Problems of Special Causes in Feedback Adjustment

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Abstract

Process adjustment is a complimentary tool to process monitoring in process control. Process adjustment directs on maintaining a process output close to a target value by manipulating another controllable variable, by which significant process improvement can be achieved. Therefore, this approach can be applied to the 'Improve' stage of Six Sigma strategy. Though the optimal control rule minimizes process variability in general, it may not properly function when special causes occur in underlying process, resulting in off-target bias and increased variability in the adjusted output process, possibly for long periods. In this paper, we consider a responsive feedback control system and the minimum mean square error control rule. The bias in the adjusted output process is investigated in a general framework, especially focussing on stationary underlying process and the special cause of level shift type. Illustrative examples are employed to illustrate the issues discussed.

1. Introduction

Process adjustment, also called engineering process control (EPC), focuses on keeping a process output close to a desired target value, i.e., minimizing the process variability around the target value by manipulation of another controllable variable. In this paper, we consider a *responsive feedback control system* in which all the effects of a change in the manipulable variable will be realized on the output within one period. It is well known that the minimum mean square error (MMSE) control rule is optimal in that it minimizes process variability [Box *et al.* (1994), Montgomery (2001), and Del Castillo (2002)]. However, when some special causes such as sudden changes in environmental conditions or mistakes by the operator occur, additional variations in

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underlying process may be introduced. In such cases, the effects of those causes may not be properly entertained by the MMSE control rule in process adjustment, resulting in off-target bias and increased variability in output process possibly for long periods.

Since early 1990s, approaches for detecting special causes in process adjustment have been introduced. The approach most attracted is the one combining statistical process control (SPC) and EPC, in which a control rule is applied to process adjustment and a control chart is applied to the adjusted output process [see, Vander Wiel *et al.* (1992), Montgomery *et al.* (1994), Capilla *et al.* (1999), and Montgomery (2001)]. In addition to that, it is to be noted that process monitoring for auto-correlated process has close similarity to process adjustment in many aspects. For special causes of level shift type, detection powers of various control charts applied to auto-correlated process were investigated and compared in terms of average run length [see, Alwan and Roberts (1988), Wardell et al. (1994), Vander Wiel (1996), and Atienza et al. (1998)].

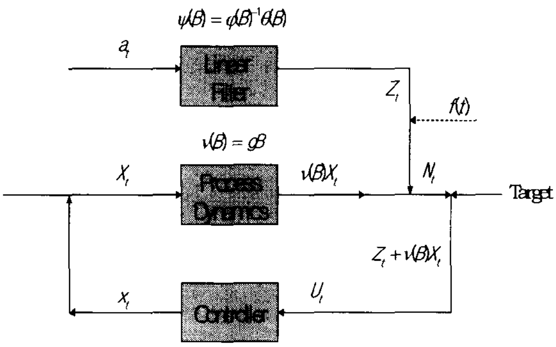
In process adjustment, since the effect of a special cause on output process is also adjusted in the process of manipulating controllable variable, the bias in the adjusted output process may be decreased over time possibly quite rapidly. Therefore, approaches using control charts may not always be quite effective in detecting special causes in short periods. Meanwhile, understanding the effects of the special causes of different types on process adjustment may provide useful information for proposing an effective detection method. In this paper, we consider the case that the underlying process is a stationary process but contaminated by special causes, and the MMSE control rule is applied to process adjustment. Three types of special causes, additive outlier (AO), innovational outlier (IO), and level shift (LS), are considered. The effects of the causes on adjusted output process are derived under a general framework, from which the special causes problems can be modeled. Finally, few illustrative examples are employed to understand and interpret the issues discussed.

2. Special Cause Problems

We consider a responsive feedback control system, as shown in Figure 1, represented by

$$U_t = Y_t + Z_t \,, \qquad (1)$$

where Z_t and U_t are the amount of deviation from target in the system output when a control action is and is not applied, respectively. In (1), $Y_t = gX_{t-1}$ is the amount of compensation on the output at time t when controllable variable is set as X_{t-1} at time $t-1$ and g is the steady state gain [Box et al. (1994)]. In this paper Z_t, X_t, and U_t will be called underlying process (or unadjusted output), input variable, and adjusted output process, respectively.



