [19], Tadanari *et al.* applied fuzzy descriptor systems to solve the problem of nonlinear model following control. Yoneyama and Ichikawa discussed the H_∞ control problem for fuzzy descriptor systems in [23]. Wang *et al.* obtained a new fuzzy controller design method for descriptor systems based on interval theory [20]. In [21], the admissibility of discrete-time fuzzy descriptor systems was studied. Since time delays and parameter uncertainties often occur during actual operation with the potential to deteriorate the stability and control performance of the system, in this paper, the goal is to extend the T-S fuzzy descriptor model to the control of nonlinear descriptor systems with time-varying delay. The robust fuzzy control for the systems will be investigated. This problem has not yet been investigated in the literature.

This paper is organized as follows. In the next section, the fuzzy descriptor model for a class of nonlinear descriptor systems with time-varying delay is dis-

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cussed. In Section 3, robust fuzzy controller design approach is given based on LMIs. In Section 4, a less conservative design method for fuzzy state feedback controller is obtained. In Section 5, a simulation example is illustrated to show the application and the effectiveness of the proposed method. Finally, concluding remarks are offered in Section 6.

2. FUZZY DESCRIPTOR MODEL OF A SYSTEMS WITH TIME VARYING DELAY

In this section, the fuzzy descriptor model is considered to express a class of solvable nonlinear descriptor systems with time-varying delay. If the uncertain system parameter information is considered, the solvable nonlinear descriptor system can be presented as an uncertain fuzzy descriptor model with time-varying delay. The ith fuzzy rule is of the following form:

R^i: if ξ_1 is M_1^i and ξ_2 is M_2^i and ... and ξ_p is M_p^i, then

$$\begin{aligned} E\dot{x}(t) &= (A_i + \Delta A_i(t))x(t) + (A_{1i} + \Delta A_{1i}(t))x(t-\tau(t)) \\ &\quad + (B_i + \Delta B_i(t))u(t), \end{aligned} \tag{1}$$

where i=1,2,...,r, M_j^i are the fuzzy sets. $x(t)=[x_1(t)\ x_2(t)\ \ldots\ x_n(t)]^T$ is the state and $u(t)$ is the control input. $\xi=[\xi_1\ \xi_2\ \ldots\ \xi_p]$ are premise variables and the premise variables are some components of $x(t)$ and other measurable system variables. $E \in \Re^{n\times n}$ may not have full rank. The fuzzy descriptor model is assumed to be locally regular, i.e. for every fuzzy rule and some $s \in C$, $\det(sE - A_i - \Delta A_i(t)) \neq 0$. $\tau(t)$ is the time-varying delay, $0 \le \tau(t) \le \tau_0 < \infty$ and $\dot{\tau}(t) \le d < 1$. A_i, A_{1i} and B_i are constant matrices of compatible dimensions. $\Delta A_i(t)$, $\Delta A_{1i}(t)$ and $\Delta B_i(t)$ represent time-varying parameter uncertainties.

$$\begin{aligned} \begin{bmatrix} \Delta A_i(t) & \Delta B_i(t) \end{bmatrix} &= H_{0i}F_{0i}(t)\begin{bmatrix} E_{0i} & E_{1i} \end{bmatrix}, \\ \Delta A_{1i}(t) &= H_{1i}F_{1i}(t)E_{2i}, \end{aligned} \tag{2}$$

where $F_{ij}^T(t)F_{ij}(t) \le I$, i = 0,1; j = 1,2,...,r. For simplicity, x, x_τ, u will be used instead of $x(t)$, $x(t-\tau(t))$ and $u(t)$.

By taking a standard fuzzy inference strategy, that is, using a singleton fuzzifier, product fuzzy inference and center average defuzzifier, the final fuzzy model of the system is inferred as follows

$$\begin{aligned} E\dot{x} &= \sum_{i=1}^{r} w_i(\xi)[(A_i + \Delta A_i(t))x \\ &\quad + (A_{1i} + \Delta A_{1i}(t))x_\tau + (B_i + \Delta B_i(t))u], \end{aligned} \tag{3}$$

where

$$w_i(\xi) = \frac{\prod_{j=1}^{p} M_j^i(\xi_j)}{\sum_{i=1}^{r}\prod_{j=1}^{p} M_j^i(\xi_j)} \ge 0, \quad \sum_{i=1}^{r} w_i(\xi) = 1. \tag{4}$$

$M_j^i(\xi_j)$ is the grade of membership of ξ_j in M_j^i.

