On-Line Parameter Estimation Scheme for Uncertain Takagi-Sugeno Fuzzy Models

Young-Wan Cho and Chang-Woo Park*

Abstract: In this paper, an estimator with an appropriate adaptive law for updating parameters is designed and analyzed based on the Lyapunov theory. The adaptive law is designed so that the estimation model follows the parameterized plant model. Using the proposed estimator, the parameters of the T-S fuzzy model can be estimated by observing the behavior of the system and it can be a basis for indirect adaptive fuzzy control.

Keywords: Parameter estimation, Takagi-Sugeno fuzzy model, fuzzy systems, adaptive control, nonlinear system.

1. INTRODUCTION

The adaptive fuzzy controllers are divided into two classes [1]. One is called the direct adaptive fuzzy control and the other is called the indirect adaptive fuzzy control. In the case of the direct adaptive fuzzy control, we view the fuzzy logic systems as controllers. However, in the case of the indirect adaptive fuzzy control, the fuzzy logic systems are used to model the plant. The controller is then constructed assuming that the fuzzy logic systems approximately represent the true plant. The appropriate adaptive law plays an important role in estimating the parameters in the fuzzy model representing the plant model, whose parameters are unknown or vary in accordance with external disturbances and parameter perturbation. Hence, the parameter estimation for the fuzzy model is essential to the indirect adaptive fuzzy control design. Hitherto, various researches on the parameter estimation of the fuzzy system from the input-output measurements have been conducted. Pedrycz previously suggested the estimation algorithm of the fuzzy relational model [2]. Sugeno proposed the parameter estimation of the so-called Takagi-Sugeno (T-S) fuzzy system [3, 4] and other researchers also participated in the estimation of the T-S fuzzy system [5]. Sugeno and Yasukawa reported qualitative modeling of a fuzzy system in [6] and some researchers attempted to estimate the fuzzy system via neural-network-based approaches [7, 8].

However, most of these are off-line algorithms and cannot be applied to situations where real-time processing is required such as adaptive control and signal processing. Even though the successive adaptive fuzzy modeling was suggested in [9], it cannot be viewed as an on-line algorithm in an actual sense since it requires an off-line learning phase before being adapted on-line. Furthermore, most of the on-line parameter estimation schemes proposed in the indirect adaptive fuzzy control [10-15] can only be applied to the specified fuzzy controllers, mainly feedback linearization based controllers. Hence, a parameter estimation scheme applicable to the general fuzzy models and controllers is needed.

To avoid these problems, this paper presents a new design and an analysis of the on-line parameter estimator for the plant model whose structure is represented by the general T-S fuzzy models.

2. T-S FUZZY MODELS AND PARAMETER ESTIMATION

As an expression model of an actual plant, we use the fuzzy implications and the fuzzy reasoning methods suggested by Takagi and Sugeno [3]. The set of fuzzy implications shown in the Takagi and Sugeno (T-S) model can express a highly nonlinear functional relation in spite of a small number of fuzzy implication rules. As a descriptive rule the T-S fuzzy model uses fuzzy implication of the following form:

$$\begin{aligned} R^i &: \textit{If } x_1(t) \textit{ is } M_1^i \textit{ and} \cdots \textit{and } x_n(t) \textit{ is } M_n^i \\ &\textit{then } \dot{\mathbf{x}}(t) = A_i x(t) + B_i \mathbf{u}(t), \end{aligned} \tag{1}$$

where R^i $(i = 1, 2, \cdots, l)$ denotes the i th implication, l is the number of fuzzy implications,

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M_j^i are fuzzy sets and $\mathbf{x}^T(t)=[x_n(t),\ x_{(n-1)}(t),\cdots,\ x_1(t)],\mathbf{u}^T(t)=[u_1(t),\ u_2(t),\cdots,\ u_m(t)]$.

The T-S fuzzy model approximates a nonlinear system with a combination of several linear systems. The overall T-S fuzzy model is formed by fuzzy partitioning of the input space. The premise of a fuzzy implication indicates a fuzzy subspace of the input space and each consequent expresses a local input-output relation in the subspace corresponding to the premise part.

Given a pair of inputs ($\mathbf{x}(t)$, $\mathbf{u}(t)$), the final output of the fuzzy system is inferred as follows:

$$\dot{x}(t)=\frac{\sum_{i=1}^{l} w_i(\mathbf{x}(t))\{A_i\mathbf{x}(t)+B_i\mathbf{u}(t)\}}{\sum_{i=1}^{l} w_i(\mathbf{x}(t))}, \tag{2}$$

where $w_i(\mathbf{x}(t))=\prod_{j=1}^{n} M_j^i(x_j(t))$, $M_j^i(x_j(t))$ is the grade of membership of $x_j(t)$ in M_j^i and it is assumed that $\sum_{i=1}^{l} w_i(\mathbf{x}(t))>0$, $w_i(\mathbf{x}(t))\geq 0$, for $i=1,2,\cdots,l$.