<Figure 1> Process adjustment in feedback control system with a special cause

2.1 MMSE Control Rule

We assumed that the underlying process Z_t can be represented by ARMA(p,q) model with *known* parameters, defined by

$$\phi(B)Z_t = \theta(B)a_t \,, \tag{2}$$

where a_t is a white noise process with mean zero and variance σ_a^2, B is the back shift operator such that $B^k Z_t = Z_{t-k}$, and operator $\phi(B) = 1-\phi_1 B - \cdots - \phi_p B^p$ and $\theta(B) = 1 - \theta_1 B - \cdots - \theta_q B^q$ are two polynomials in B with orders p and q. We also assume that $\phi(B)$ and $\theta(B)$ have no common factors and that all of their roots are outside the unit circle.

For model (2) with known parameters as assumed above, the one-step-ahead MMSE forecast (MMSEF) of Z_t at time $t-1$, denoted by $Z_{t-1}(1)$, and its error can be expressed as

$$\begin{aligned} Z_{t-1}(1) &= \psi^{(1)}(B)\,a_t \\ &= \psi_1 a_{t-1} + \psi_2 a_{t-2} + \cdots \,, \\ e_{t-1}(1) &= a_t \,, \end{aligned} \tag{3}$$

where

$\psi(B) = \phi(B)^{-1}\theta(B) = 1 + \psi^{(1)}(B)$ $= 1 + \psi_1 B + \psi_2 B^2 + \cdots$. The MMSEF (3) can be expressed in terms of present and past observations as

$$\begin{aligned} Z_{t-1}(1) &= \pi^{(1)}(B) Z_t \\ &= \pi_1 Z_{t-1} + \pi_2 Z_{t-2} + \cdots, \end{aligned} \tag{4}$$

where

$\pi(B) = \theta(B)^{-1}\phi(B) = 1 - \pi^{(1)}(B)$ $= 1 - \pi_1 B - \pi_2 B^2 - \cdots$.

When no special cause occurs in the underlying process, it is well known that input variable set at time $t-1$ as

$$X_{t-1} = -Z_{t-1}(1)/g \tag{5}$$

is the MMSE control rule that minimizes the output variability. That is, the adjusted output at time t will then be a white noise,

$$U_t = e_{t-1}(1) = a_t. \quad (6)$$

Therefore, by applying process adjustment, the variation of the output process is reduced from the variance of unadjusted output, σ_Z^2, to the variance of adjusted output, σ_a^2. For example, when Z_t follows an AR(1) model with a parameter ϕ, the variance of Z_t is $\sigma_Z^2 = \sigma_a^2/(1-\phi^2)$ and thus process adjustment reduces the variability of output process as much as

$$r = 1 - \sigma_a^2/\sigma_Z^2 = \phi^2$$

where r is the magnitude of reduction rate. Since $|\phi| < 1$, the degree of reduction in terms of variance of the output process is relatively low for weakly auto-correlated series (i.e., $\phi \approx 0$), while the reduction rate is high for strongly correlated series. For example, for $\phi = 0.2$ we may expect only 4% reduction, but for $\phi = 0.8$ we can expect about 64% reduction in variability.

In practice, the MMSE controller (5) can be expressed in terms of adjusted outputs by

$X_{t-1} = -\psi^{(1)}(B)\,U_t/g$, for which (3) and (6) are used in (5) [Box et al. (1994), Box and Lucceno (1997)].

When special causes occur in the system, the MMSE controller may not properly compensate the effects of such interventions, resulting in off-target biases and increased variability in the adjusted output process U_t for the time being. In order to understand the effects of special causes on process adjustment, the biases in the adjusted output will be explicitly derived in next section.

2.2 Effects of Special Causes on Adjusted Output

If a special cause occurs in the system, the underlying process is contaminated by the effects of the cause and the contaminated process, denoted by N_t, can be represented as

$$N_t = Z_t + \omega\xi(B)I_t(T), \quad (7)$$

where T is the time of occurrence, ω is the impact parameter of the cause, and $I_t(T) = 1$ if $t = T$ and 0 if $t \neq T$ signifies the pulse indicator at time T. In (7), $\omega\xi(B)I_t(T)$ represents contaminating effects of a special cause on underlying process where $\xi(B) = 1 + \xi_1 B + \xi_2 B^2 + \cdots$ denotes the type or pattern of the cause effects on (pure) underlying process Z_t. Adopting outlying patterns in time series, we consider three types of special causes such as

$\xi(B) = 1$ for AO type,

$\xi(B) = \psi(B)$ for IO type,

$\xi(B) = 1/(1-B)$ for LS type

[see, Tsay (1986), Chen and Liu (1993a,b)].