Consider a nonlinear descriptor system

$$E\dot{x}(t) = f(x(t), u(t)), \tag{5}$$

where $E \in \Re^{n\times n}$, det (E)=0. The following definition regarding the solvability of the nonlinear descriptor system was given in [12].

Definition 1: If for any piecewise continuous input $u(t)$ and any initial state x_0, there always exists a unique differentiable solution $x(t)$ with $x(0)=x_0$, then the system (5) is called solvable.

The purpose of this paper is to adopt the fuzzy approach to control nonlinear descriptor systems with time-varying delay and design a robust state feedback controller for the systems. The analysis is developed under the assumption that the nonlinear descriptor system is solvable.

A Lemma will be used in the proof of the main results.

Lemma 1 [22]: Let A, D, E and F be real matrices of appropriate dimensions with $F^T F \le I$. Then for any P >0 and scalar ε >0, if $P - \varepsilon DD^T > 0$, we have

$$\begin{aligned} &(A + DFE)^T P^{-1}(A + DFE) \\ &\le A^T(P - \varepsilon DD^T)^{-1}A + \varepsilon^{-1}E^T E. \end{aligned} \tag{6}$$

3. ROBUST FUZZY CONTROLLER DESIGN

In this section, the robust stability of the unforced solvable nonlinear descriptor system with time-varying delay is analyzed first. Then, a robust fuzzy state feedback controller design method is obtained based on LMIs.

Consider the following unforced system with time-varying delay

$$E\dot{x} = \sum_{i=1}^{r} w_i(\xi)\begin{bmatrix} (A_i + H_{0i}F_{0i}(t)E_{0i})x \\ +(A_{1i} + H_{1i}F_{1i}(t)E_{2i})x_\tau \end{bmatrix}. \tag{7}$$

Select the Lyapunov function as

$$v(x) = x^T E^T V x + \frac{1}{1-d}\int_{t-\tau(t)}^{t} x^T(\sigma)Sx(\sigma)d\sigma, \tag{8}$$

where $E^T V = V^T E \ge 0$, S is a symmetric positive definite matrix. Theorem 1 gives a sufficient condition for the robust stability of system (7).

Theorem 1: The unforced nonlinear descriptor system (7) with time-varying delay is robustly stable if there exist nonsingular matrices $S>0$, V and constants $\varepsilon_i>0$ such that for $i=1,2,\ldots,r$

$$E^T V = V^T E \geq 0 \tag{9}$$

$$\begin{bmatrix} A_i^T V + V^T A_i + S/(1-d) & * & * & * & * & * \\ E_{0i} & -I & * & * & * & * \\ A_{1i}^T V & 0 & -S & * & * & * \\ H_{0i}^T V & 0 & 0 & -I & * & * \\ H_{1i}^T V & 0 & 0 & 0 & -\varepsilon_i I & * \\ 0 & 0 & \varepsilon_i^{0.5} E_{2i} & 0 & 0 & -I \end{bmatrix} < 0 \tag{10}$$

where * is used as an ellipse for terms induced by symmetry.

Proof: The derivative of the Lyapunov function (8) along the trajectory of (7) is

$$\dot{v}(x) = \sum_{i=1}^{r} w_i(\xi) \left\{ \begin{matrix} x^T \begin{bmatrix} (A_i + H_{0i}F_{0i}(t)E_{0i})^T V \\ +V^T (A_i + H_{0i}F_{0i}(t)E_{0i}) \end{bmatrix} x \\ +2x^T V^T (A_{1i} + H_{1i}F_{1i}(t)E_{2i}) x_\tau \end{matrix} \right\}$$

$$+\frac{1}{1-d} x^T S x - \frac{1-\dot{\tau}(t)}{1-d} x_\tau^T S x_\tau$$

$$\leq \sum_{i=1}^{r} w_i(\xi) \left\{ \begin{matrix} x^T \begin{bmatrix} A_i^T V + V^T A_i + V^T H_{0i} H_{0i}^T V + E_{0i}^T E_{0i} \\ +V^T \begin{pmatrix} A_{1i} + H_{1i} \\ F_{1i}(t) E_{2i} \end{pmatrix} S^{-1} \begin{pmatrix} A_{1i} + H_{1i} \\ F_{1i}(t) E_{2i} \end{pmatrix}^T V \end{bmatrix} x \\ +x_\tau^T S x_\tau \end{matrix} \right\}$$