To develop the parameter estimator for the T-S fuzzy modeled plant, we start with the plant parameterization as

$$\begin{aligned}\dot{x}&=\frac{\sum_{i=1}^{n} w_i(\mathbf{x})\{A_i\mathbf{x}+B_i\mathbf{u}+A_s\mathbf{x}-A_s\mathbf{x}\}}{\sum_{i=1}^{n} w_i(\mathbf{x})}\\&=A_s\mathbf{x}+\frac{\sum_{i=1}^{n} w_i(\mathbf{x})((A_i-A_s)\mathbf{x}+B_i\mathbf{u})}{\sum_{i=1}^{n} w_i(\mathbf{x})},\end{aligned} \tag{3}$$

where A_s is an arbitrary stable matrix, i.e., having all its eigenvalues in the left half plane.

Now, we define the estimation model as

$$\dot{\hat{\mathbf{x}}}=A_s\hat{\mathbf{x}}+\frac{\sum_{i=1}^{n} w_i(\mathbf{x})((\hat{A}_i-A_s)\mathbf{x}+\hat{B}_i\mathbf{u})}{\sum_{i=1}^{n} w_i(\mathbf{x})}, \tag{4}$$

where $\hat{A}_i(t)$, $\hat{B}_i(t)$ are the estimates of $A_i(t)$, $B_i(t)$ at time t to be generated by an adaptive law, and $\hat{\mathbf{x}}(t)\in R^n$ is the estimate of the vector $\mathbf{x}(t)$.

By defining the estimation error vector ε as

$$\varepsilon\equiv\mathbf{x}-\hat{\mathbf{x}}$$

we obtain

$$\begin{aligned}\dot{\varepsilon}&=\dot{\mathbf{x}}-\dot{\hat{\mathbf{x}}}\\&=A_m\varepsilon-\frac{\sum w_i\tilde{A}_i}{\sum w_i}\mathbf{x}-\frac{\sum w_i\tilde{B}_i}{\sum w_i}\mathbf{u},\end{aligned} \tag{5}$$

where $\tilde{A}_i\equiv\hat{A}_i-A_i$, $\tilde{B}_i\equiv\hat{B}_i-B_i$.

Let us now consider the series-parallel model design and use (5) to derive the adaptive law for estimating the elements of A_i, B_i. We assume that the adaptive law has the general structure

$$\begin{aligned}\dot{\hat{A}}_i&=F_i(\mathbf{x},\hat{\mathbf{x}},\varepsilon,\mathbf{u}),\\\dot{\hat{B}}_i&=G_i(\mathbf{x},\hat{\mathbf{x}},\varepsilon,\mathbf{u}),\end{aligned} \tag{6}$$

where F_i and G_i ($i=1,\cdots,l$) are functions of known signals that are to be chosen so that the equilibrium

$$\hat{A}_{ie}=A_i,\ \hat{B}_{ie}=B_i,\ \varepsilon_e=0 \tag{7}$$

of (5), (6) has some desired stability properties. By choosing the following function as the Lyapunov function candidate,

$$V(\varepsilon,\tilde{A}_i,\tilde{B}_i)=\varepsilon^T P\varepsilon+\sum_{i=1}^{l} tr(\frac{\tilde{A}_i^T P\tilde{A}_i}{r_{1i}})+\sum_{i=1}^{l} tr(\frac{\tilde{B}_i^T P\tilde{B}_i}{r_{2i}}),$$

where $tr(A)$ denotes the trace of a matrix A, r_{1i}, $r_{2i}>0$ are constant scalars, and $P=P^T>0$ is chosen as the solution of the Lyapunov equation

$$A_s^T P+PA_s=-I, \tag{8}$$

whose existence is guaranteed by the stability assumption of A_s.

we obtain the time derivative $\dot{V}$ of V along the trajectory of (5), (6), which is given by L

$$\begin{aligned}\dot{V}&=\dot{\varepsilon}^T P\varepsilon+\varepsilon^T P\dot{\varepsilon}+\sum_{i=1}^{N} tr(\frac{\dot{\tilde{A}}_i^T P\tilde{A}_i}{r_{1i}}+\frac{\tilde{A}_i^T P\dot{\tilde{A}}_i}{r_{1i}})\\&+\sum_{i=1}^{N} tr(\frac{\dot{\tilde{B}}_i^T P\tilde{B}_i}{r_{2i}}+\frac{\tilde{B}_i^T P\dot{\tilde{B}}_i}{r_{2i}}),\end{aligned} \tag{9}$$