We now consider such a situation that a special cause has not been detected and thus the MMSE control rule (5) is applied to a contaminated process N_t in process adjustment. Then, at time $t-1$, the one- step-ahead forecast of underlying process value, denoted as $\hat{Z}_{t-1}(1)$, will be computed with contaminated series N by $\hat{Z}_{t-1}(1)=\pi^{(1)}(B)N_t$, as done in (4). Using the fact that $B^k I_t(T)=I_{t-k}(T)$ and thus $\xi(B)I_t(T)=\xi_{t-T}$, the forecast based on contaminated series can be expressed in terms of the forecast based on pure underlying series Z as follows:

$$\hat{Z}_{t-1}(1)=Z_{t-1}(1) + \omega\sum_{i=1}^{t-T}\pi_i\xi_{t-T-i}. \qquad (8)$$

From now on, the hat '^' above any character signifies the use of contaminated series N in computation of the statistic. From (8), the bias in the one-step-ahead forecast led by special causes of each types can be explicitly expressed. Using the identity $\psi(B)\pi(B)=1$ and an useful expression therefrom,

$$\psi_k=\sum_{j=1}^{k}\psi_{k-j}\pi_j,$$

the biases in forecast are computed as $\omega\pi_{t-T}$ for AO type, $\omega\psi_{t-T}$ for IO type, and $\omega\sum_{i=1}^{t-T}\pi_i$ for LS type special cause, respectively.

It is important to note that process adjustment employs a system of statistical forecasting, and thus special causes shall produce some carry-over effects on process control when they are not properly accounted for. For such situation, process adjustment based on contaminated underlying series would produce biases in the adjusted outputs in some degree. When MMSE controller in (5) is applied to contaminated series, input variable X will be set as $\hat{X}_{t-1}=-\hat{Z}_{t-1}(1)/g$ and the influence of special causes on input variable can be explicitly expressed, in terms of X_{t-1}, as

$$\begin{aligned}\hat{X}_{t-1}&=-(Z_{t-1}(1) + \omega\sum_{i=1}^{t-T}\pi_i\xi_{t-T-i})/g \\ &=X_{t-1}-(\omega/g)\sum_{i=1}^{t-T}\pi_i\xi_{t-T-i}.\end{aligned} \qquad (9)$$

The adjusted output, when the MMSE rule (9) is applied to contaminated process, can be written as $\hat{U}_t=N_t+g\hat{X}_{t-1}$. By simple derivation, the adjusted output can be expressed, in terms of U_t, as

$$\hat{U}_t=U_t+\omega(\xi_{t-T}-\sum_{i=1}^{t-T}\pi_i\xi_{t-T-i}), \qquad (10)$$

where $U_t=e_{t-1}(1)=a_t$ in (6) is used in derivation of (10). It is to be noted that at time t, the magnitudes of bias in the adjusted output are ω for all types of special causes. However, for time $t>T$, the bias that is given in the

<Table 1> Bias in the adjusted output for each type of special causes

Type / Model	Bias, where $\lambda=1-\theta$ and $\delta=\phi-\theta$, for $t(>T)$			
	General Form	AO	IO	LS
ARMA(p,q)	$\omega(\xi_{t-T}-\sum_{i=1}^{t-T}\pi_i\xi_{t-T-i})$	$-\omega\pi_{t-T}$	0	$\omega(1-\sum_{i=1}^{t-T}\pi_i)$
AR(1)	$\omega(\xi_{t-T}-\phi\xi_{t-T-i})$	$-\omega\phi I_t(T+1)$	0	$\omega(1-\phi)$
MA(1)	$\omega(\xi_{t-T}+\sum_{i=1}^{t-T}\theta^i\xi_{t-T-i})$	$\omega\theta^{t-T}$	0	$\omega(1-\theta^{t-T+1})/\lambda$
ARMA(1,1)	$\omega(\xi_{t-T}-\delta\sum_{i=1}^{t-T}\theta^{i-1}\xi_{t-T-i})$	$-\omega\delta\theta^{t-T-1}$	0	$\omega\{1-\delta(1-\theta^{t-T})/\lambda\}$

second term of (10) can be explicitly written, for each type of special cause, as $-\omega\pi_{t-T}$ for AO type, zero for IO type, and $\omega(1-\sum_{i=1}^{t-T}\pi_i)$ for LS type, which are summarized in Table 1. We thus showed that special causes of different types produce distinct patterns of biases in the adjusted output, from which some statistics and procedures for identifying the types of special causes occurred can be constructed.