$$+\frac{1}{1-d} x^T S x - x_\tau^T S x_\tau .$$

From Lemma 1 and (4), we have

$$\dot{v}(x) \leq \sum_{i=1}^{r} w_i(\xi) x^T \begin{bmatrix} A_i^T V + V^T A_i + V^T H_{0i} H_{0i}^T V + E_{0i}^T E_{0i} \\ +V^T A_{1i} (S - \varepsilon_i E_{2i}^T E_{2i})^{-1} A_{1i}^T V + \\ \varepsilon_i^{-1} V^T H_{1i} H_{1i}^T V + \frac{1}{1-d} S \end{bmatrix} x .$$

According to the Schur complement, $w_i(\xi)>0$ and (10) indicate $\dot{v}(x)<0$, thus system (7) is robustly asymptotically stable. □

Next, the fuzzy controller will be designed to stabilize system (3). The fuzzy controller shares the same fuzzy sets with the fuzzy model in the premise parts and has local linear controllers in the consequent parts. The ith fuzzy rule of the fuzzy controller is of the following form

R_c^i: if ξ_1 is M_1^i and ξ_2 is M_2^i and ... and ξ_p is M_p^i, then

$$u = K_i x,\ i=1,2,\ldots,r, \tag{11}$$

where K_i are the local linear feedback gains. The final form of the fuzzy controller is

$$u = \sum_{i=1}^{r} w_i(\xi) K_i x . \tag{12}$$

By substituting (12) into (3), the closed-loop nonlinear descriptor system with time-varying delay can be represented as

$$E\dot{x} = \sum_{i=1}^{r}\sum_{j=1}^{r} w_i(\xi) w_j(\xi) \begin{bmatrix} (G_{ij} + H_{0i}F_{0i}(t)D_{ij}) x \\ +(A_{1i} + H_{1i}F_{1i}(t)E_{2i}) x_\tau \end{bmatrix} \tag{13}$$

where $G_{ij} = A_i + B_i K_j$, $D_{ij} = E_{0i} + E_{1i} K_j$.

Theorem 2: For the nonlinear descriptor system (3), there exists a fuzzy state feedback controller such that the closed-loop system is robustly asymptotically stable, if there exist nonsingular matrices $Q>0$, X, Y_i and constants $\varepsilon_i>0$ satisfying the following conditions

$$X^T E^T = EX \geq 0 , \tag{14}$$

$$\begin{bmatrix} \Theta_{ij} & * & * & * & * & * \\ E_{0i}X + E_{1i}Y_j & -I & * & * & * & * \\ X^T A_{1i}^T & 0 & -Q & * & * & * \\ H_{0i}^T & 0 & 0 & -I & * & * \\ H_{1i}^T & 0 & 0 & 0 & -\varepsilon_i I & * \\ 0 & 0 & \varepsilon_i^{0.5} E_{2i} X & 0 & 0 & -I \end{bmatrix} < 0, \tag{15}$$

where

$$\Theta_{ij} = X^T A_i^T + A_i X + Y_j^T B_i^T + B_i Y_j + \frac{1}{1-d} Q, \tag{16}$$

$i,j=1,2,\ldots,r$. The state feedback gains are constructed as $K_i = Y_i X^{-1}$.

Proof: Applying Theorem 1 to system (13), then introducing new variables $X = V^{-1}$, $Y_j = K_j X$ and $Q = V^{-T} S V^{-1}$, from the Schur complement, the proof can be completed directly. □