where

$$\dot{\varepsilon}^T P\varepsilon+\varepsilon^T P\dot{\varepsilon}=\varepsilon^T(A_s^T P+PA_s)\varepsilon-2\varepsilon^T P\frac{\sum w_i\tilde{A}_i}{\sum w_i}\mathbf{x}-2\varepsilon^T P\frac{\sum w_i\tilde{B}_i}{\sum w_i}\mathbf{u}$$

and

$$tr(\frac{\dot{\tilde{A}}_i^T P\tilde{A}_i}{r_{1i}}+\frac{\tilde{A}_i^T P\dot{\tilde{A}}_i}{r_{1i}})=2tr(\frac{\tilde{A}_i^T PF_i}{r_{1i}}),$$

$$tr(\frac{\dot{\tilde{B}}_i^T P\tilde{B}_i}{r_{2i}}+\frac{\tilde{B}_i^T P\dot{\tilde{B}}_i}{r_{2i}})=2tr(\frac{\tilde{B}_i^T PG_i}{r_{2i}}).$$

Therefore,

$$\dot{V} = -\varepsilon^T\varepsilon-2\varepsilon^T P\frac{\sum w_i\tilde{A}_i}{\sum w_i}\mathbf{x}-2\varepsilon^T P\frac{\sum w_i\tilde{B}_i}{\sum w_i}\mathbf{u} + \sum 2tr(\frac{\tilde{A}_i^T PF_i}{r_{1i}})+\sum 2tr(\frac{\tilde{B}_i^T PG_i}{r_{2i}}). \tag{10}$$

By using the following properties of trace to manipulate (10):

(i) $tr(AB)=tr(BA)$

(ii) $tr(A+B)=tr(A)+tr(B)$ for any $A, B\in R^{n\times n}$

(iii) $tr(yx^T)=x^T y$ for any $x, y\in R^{n\times 1}$.

After some straightforward manipulation on (10) using the properties of trace given in the above, we have

$$\dot{V} = -\varepsilon^T\varepsilon+2tr(\sum\frac{\tilde{A}_i^T PF_i}{r_{1i}}-\frac{\sum w_i\tilde{A}_i^T}{\sum w_i}P\varepsilon\mathbf{x}^T + \sum\frac{\tilde{B}_i^T PG_i}{r_{2i}}-\frac{\sum w_i\tilde{B}_i^T}{\sum w_i}P\varepsilon\mathbf{u}^T). \tag{11}$$

The obvious choice for F_i, G_i to make $\dot{V}$ negative is

$$\sum\frac{\tilde{A}_i^T PF_i}{r_{1i}}=\frac{\sum w_i\tilde{A}_i^T}{\sum w_i}P\varepsilon\mathbf{x}^T,$$

$$\sum\frac{\tilde{B}_i^T PG_i}{r_{2i}}=\frac{\sum w_i\tilde{B}_i^T}{\sum w_i}P\varepsilon\mathbf{u}^T.$$

That is,

$$\dot{\hat{A}}_i=F_i=r_{1i}\frac{w_i}{\sum w_i}\varepsilon\,\mathbf{x}^T, \tag{12a}$$

$$\dot{\hat{B}}_i=G_i=r_{2i}\frac{w_i}{\sum w_i}\varepsilon\,\mathbf{u}^T. \tag{12b}$$

The signals for driving the adaptation law (12a) and (12b) of the parameter estimator are known or available for measurement. Therefore, the estimation law for the estimation model (4) can be implemented. The overall estimation scheme is shown in Fig. 1.

We establish the following theorem, which shows the properties of the adaptive law (12).

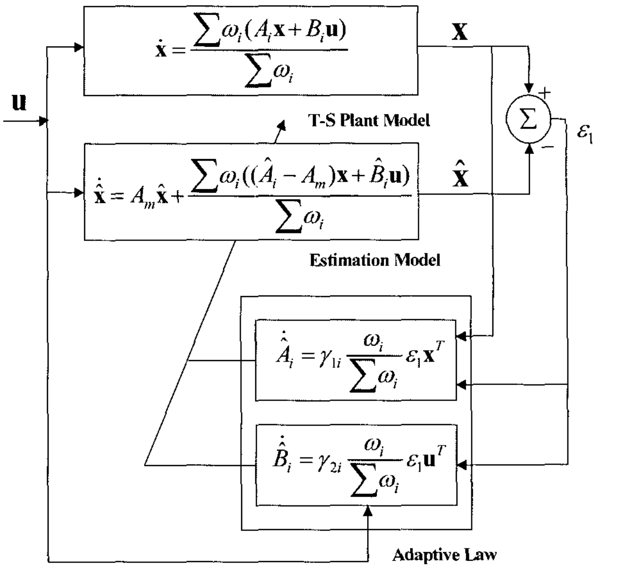


Fig. 1. T-S fuzzy model parameter estimator.