Bias in Table 1 can be interpreted as the magnitude of mean level shift in output process, and thus the effects of a special cause of each type on the output process can be explicitly explained. For an AO type special cause, the bias in U_t at the occurrence time is ω but it decreases with opposite sign because $-\omega\pi_{t-T}$ converges to zero as $t\rightarrow\infty$ from the fact that stationarity of Z_t guarantees $\pi_j\rightarrow 0$ as $j\rightarrow\infty$. Meanwhile, an IO type special cause produces effects that follow the pattern of dependent structure of the underlying process Z_t, i.e., the ψ-weights of the model. Therefore, MMSEF based on past contaminated series N becomes MMSEF of future contaminated underlying process N_t, rather than Z_t, resulting in no bias in the adjusted output process, as given in Table 1.

For LS type special cause, the mean level shift, that is, bias $\omega(1-\sum_{i=1}^{t-T}\pi_i)$, in the adjusted output process is ω at the occurrence time T, and converges to some non-zero finite value as time goes on (i.e., $t\rightarrow\infty$), because $\sum_{i=1}^{\infty}\pi_i=1$ isn't satisfied for stationary process. That is, when underlying process is stationary, the effects of a special cause of LS type would not vanish eventually, resulting in permanent mean shift in the adjusted output process. Therefore, some additional set-up adjustment procedure may be needed to account for the effects to be remained in the output, which will be reported in a separate paper.

3. Illustrative Examples

In Section 2, the effects of special causes on process adjustment are derived for general ARMA(p,q) underlying process model and three types of special causes are considered. We now investigate the effects on the adjusted outputs for three illustrative models: AR(1), MA(1), and ARMA(1,1), in details.

AR(1) Process

The AR(1) process with parameter ϕ for $|\phi|<1$ is a stationary process. This process is considered appropriate to many real situations, especially for positive parameter values [Atienza et al. (1998), Montgomery (2001)]. Using the fact that $\pi_j=\phi$ for $j=1$ and 0 for $j\geq 2$ and $\psi_j=\phi^j$, at time t the bias in the adjusted output brought by the special cause of general type can be derived as

$$\omega(\xi_{t-T}-\phi\xi_{t-T-i}).$$

For an AO type special cause, because of the Markov property of AR(1) model, the biases in adjusted outputs are ω and $-\omega\phi$ at times T and $T+1$, respectively, but zero for $t(\geq T+2)$. That is, the effects on the outputs last only two periods with opposite direction (sign). For IO type, the bias is ω at time T, but zero at times $t(\geq T+1)$, from the reason explained in Section 2.2.

Meanwhile, Markov property of AR(1) implies that a special cause of LS type produces bias of ω at time T, and non-zero constant bias $\omega(1-\phi)$ for all time points after T. Therefore, MMSE controller applied to the contaminated underlying process does not compensate all the effects of LS type cause, resulting in permanent mean shift in the adjusted output process. For this case, detection of an LS type special cause and elimination of the source of the cause (or re-setting up input variable) will be crucial to reduce losses from the increased variation in the adjusted output process.

MA(1) Process

The MA(1) process with parameter θ is a stationary process of another kind. Though MA(1) process may not be encountered often in manufacturing or chemical processes, it may be appropriate for processes related to business and economy. Using the fact that $\pi_j=\phi$ and $\psi_j=\theta^j$ for $j=1$ and 0 for $j\geq 2$, the bias in the adjusted output can be derived as

$$\omega(\xi_{t-T}+\sum_{i=1}^{t-T}\theta^i\xi_{t-T-i}),$$

as shown in Table 1.

For AO type special cause, the biases in adjusted outputs are ω at the occurrence time T and the bias $\omega\theta^{t-T}$ decreases geometrically as time t runs away from time T, vanishing eventually. That is, depending on θ, bias may not be negligible for a while just after the occurrence time. For IO type, the bias is ω at time T, but zero at time $t(\geq T+1)$.

The bias in the adjusted outputs incurred by a LS type special cause is ω at time T and $\omega(1-\theta^{t-T+1})/\lambda$ for $t(\geq T+1)$, where $\lambda=1-\theta$. Thus, the bias decreases as time goes on and converges to ω/λ eventually. That is, an LS type special cause can not be completely compensated by the MMSE control and permanent mean shift is brought into the adjusted output process.