4. LESS CONSERVATIVE LMI DESIGN OF ROBUST FUZZY CONTROLLER

In this section, the fuzzy rule weights $w_i(\xi)$ will be applied to obtain a less conservative fuzzy controller design method. (13) can be rewritten as the following form

$$
\begin{aligned}
E\dot{x} = \sum_{i=1}^{r} w_i^2(\xi)[(G_{ii} + H_{0i}F_{0i}(t)D_{ii})x \\
+ (A_{1i} + H_{1i}F_{1i}(t)E_{2i})x_\tau] \qquad (17) \\
+ \sum_{i<j\le r} w_i(\xi)w_j(\xi)\{[(G_{ij} + G_{ji}) \\
+ (H_{0i}F_{0i}(t)D_{ij} + H_{0j}F_{0j}(t)D_{ji})]x \\
+ [(A_{1i} + A_{1j}) + (H_{1i}F_{1i}(t)E_{2i} + H_{1j}F_{1j}(t)E_{2j})]x_\tau\}.
\end{aligned}
$$

Let

$$
\Gamma_{ij} = G_{ij} + G_{ji}, \quad \Lambda_{ij} = A_{1i} + A_{1j},
$$

$$
H_{0ij} = [H_{0i} \quad H_{0j}], \quad H_{1ij} = [H_{1i} \quad H_{1j}],
$$

$$
O_{ij} = \begin{bmatrix} D_{ij} \\ D_{ji} \end{bmatrix}, \quad E_{2ij} = \begin{bmatrix} E_{2i} \\ E_{2j} \end{bmatrix},
$$

$$
F_{0ij} = \begin{bmatrix} F_{0i}(t) & 0 \\ 0 & F_{0j}(t) \end{bmatrix}, \quad F_{1ij} = \begin{bmatrix} F_{1i}(t) & 0 \\ 0 & F_{1j}(t) \end{bmatrix},
$$

Then (17) is

$$
\begin{aligned}
E\dot{x} = \sum_{i=1}^{r} w_i^2(\xi) \begin{bmatrix} (G_{ii} + H_{0i}F_{0i}(t)D_{ii})x \\ +(A_{1i} + H_{1i}F_{1i}(t)E_{2i})x_\tau \end{bmatrix} \\
+ \sum_{i<j\le r} w_i(\xi)w_j(\xi)[(\Gamma_{ij} + H_{0ij}F_{0ij}(t)O_{ij})x \qquad (18) \\
+ (\Lambda_{ij} + H_{1ij}F_{1ij}(t)E_{2ij})x_\tau].
\end{aligned}
$$

The main result is given as follows.

Theorem 3: For the nonlinear descriptor system (3), there exists a fuzzy state feedback controller such that the closed-loop system is robustly asymptotically stable, if there exist nonsingular matrices $Q>0$, X, Y_i, R_{ij} ($i<j\le r$) and constants $\varepsilon_i>0$, $\lambda_j>0$ satisfying the following conditions

$$
X^T E^T = EX \ge 0, \qquad (19)
$$

$$
\begin{bmatrix}
\Theta_{ii} + R_{ii} & * & * & * & * & * \\
E_{0i}X + E_{1i}Y_i & -I & * & * & * & * \\
X^T A_{1i}^T & 0 & -Q & * & * & * \\
H_{0i}^T & 0 & 0 & -I & * & * \\
H_{1i}^T & 0 & 0 & 0 & -\varepsilon_i I & * \\
0 & 0 & \varepsilon_i^{0.5}E_{2i}X & 0 & 0 & -I
\end{bmatrix} < 0, \qquad (20)
$$

$$
\begin{bmatrix}
\Theta_{ij} + \Theta_{ji} + 2R_{ij} & * & * & * & * & * & * & * & * & * \\
E_{0i}X + E_{1i}Y_j & -I & * & * & * & * & * & * & * & * \\
E_{0j}X + E_{1j}Y_i & 0 & -I & * & * & * & * & * & * & * \\
X^T \Lambda_{ij}^T & 0 & 0 & -2Q & * & * & * & * & * & * \\
H_{0i}^T & 0 & 0 & 0 & -I & * & * & * & * & * \\
H_{0j}^T & 0 & 0 & 0 & 0 & -I & * & * & * & * \\
H_{1i}^T & 0 & 0 & 0 & 0 & 0 & -\lambda_i I & * & * & * \\
H_{1j}^T & 0 & 0 & 0 & 0 & 0 & 0 & -\lambda_i I & * & * \\
0 & 0 & 0 & \lambda_i^{0.5}E_{2i}X & 0 & 0 & 0 & 0 & -I & * \\
0 & 0 & 0 & \lambda_i^{0.5}E_{2j}X & 0 & 0 & 0 & 0 & 0 & -I
\end{bmatrix} \le 0, \qquad (21)
$$