Theorem 1: Consider the plant model (2) and the estimation model (4) with the estimation law (12). Assume $\mathbf{u}\in\pounds_\infty$. and then, the adaptive law (12) guarantees that $|\varepsilon(t)|\to 0$ as $t\to\infty$ and $\|\dot{\hat{A}}_i(t)\|\to 0$, $\|\dot{\hat{B}}_i(t)\|\to 0$ as $t\to\infty$.

Proof: From the adaptive law (12), it directly follows that the time derivative $\dot{V}$ of V along the solution trajectory of (5), (6) satisfies

$$\dot{V} = -\varepsilon^T\varepsilon\le 0. \tag{13}$$

Since the function V is a Lyapunov function for the system (5), (6) where $\mathbf{x}$ and $\mathbf{u}$ are treated as independent bounded functions of time, and $,\dot{V}\le 0$ we conclude that the equilibrium given by (7) is uniformly stable, which implies that the trajectory $\varepsilon(t)$, $\hat{A}_i(t)$, $\hat{B}_i(t)$ is bounded for all $t>0$.

Because $\varepsilon=\mathbf{x}-\hat{\mathbf{x}}$ and $\mathbf{x}\in\pounds_\infty$, we also have $\hat{\mathbf{x}}\in\pounds_\infty$. Therefore, all signals in (5) and (6) are uniformly bounded.

From (8) and (13), we conclude that because V is bounded from below and it has a limit, i.e.,

$$\lim_{t\to\infty}V(\varepsilon(t),\hat{A}_i(t),\hat{B}_i(t))=V_\infty<\infty. \tag{14}$$

From (13) and (14), it follows that

$$\int_0^\infty\varepsilon^T\varepsilon\,d\tau=-\int_0^\infty\dot{V}\,d\tau=(V_0-V_\infty), \tag{15}$$

where $V_0=V(\varepsilon(0),\hat{A}_i(0),\hat{B}_i(0))$ which implies that

$\varepsilon \in \pounds_2$. Because $0 \le w_i(\mathbf{x}) \le 1$, and $\mathbf{u}, \hat{A}_i, \hat{B}_i, \hat{\mathbf{x}}$, $\varepsilon \in \pounds_\infty$, it follows from (5) that $\dot{\varepsilon} \in \pounds_\infty$, which, together with $\varepsilon \in \pounds_2$, implies that $\varepsilon \to 0$ as $t \to \infty$, which, in turn, implies that $\dot{\hat{A}}_i(t)$, $\dot{\hat{B}}_i(t) \to 0$ as $t \to \infty$.

3. ADAPTIVE FUZZY CONTROL DESIGN

In the previous section, we have presented an online parameter estimation for the general T-S fuzzy models. Based on the analysis, in this section, we adopt the proposed estimator with the existing fuzzy state feedback controller, and an indirect adaptive fuzzy controller for the given unknown T-S fuzzy modeled plant is designed.

3.1. Indirect adaptive fuzzy state feedback control structure

Consider the fuzzy model representing a nonlinear SISO system with the following form of fuzzy rules.

i-th plant rule:

$$\begin{aligned} &IF\ x\ is\ M_1^i\ and\ \dot{x}\ is\ M_2^i\ and \cdots and\ x^{(n-1)}\ is\ M_n^i \\ &THEN\ x^{(n)} = \mathbf{a}_i^T \cdot \mathbf{x} + b_i u \quad or \quad \dot{\mathbf{x}} = A_i \mathbf{x} + B_i u \\ &\qquad i = 1, 2, \cdots, l \end{aligned} \tag{16}$$

where, state $\mathbf{x}^T = [x^{(n-1)}, x^{(n-2)}, \cdots, \dot{x}, x] = [x_n, x_{n-1}, \cdots, x_1]$ and input $u \in R^1$ are available for measurement, $\mathbf{a}_i^T = [a_n^i, a_{n-1}^i, \cdots, a_1^i]$, $b^i \in R^1$ ($i = 1, \cdots, l$) are unknown, and $u \in \pounds_\infty$, $w_i(\mathbf{x}) = \prod_{j=1}^{p} M_j^i(\mathbf{x})$. M_j^i is the fuzzy set and l is the number of fuzzy rules and

$$A_i = \begin{bmatrix} a_n^i & a_{n-1}^i & \cdots & a_2^i & a_1^i \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}, \quad B_i = \begin{bmatrix} b_i \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