ARMA(1,1) Process

In process adjustment, IMA(1,1) process is of particular importance, because if they are left unadjusted there is no guarantee that they will return to the target in a finite periods. We consider ARMA(1,1) process as an illustrative example, because when the AR parameter ϕ converges to 1 the process becomes IMA(1,1) process. For this process, $\psi_j=\delta\phi^{j-1}$ and $\pi_j=\delta\theta^{j-1}$ for $j\geq 1$, where $\delta=\phi-\theta$. From equation (10), the bias in the adjusted output is simplified as

$$\omega(\xi_{t-T}-\delta\sum_{i=1}^{t-T}\theta^{i-1}\xi_{t-T-i}),$$

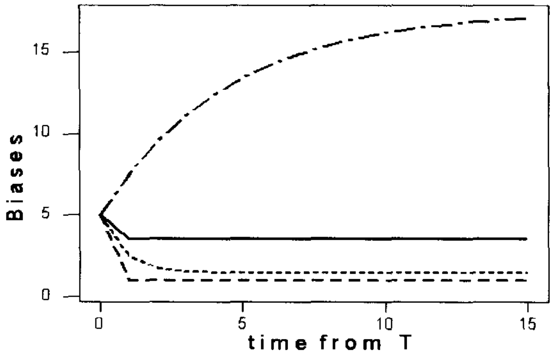
which is given in Table 1.

It is to be noted again that the biases at the occurrence time T are ω for all three types of special causes. Now, consider the effects of special causes on the adjusted output at time t after the occurrence time T. For AO type special cause, the bias is $-\omega\delta\theta^{t-T-1}$, which decreases geometrically and converges to zero, i.e., vanishes, as time goes on. For IO type, there will be no bias after T.

For the LS type special cause, MMSE control rule produces bias in the adjusted output, as given in the Table 1, as $\omega\{1-\delta(1-\theta^{t-T})/\lambda\}$, which converges to a non-zero permanent mean shift $\omega(1-\phi)/(1-\theta)$, eventually. That is, for ARMA(1,1) model, it is understood that the effects of LS type cause cannot be entirely compensated by the MMSE control rule.

A Numerical Example

As an numerical example, we consider four models such as two AR(1) model with $\phi=$ 0.3 and 0.8, and two ARMA(1,1) models with parameters of $(\phi,\ \theta)=$ (0.8, 0.3) and (0.3, 0.8). The biases at times $t \geq T$ led by a LS type special cause with $\omega = 5.0$ are computed for each model and given in Figure 2.



<Figure 2> Biases in adjusted output in times from T : { $\omega = 5.0$ and $(\phi, \theta) =$ (0.3, 0.0), solid; (0.8, 0.0), dash; (0.8, 0.3), dot; (0.3, 0.8), dash 1-dot line}

For AR(1) model, the effects (mean shifts) of the LS type cause are 0.5 $(= \omega)$ at time T for both models and 3.5 and 1.0 $(= 1 - \phi)$ for model with parameter ϕ = 0.3 and 0.8, respectively, which are permanent mean level changes in process outputs. We note that a model with greater parameter value has smaller effects in process outputs.

For ARMA(1,1) model, the biases in the adjusted output depend on the parameter values, ϕ and θ. The biases given in Table 1 depend mainly on $\phi - \theta$ and $\lambda = 1 - \theta$, and thus relative size of ϕ and θ is crucial for the pattern of biases. When $(\phi, \theta) = (0.8, 0.3)$, $\phi - \theta = 0.5$ and thus the bias decreases to the limit value $\omega(1-\phi)/(1-\theta)$, which is smaller than $\omega = 5.0$. Meanwhile, for the model with $(\phi, \theta) = (0.3, 0.8)$, the bias increases to the limit that is larger than the impact parameter value $\omega = 5.0$, In summary, the performance of process adjustment may depend on the degree of dependency in the underlying process.

4. Summary and Conclusion

We considered a responsive feedback control system, in which the underlying process follows a stationary ARMA process. It is well known that the MMSE control rule is optimal in that it minimizes the variability of the adjusted output process. In practice, special causes may occur in the system and application of the MMSE control rule without accounting for the occurrence of the cause may lead to off-target biases and increased variability in the adjusted output.

In this paper, we introduced a general framework for the special cause problems in process adjustment. By adopting three types of special causes, namely AO, IO, and LS, we derived the impacts of special causes on the adjusted output process for stationary underlying process, in general sense. In summary, it is shown that the effects of special causes at the occurrence time T are ω for all types, but the patterns of effects for $t(>T)$ are distinct from the three types of special causes.

For AO type special cause, the effects are shown to be vanished eventually. That is, though it may take some periods, the impact of the cause also will be entirely compensated in process of adjustment. For IO type cause, there will be no bias in the adjusted output process after T because the effects follow the pattern of dependent structure of the underlying process. For LS type special cause, it is shown that MMSE control rule produces biases in the adjusted output, which converges to a level of permanent mean shift, eventually. That is, the effects of LS type cause cannot be entirely compensated by the MMSE control rule.

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