for $i<j\le r$, and

$$
R = \begin{bmatrix}
R_{11} & R_{12} & \cdots & R_{1r} \\
R_{12} & R_{22} & \cdots & R_{2r} \\
\vdots & \vdots & \ddots & \vdots \\
R_{1r} & R_{2r} & \cdots & R_{rr}
\end{bmatrix} > 0, \qquad (22)
$$

where Θ_{ij} are given by (16). Furthermore, the state feedback gains are constructed as

$$
K_i = Y_i X^{-1}. \qquad (23)
$$

Proof: Let

$$
\begin{aligned}
&\Phi(G_{ii}, H_{0i}, D_{ii}, A_{1i}, E_{2i}, H_{1i}, S, \varepsilon_i, V) \\
&= G_{ii}^T V + V^T G_{ii} + V^T H_{0i}H_{0i}^T V + D_{ii}^T D_{ii} \\
&\quad + V^T A_{1i}(S - \varepsilon_i E_{2i}^T E_{2i})^{-1} A_{1i}^T V + \varepsilon_i^{-1} V^T H_{1i} H_{1i}^T V.
\end{aligned}
$$

From the Lyapunov function (8), we get

$$
\begin{aligned}
\dot{v}(x) \le \sum_{i=1}^{r} w_i^2(\xi) \begin{bmatrix} x^T \Phi(G_{ii}, H_{0i}, D_{ii}, A_{1i}, E_{2i}, H_{1i}, S, \varepsilon_i, V)x \\ + x_\tau^T S x_\tau \end{bmatrix} \\
+ \frac{1}{1-d} x^T S x - x_\tau^T S x_\tau + \sum_{i<j\le r} w_i(\xi)w_j(\xi) \\
\left[x^T \Phi(\Gamma_{ij}, H_{0ij}, O_{ij}, \Lambda_{ij}, E_{2ij}, H_{1ij}, 2S, \lambda_i, V)x + x_\tau^T (2S) x_\tau\right].
\end{aligned}
$$

Note that

$$\sum_{i=1}^{r} w_i^2(\xi)x_\tau^T S x_\tau + \sum_{i<j\le r} w_i(\xi)w_j(\xi)x_\tau^T(2S)x_\tau = x_\tau^T S x_\tau .$$

Thus,

$$\dot{v}(x)\le \sum_{i=1}^{r} w_i^2(\xi)\begin{bmatrix} x^T\Phi(G_{ii},H_{0i},D_{ii},A_{1i},E_{2i},H_{1i},S,\varepsilon_i,V)x \\ +\dfrac{1}{1-d}x^TSx \end{bmatrix}$$

$$+\sum_{i<j\le r} w_i(\xi)w_j(\xi)\begin{Bmatrix} x^T\Phi(\Gamma_{ij},H_{0ij},O_{ij},\Lambda_{ij},E_{2ij},H_{1ij},2S,\lambda_i,V)x \\ +\dfrac{1}{1-d}x^T(2S)x \end{Bmatrix}.$$

Introducing new matrix variables [10] T_{ij}, if the following conditions hold

$$\Phi(G_{ii},H_{0i},D_{ii},A_{1i},E_{2i},H_{1i},S,\varepsilon_i,V)+\frac{1}{1-d}S<-T_{ii},$$

$$\Phi(\Gamma_{ij},H_{0ij},O_{ij},\Lambda_{ij},E_{2ij},H_{1ij},2S,\lambda_i,V)+\frac{2}{1-d}S\le -2T_{ij}(i<j\le r).$$

Then

$$\dot{v}(x)\le -\sum_{i=1}^{r} w_i^2(\xi)x^TT_{ii}x-2\sum_{i<j\le r} w_i(\xi)w_j(\xi)x^TT_{ij}x$$

$$=-\begin{bmatrix} w_1(\xi)x \\ w_2(\xi)x \\ \vdots \\ w_r(\xi)x \end{bmatrix}^T \begin{bmatrix} T_{11} & T_{12} & \cdots & T_{1r} \\ T_{12} & T_{22} & \cdots & T_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ T_{1r} & T_{2r} & \cdots & T_{rr} \end{bmatrix} \begin{bmatrix} w_1(\xi)x \\ w_2(\xi)x \\ \vdots \\ w_r(\xi)x \end{bmatrix}.$$