The T-S fuzzy rules can be inferred as

$$\begin{aligned} x^{(n)} &= \frac{\sum_{i=1}^{l} w_i(\mathbf{x})\{\mathbf{a}_i^T \cdot \mathbf{x} + b_i u\}}{\sum_{i=1}^{r} w_i(\mathbf{x})} \\ &= \sum_{i=1}^{l} h_i(\mathbf{x})\{\mathbf{a}_i^T \cdot \mathbf{x} + b_i u\} \end{aligned} \tag{17a}$$

or equivalently,

$$\dot{x} = \frac{\sum_{i=1}^{l} w_i(\mathbf{x}) \left\{ \begin{bmatrix} a_n^i & a_{n-1}^i & \cdots & a_2^i & a_1^i \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} b_i \\ 0 \\ \vdots \\ 0 \end{bmatrix} \mathbf{u} \right\}}{\sum_{i=1}^{l} w_i(\mathbf{x})} \tag{17b}$$

where $w_i(\mathbf{x}) = \prod_{j=1}^{n} M_{ij}(\mathbf{x}^{(j-1)})$, $h_i(\mathbf{x}) = \frac{w_i(\mathbf{x})}{\sum_{i=1}^{r} w_i(\mathbf{x})}$.

We adopt the fuzzy state feedback controller shown in Theorem 2 to stabilize (17).

Theorem 2 [18]: If we choose the following controller for the plant represented by T-S fuzzy model (17),

$$\begin{aligned} u &= \frac{\mathbf{a}_d^T \cdot \mathbf{x} - \sum_{i=1}^{r} h_i(\mathbf{x})\, \mathbf{a}_i^T \cdot \mathbf{x}}{\sum_{i=1}^{r} h_i(\mathbf{x}) b_i} \\ &= \frac{\sum_{i=1}^{r} h_i(\mathbf{x})(\mathbf{a_d}^{\mathbf{T}} - \mathbf{a_i}^{\mathbf{T}}) \cdot \mathbf{x}}{\sum_{i=1}^{r} h_i(\mathbf{x}) b_i}, \end{aligned} \tag{18}$$

where we use the same $\mathbf{a}_i$, b_i and $h_i(\mathbf{x})$ with the fuzzy model (16) for all i, and $\mathbf{a}_d \in R^n$ is chosen such that the exact linearized system (19)

$$x^{(n)} = \mathbf{a}_d^T \cdot \mathbf{x} \tag{19}$$

is asymptotically stable.

Then the given nonlinear fuzzy system is transformed into the desired linear system as in (19).

Proof: Refer to [18].

However, the parameter vectors, $\mathbf{a}_i$ and b_i are unknown and then, they are tuned by $\hat{\mathbf{a}}_i$ and $\hat{b}_i$ via the proposed estimation methods as follows.

$$u = \frac{\sum_{i=1}^{r} h_i(\mathbf{x})(\mathbf{a_d} - \hat{\mathbf{a}}_i)^T \cdot \mathbf{x}}{\sum_{i=1}^{r} h_i(\mathbf{x}) \hat{b}_i} \tag{20}$$

By considering the plant parameterization

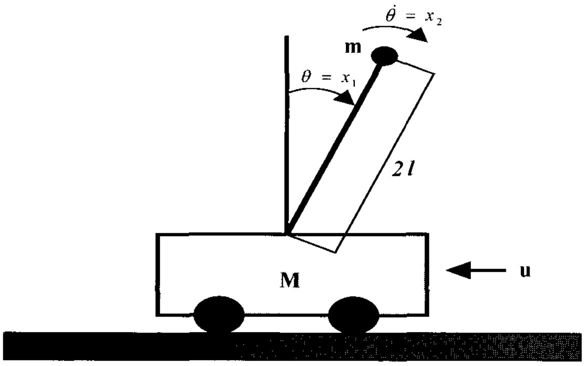


Fig. 2. The inverted pendulum system.

$$\dot{x} = A_s\,\mathbf{x} + \frac{\sum_{i=1}^{l} w_i(\mathbf{x})((A_i - A_s)\mathbf{x} + B_i\,u)}{\sum_{i=1}^{l} w_i(\mathbf{x})} \tag{21}$$

and A_s is a stable matrix.