Hence, if

$$E^TV=V^TE\ge 0,$$

and

$$T=\begin{bmatrix} T_{11} & T_{12} & \cdots & T_{1r} \\ T_{12} & T_{22} & \cdots & T_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ T_{1r} & T_{2r} & \cdots & T_{rr} \end{bmatrix}>0.$$

then the closed-loop system is asymptotically stable. Let $X=V^{-1}$, $Y_i=K_iV^{-1}$, $Q=V^{-T}SV^{-1}$ and $R_{ij}=V^{-T}T_{ij}V^{-1}$, the conclusion can be obtained directly. □

Remark : If E of system (1) is set as I, Theorem 3 gives a fuzzy controller design approach to robustly stabilize the normal nonlinear system with time varying delay.

5. ILLUSTRATIVE EXAMPLE

To demonstrate the fuzzy controller design procedure, consider the following fuzzy model of nonlinear descriptor system with time-varying delay.

if x_1 is P, then

$$E\dot{x}=(A_1+\Delta A_1(t))x+(A_{11}+\Delta A_{11}(t))x_\tau+(B_1+\Delta B_1(t))u,$$

if x_1 is N, then

$$E\dot{x}=(A_2+\Delta A_2(t))x+(A_{12}+\Delta A_{12}(t))x_\tau+(B_2+\Delta B_2(t))u.$$

where, the membership functions of 'P', 'N' are given as follows the effectiveness of the method.

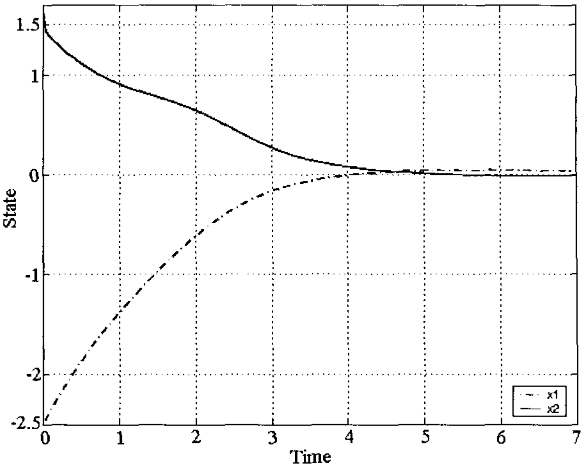


Fig. 1. Simulation result of the system state.

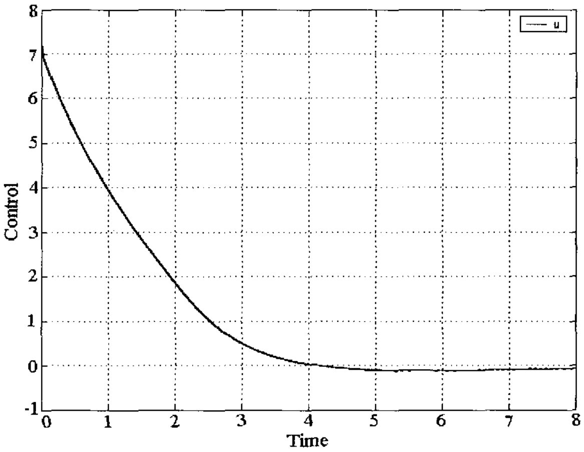


Fig. 2. Simulation result of the control input.

$$w_1(x_1)=1-\frac{1}{1+e^{-2x_1}},\ w_2(x_1)=1-w_1(x_1).$$

$\Delta A_i(t)$, $\Delta A_{1i}(t)$, $\Delta B_i(t)$ are given by (2), and

$$E=\begin{bmatrix}1&0\\0&0\end{bmatrix},\ A_1=\begin{bmatrix}0&1\\-1&-2\end{bmatrix},\ A_2=\begin{bmatrix}0&1\\-2&-2\end{bmatrix},$$

$$B_1=\begin{bmatrix}0\\0.1\end{bmatrix},\quad B_2=\begin{bmatrix}0\\1\end{bmatrix},\quad A_{11}=\begin{bmatrix}0&0\\0.2&0.1\end{bmatrix},$$

$$A_{12}=\begin{bmatrix}0&0\\0.1&0.5\end{bmatrix},$$

$$H_{01}=\begin{bmatrix}0.1\\0.2\end{bmatrix},\ H_{02}=\begin{bmatrix}0\\0.1\end{bmatrix},\ H_{11}=H_{12}=\begin{bmatrix}0.1\\0.5\end{bmatrix},$$

$E_{01}=E_{02}=[1\ \ 0]$, $E_{21}=E_{22}=[0.1\ \ 0]$,

$E_{11}=0.3$, $E_{12}=0.2$, $\tau(t)=2+0.5\sin t$, $d=0.5$.

Next, a less conservative fuzzy controller will be designed to stabilize the nonlinear descriptor system. Substituting the above parameters into Theorem 3, the solutions of (20), (21) and (22) can be obtained using the LMI toolbox in MATLAB, while the solutions simultaneously satisfy (19).

$$X=\begin{bmatrix}0.6657 & 0\\ -0.9741 & 0.8419\end{bmatrix},\ Q=\begin{bmatrix}0.4818 & -0.1729\\ -0.1729 & 0.7384\end{bmatrix},$$

$$R_{11}=\begin{bmatrix}0.4967 & -0.4420\\ -0.4420 & 0.7298\end{bmatrix},\ R_{12}=\begin{bmatrix}0.1137 & 0.0247\\ 0.0247 & 0.0961\end{bmatrix},$$

$$R_{22}=\begin{bmatrix}0.4117 & 0.0868\\ 0.0868 & 1.0064\end{bmatrix},\ Y_1=\begin{bmatrix}-2.4391 & 0.8033\end{bmatrix},$$

$$Y_2=\begin{bmatrix}-1.3148 & -0.3358\end{bmatrix}.$$

Then the two state feedback gains are obtained as

$$K_1=\begin{bmatrix}-2.2679 & 0.9541\end{bmatrix},$$

$$K_2=\begin{bmatrix}-2.5588 & -0.3988\end{bmatrix}.$$

According to Theorem 3, the closed-loop system is asymptotically stable via the fuzzy controller

$$u=\sum_{i=1}^{2} w_i(x_1)K_i x.$$

Let $F_{ij}(t)=\sin(t)$, $i=0,1, j=1,2$. Fig. 1 shows the simulation results when the initial condition of the nonlinear descriptor system is constrained as $[x_1(0)\ \ x_2(0)]^T=[-2.5\ \ 1.64]^T$ and Fig. 2 shows the trajectory of fuzzy control u.

From the simulation results, it can be concluded that when time-varying delay and modeling errors are considered, the solvable nonlinear descriptor system is indeed robustly asymptotically stable under the fuzzy control strategy presented in this paper.

6. CONCLUSIONS

A novel method has been presented to control nonlinear descriptor systems with time-varying delay in this paper. By considering the modeling errors and system uncertainties, a robust fuzzy state feedback controller design method is obtained in terms of LMIs. With the effective application of weights, a less conservative fuzzy controller design method is given. The relaxed design method includes the interactions of the different subsystems into one matrix, and the LMIs can be solved by applying standard methods. An example is given to illustrate the application and the effectiveness of the method.

REFERENCES

[1] S. G. Cao, N. W. Rees, and G. Feng, "Analysis and design for a class of complex control systems part I, II: fuzzy modeling and identification, fuzzy controller design," *Automatica*, vol. 33, no. 6, pp. 1017-1040, June 1997.

[2] Y. Y. Cao and P. M. Frank, "Analysis and synthesis of nonlinear time-delay systems via fuzzy control approach," *IEEE Trans. on Fuzzy Systems*, vol. 8, no. 2, pp. 200-211, April 2000.

[3] B. S. Chen, S. T. Chung, and J. U. Huey, "Robustness design of nonlinear dynamic systems via fuzzy linear control," *IEEE Trans. on Fuzzy Systems*, vol. 7, no. 5, pp. 571-584, October 1999.

[4] B. S. Chen, S. T. Iseng, and J. U. Huey, "Mixed H_2/H_∞ fuzzy output feedback control design for nonlinear dynamic systems: a LMI approach," *IEEE Trans. on Fuzzy Systems*, vol. 8, no. 3, pp. 249-265, June 2000.