We define the estimation model as

$$\dot{\hat{\mathbf{x}}} == A_s\hat{\mathbf{x}} + \frac{\sum_{i=1}^{l} w_i(\mathbf{x})((\hat{A}_i(t) - A_s)\mathbf{x} + \hat{B}_i(t)u)}{\sum_{i=1}^{l} w_i(\mathbf{x})}. \tag{22}$$

By following the estimator design methods, we can derive an adaptive law shown in Lemma 1 to make the estimation error $\varepsilon = \mathbf{x} - \hat{\mathbf{x}}$ be zero and with the adaptive law; the estimation model (22) can be the same as (23) by adopting the control input (20) to (22).

$$x^{(n)} = \mathbf{a}_d{}^T \cdot \mathbf{x} \tag{23}$$

Lemma 1: Consider the plant model (21) and the estimation model (22). Assume that $u \in \pounds_\infty$, and then, the following adaptive law (24),

$$\dot{\mathbf{a}}_i^T = \dot{\hat{\mathbf{a}}}_i^T = \mathbf{f}_i^T = r_{1i}\frac{w_i}{\sum w_i}\mathbf{p}_1^T\varepsilon_1\,\mathbf{x}^T, \tag{24a}$$

$$\dot{b}_i = \dot{\hat{b}}_i = g_i = r_{2i}\frac{w_i}{\sum w_i}\mathbf{p}_1^T\varepsilon_1\,u, \tag{24b}$$

where $\mathbf{p}_1$ is the first column of P guarantees that

$$|\varepsilon_1(t)| \to 0 \quad as \quad t \to \infty \tag{25a}$$

and

$$\left\|\dot{\mathbf{a}}_i(t)\right\| \to 0\ ,\left\|\dot{b}_i(t)\right\| \to 0 \quad as \quad t \to \infty \tag{25b}$$

Proof: The proof is given in Appendix A.

3.2. Control simulations

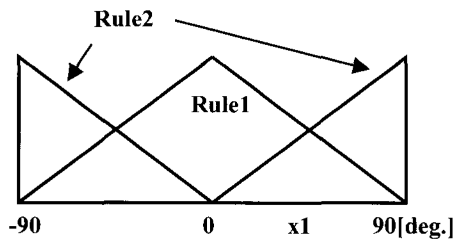


Fig. 3. Membership functions.

Consider the problem of balancing an inverted pendulum on a cart shown in Fig. 2. The equations of motion for the pendulum are

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f(\mathbf{x}) + g(\mathbf{x}) + d(t) \\ &= \frac{g\,\sin(x_1) - amlx_2^2(2x_1)/2 - a\cos(x_1)u}{4l/3 - a\,ml\cos^2(x_1)} + d(t), \end{aligned} \tag{26}$$

where $\mathbf{x} = [x_1 \quad x_2]^T$ in which x_1 denotes the angle (in radians) of the pendulum from the vertical and x_2 is the angular velocity. $g = 9.8m/s^2$ is the gravity constant, m is the mass of the pendulum, M is the mass of the cart, $2l$ is the length of the pendulum, u is the control force applied to the cart (in Newtons). $d(t)$ is the external disturbance and $a = \frac{1}{m+M}$. We choose $m = 2.0kg$, $M = 8.0kg$ and $2l = 1.0m$ in the simulation.

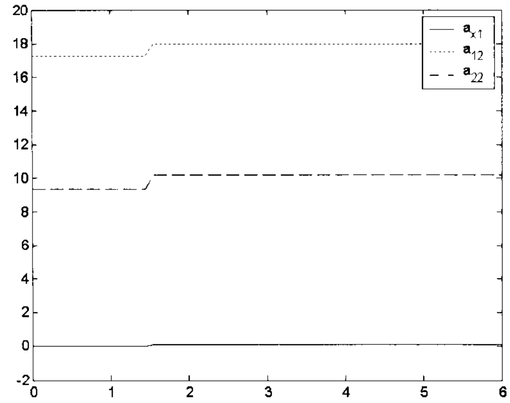
The dynamic equations (26) can be approximated by the following two fuzzy rules [19] and the membership functions used in this fuzzy model are shown in Fig. 3.

$$\begin{aligned} &Rule\ 1: IF\ \ x\ is\ about\ 0 \\ &\qquad THEN\ \ddot{x} = \mathbf{a}_1^T \cdot \mathbf{x} + b_1\,u \\ &Rule\ 2: IF\ \ x\ is\ about\ \pm\frac{\pi}{2}(\ |x| < \frac{\pi}{2}) \\ &\qquad THEN\ \ddot{x} = \mathbf{a}_2^T \cdot \mathbf{x} + b_2\,u \end{aligned} \tag{27}$$

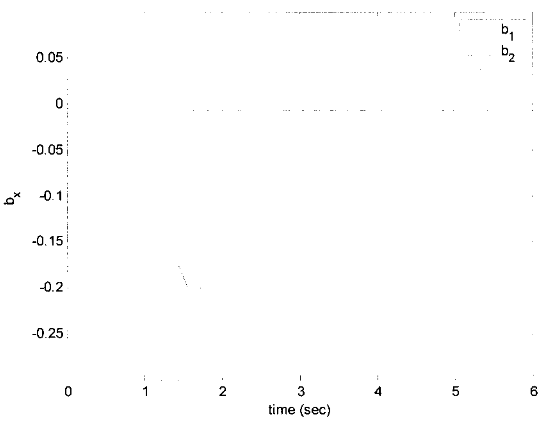
(27) can be inferred as

$$\begin{aligned} x^{(n)} &= \frac{\sum_{i=1}^{2} w_i(\mathbf{x})\{\,\mathbf{a}_i^T \cdot \mathbf{x} + b_i\,u\,\}}{\sum_{i=1}^{2r} w_i(\mathbf{x})} \\ &= \sum_{i=1}^{r} h_i(x)\{\,\mathbf{a}_i^T \cdot \mathbf{x} + b_i\,u\,\}, \end{aligned} \tag{28}$$

where $w_i(\mathbf{x}) = \prod_{j=1}^{2} M_{ij}(x^{(j-1)})$, $h_i(\mathbf{x}) = \frac{w_i(\mathbf{x})}{\sum_{i=1}^{2} w_i(\mathbf{x})}$,



(a) Parameter a_{XX} .



(b) Parameter b_X .

Fig. 4. Plant parameter variation.

and ,

$$\mathbf{a_1} = \left[0 \quad \frac{g}{4l/3 - aml} \right] = [0 \quad 17.29],$$

$$\mathbf{a_2} = \left[0 \quad \frac{2g}{\pi\,(4l/3 - aml\beta^2)} \right] = [0 \quad 9.35],$$

$$b_1 = -\frac{a}{4l/3 - aml} = -0.1765 ,$$

$$b_2 = -\frac{a\beta}{4l/3 - aml\beta^2} = -0.0052 ,$$

$$\beta = \cos 88°.$$

We construct the control effort as

$$u = \frac{\sum_{i=1}^{2} h_i(\mathbf{x})(\mathbf{a_d} - \hat{\mathbf{a}}_i)^T \cdot \mathbf{x}}{\sum_{i=1}^{2} h_i(\mathbf{x})\hat{b}_i}, \tag{29}$$

where $\mathbf{a_d}$ = [-1 -1].

Table 1. Variation of plant parameters

	Variation of plant parameters
CASE 1	$0 \le t \le 5 : a_{11} = 0 , a_{12} = 17.29, a_{21} = 0$, $a_{22} = 9.35 , b_1 = -0.1765 , b_2 = -0.0052$
CASE 2	$0 \le t \le 1.5 : a_{11} = 0 , a_{12} = 17.29, a_{21} = 0$, $a_{22} = 9.35 , b_1 = -0.1765 , b_2 = -0.0052$ $1.5 \le t \le 5 : a_{11} = 0.1 , a_{12} = 18, a_{21} = 0.1$, $a_{22} = 10.2 , b_1 = -0.2 , b_2 = -0.007$

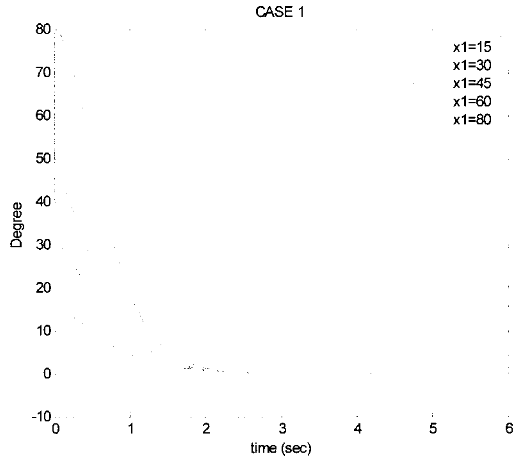


Fig. 5. Simulation result of design example (CASE1).

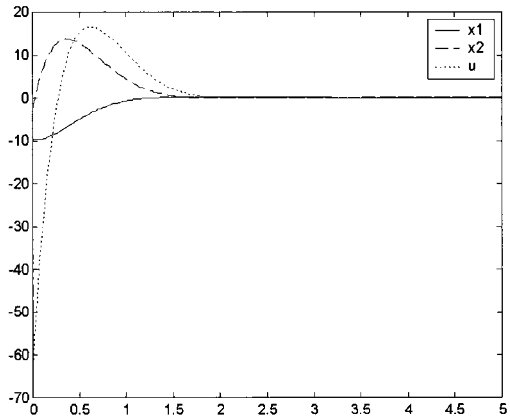


Fig. 6. Simulation result of design example (CASE1).