[5] C. L. Chen, P. C. Chen, and C. K. Chen, "Analysis and design of fuzzy control systems," *Fuzzy Sets and Systems*, vol. 57, no. 2, pp. 125-140, July 1993.

[6] J. D. Cobb, "Controllability, observability and duality in singular systems," *IEEE Trans. on Automatic Control*, vol. 29, no. 12, pp. 1076-1082, December 1984.

[7] L. Dai, *Singular Control Systems*, Springer, Berlin, 1989.

[8] R. Gani and I. T. Cameron, "Modeling for dynamic simulation of chemical processes: the index problem," *Chemical Engineering and Science*, vol. 47, no. 5, pp. 1311-1315, April 1992.

[9] N. Harris and M. Clamroch, "Singular systems of differential equations as dynamic models for constrained robot systems," *Proc. IEEE Conf. Robotics and Automation*, pp. 21-28, 1986.

[10] E. T. Kim and H. J. Lee, "New approaches to relaxed quadratic stability condition of fuzzy control systems," *IEEE Trans. on Fuzzy Systems*, vol. 8, no. 5, pp. 523-534, October 2000.

[11] F. L. Lewis, "A survey of linear singular systems," *Circuits, Systems, and Signal Processing*, vol. 5, no. 1, pp. 3-36, January 1986.

[12] X. P. Liu, "Robust stabilization of nonlinear singular systems," *Proc. of the 34th Conf. Decision and Control*, pp. 2375-2376, 1995.

[13] X. J. Ma and Z. Q. Sun, "Output tracking and regulation of Nonlinear System Based on Takagi-Sugeno Fuzzy Model," *IEEE Trans. on Systems Man Cybernetics*, vol. 30, no. 1, pp. 47-59, February 2000.

[14] R. W. Newcomb, "The semistate description of nonlinear time variable circuits," *IEEE Trans. on Circuits and Systems*, vol. 28, no. 1, pp. 62-71, January 1981.

[15] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its application to modeling and control," *IEEE Trans. on Systems, Man and Cybernetics*, vol. 15, no. 1, pp. 116-132, January 1985.

[16] K. Tanaka and M. Sugeno, "Stability analysis and design of fuzzy control systems," *Fuzzy Sets and Systems*, vol. 45, no. 1, pp. 135-156, January 1992.

[17] T. Taniguchi, K. Tanaka, K. Yamafuji, and H. O. Wang, "Fuzzy descriptor systems: stability analysis and design via LMIs," *Proc. American*

Control Conference, pp. 1827-1831, 1999.

[18] T. Taniguchi, K. Tanaka, and H. O. Wang, "Fuzzy descriptor systems and fuzzy controller designs," *Proc. of the 8th International Fuzzy System Association World Congress*, pp. 655-659, 1999.

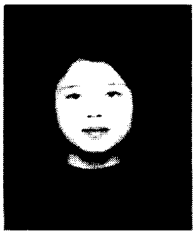
[19] T. Taniguchi, K. Tanaka, and H. O. Wang, "Fuzzy descriptor systems and nonlinear model following control," *IEEE Trans. on Fuzzy Systems*, vol. 8, no. 4, pp. 442-432, August 2000.

[20] Y. Wang, Q. L. Zhang, and W. Q. Liu, "Stability analysis and design for T-S fuzzy descriptor systems," *Proc. of the 40th Conf. Decision and Control*, pp. 3962-3967, 2001.

[21] Y. Wang, Q. L. Zhang, and X. D. Liu, "Robustness design of uncertain discrete-time fuzzy descriptor systems with guaranteed admissibility," *Proc. American Control Conference*, pp. 1699-1704, 2002.

[22] L. Xie and E. S. Carlos, "Criteria for robust stabilization of uncertain linear systems with time-varying state delays," *Proc. IFAC 13th Triennial World Congress*, pp. 137-142, 1996.

[23] J. Yoneyama and A. Ichikawa, "H_∞ control for Takagi-Sugeno fuzzy descriptor systems," *Proc. IEEE Conf. Systems, Man and Cybernetics*, pp. III-28-III-33, 1999.



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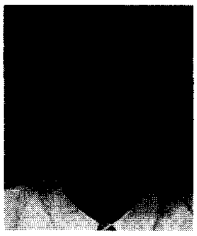

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