The plant parameters $\hat{\mathbf{a}}_i$, $\hat{b}_i$ are adjusted on-line by adaptive law (24) where the adaptation rates $r_{11} = 50$, and $r_{21} = 0.5$ are used and $p_1 = [\ 0.1562 \quad 0.1250]^T$, the column of the positive definite matrix P is obtained by solving the Lyapunov equation (8) with stable matrix, $A_s = \begin{bmatrix} -4 & -4 \\ 1 & 0 \end{bmatrix}$.

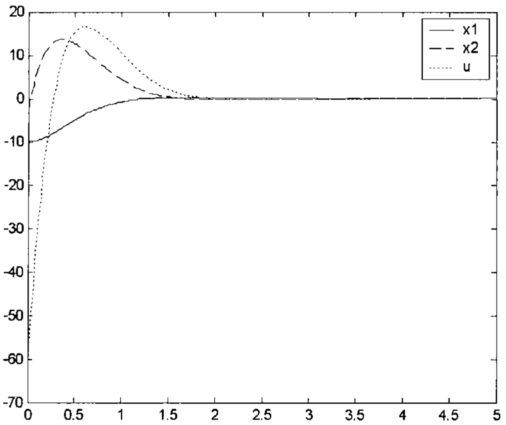


Fig. 7. Simulation result of design example (CASE2).

4. CONCLUSIONS

The adaptive law adjusting the parameters of the T-S fuzzy models has been formulated based on the Lyapunov theory so that the parameter estimation was guaranteed. Hence, it can be used in cases where the plant parameters in the T-S fuzzy model are uncertain.

APPENDIX

Since the estimation for the controllable canonical form (16) of T-S fuzzy models is a special case of the general T-S fuzzy model estimation given in section 2, by following the similar procedure, we can easily derive the adaptive law (24).

The estimation error vector ε defined as

$$\varepsilon \equiv \mathbf{x} - \hat{\mathbf{x}}$$

satisfies

$$\begin{aligned}\dot{\varepsilon} &= \dot{\mathbf{x}} - \dot{\hat{\mathbf{x}}} \\ &= A_m\varepsilon - \frac{\sum w_i [\tilde{\mathbf{a}}_i \;\; 0 \;\; \cdots \;\; 0]^T}{\sum w_i}\mathbf{x} \\ &\quad - \frac{\sum w_i [\tilde{b}_i \;\; 0 \;\; \cdots \;\; 0]^T}{\sum w_i} u,\end{aligned} \tag{30}$$

where $\tilde{\mathbf{a}}_i^T \equiv \hat{\mathbf{a}}_i^T - \mathbf{a}_i^T$, $\tilde{b}_i \equiv \hat{b}_i - b_i$.

Let us choose the following function as a Lyapunov function candidate

$$V(\varepsilon_1, \tilde{\mathbf{a}}_i, \tilde{b}_i) = \varepsilon_1^T P \varepsilon_1 + \sum_{i=1}^{l} \frac{\tilde{\mathbf{a}}_i^T \tilde{\mathbf{a}}_i}{r_{1i}} + \sum_{i=1}^{l} \frac{\tilde{b}_i^2}{r_{2i}}, \tag{31}$$

where r_{1i}, $r_{2i} > 0$ are constants, and $P = P^T > 0$ is chosen as the solution of the Lyapunov equation

$$A_s^T P + P A_s = -I .$$

After some straightforward manipulation, we obtain the time derivative of V as (32).

$$\begin{aligned}\dot{V} &= -\varepsilon^T \varepsilon - 2\frac{\sum w_i \mathbf{p}_1^T \varepsilon_1 \mathbf{x}^T \tilde{\mathbf{a}}_i}{\sum w_i} \\ &\quad - 2\frac{\sum w_i \tilde{b}_i}{\sum w_i}\mathbf{p}_1^T \varepsilon_1 u + \sum_{i=1}^{N} 2\frac{\dot{\tilde{\mathbf{a}}}_i^T \tilde{\mathbf{a}}_i}{r_{1i}} + \sum_{i=1}^{N} 2\frac{\tilde{b}_i \dot{\tilde{b}}_i}{r_{2i}},\end{aligned} \tag{32}$$

where $\mathbf{p}_1$ is the first column of P.

The obvious choice to make $\dot{V}$ negative is

$$\dot{\tilde{\mathbf{a}}}_i^T = \dot{\hat{\mathbf{a}}}_i^T = \mathbf{f}_i^T = r_{1i}\frac{w_i}{\sum w_i}\mathbf{p}_1^T \varepsilon_1 \mathbf{x}^T, \tag{33a}$$

$$\dot{\tilde{b}}_i = \dot{\hat{b}}_i = g_i = r_{2i}\frac{w_i}{\sum w_i}\mathbf{p}_1^T \varepsilon_1 u . \tag{33b}$$

By following the procedure similar to that of the proof of Theorem 1, we can easily prove (25) using Barbalat's lemma [20].

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