

# Uncertainty in the Calibration of Coaxial Thermal Noise Sources using a Noise Figure Measuring Equipment

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## Abstract

In this paper, the uncertainty in the calibration of coaxial thermal noise sources using a noise figure measuring (NFM) equipment is evaluated. Contributions to the uncertainty such as the calibration uncertainty of the standard noise source, mismatch, measurement of adapter efficiency, ambient temperature variation, and repeatability are evaluated in the frequency range of 10 MHz to 18 GHz. Results show that the expanded uncertainty ( $k=2$ ) is 0.23 dB for the noise sources of 5 dB and 15 dB ENR, and 0.27 dB for those of 21 dB.

**Key words** : Excess Noise Ratio, Noise Temperature, Coaxial Thermal Noise Source, Uncertainty.

## I. Introduction

Thermal noise sources have been used to characterize the noise performance of the electrical and electronic systems. One of them in common use is a diode noise source of which output depends on the noise generated in a solid state diode<sup>[1]</sup>. When the diode is biased, the output noise will be greater than  $kT_C B$ , the noise power at 'off' state, due to the avalanche noise generation in the diode; when unbiased the output will be the thermal noise generated in the attenuator,  $kT_C B$ , where  $k=1.380 \times 10^{-23}$  J/K is the Boltzmann constant,  $T_C$  is the effective noise temperature of the noise source at the 'off' state, and  $B$  is the bandwidth of the measuring system. These levels are denoted by  $T_H$  and  $T_C$ , corresponding to the terms 'hot' and 'cold' state, respectively. The terms can also be used to refer to the noise temperature of two independent noise sources: if the output power of one noise source is higher than that of the other noise source, then the higher one is referred to as a 'hot' noise source and the lower one a 'cold' source.

Each diode noise source has its own excess noise ratio(ENR) calibrated at a certified laboratory. To calibrate the noise temperature of a noise source under test, a specially designed reference noise source is used at a national metrology laboratory. Generally a nitrogen-cooled cold source and an ambient source, or a heated noise source and an ambient noise source are used as a pair. For the calibration laboratories of domestic

industries, a diode noise source and a noise figure measuring(NFM) equipment are usually used to characterize the noise performance of electrical and electronic devices, modules, or systems. If a calibration laboratory is equipped with hot and cold standard noise sources and an NFM equipment, one can readily calibrate a noise source. The term, a standard noise source, means that its ENR has been calibrated at a laboratory with higher level of traceability of the national standards.

Traditionally, the  $Y$ -factor method has been used to determine the noise performance of an electrical and electronic equipment<sup>[1]</sup>. In this paper, we describe the calibration method of the coaxial noise source using a commercial NFM equipment, and calibrated hot/cold noise sources. In the following section, the uncertainty of the overall calibration system is thoroughly analyzed in the frequency range of 10 MHz to 18 GHz. Finally, we will conclude by addressing future works to reduce the calibration uncertainty.

## II. The Excess Noise Ratio of a Thermal Noise Source

The excess noise ratio(ENR) of a thermal noise source is defined as [2]

$$\text{ENR}_{\text{dB}} = 10 \log \left( \frac{T_H - T_C}{T_0} \right) \quad (1)$$

where  $T_0$  is the reference temperature, 290 K. Noise temperatures  $T_H$  and  $T_C$  corresponding to hot and cold

states, respectively, should be an effective temperature. The term "effective" indicates that the reflection coefficient of the noise source was taken into account. This definition of ENR supersedes an earlier definition  $ENR=(T_H-T_0)/T_0$  or  $ENR_{dB}^{CAL}=\log\{(T_H-T_0)/T_0\}$ , which implicitly assumes that  $T_C$  is always to be  $T_0$ . Nevertheless, the calibrated ENR of a noise source is always referenced to  $T_C=T_0=290$  K. If  $T_C$  is not  $T_0$ , the ENR is not correct. In such case, the physical temperature of the noise source should be measured and the temperature correction is applied as [2].

$$ENR_{dB}^{CORR} = 10 \log \left\{ \left[ \frac{T_H - T_0}{T_0} \right] + \left[ \frac{T_0 - T_C}{T_0} \right] \right\}$$

$$= 10 \log \left\{ 10^{\frac{ENR_{dB}^{CAL}}{10}} + \left[ \frac{T_0 - T_C}{T_0} \right] \right\} \quad (2)$$

where  $ENR_{dB}^{CORR}$  is the corrected ENR value and  $ENR_{dB}^{CAL}$  is the original calibrated ENR values. In (2), the second term in the bracket is the factor to compensate the deviation of  $T_C$  from  $T_0$ .

### III. Measurement

#### 3-1 Measurement System

The measurement setup for calibrating a noise source is shown in Fig. 1. The equipments required are as follows: a signal generator, an NFM equipment, and a set of hot and cold noise sources. A network analyzer which operates in the frequency range of interest is used for measuring the efficiency of adapters. Inevitably, ancillary items such as adapters and cables are necessary for the measurement system.

The measurement procedure to calibrate the ENR of a noise source is as follows:

- (a) Calibrate the network analyzer in a 'full 2-port' mode using an appropriate calibration kit and measure the insertion loss,  $\alpha_L$ , of the adapter used to connect a noise source under test or

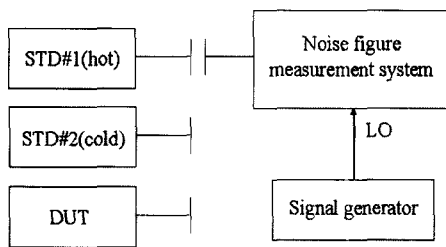


Fig. 1. The measurement setup for calibrating a noise source.

generally device under test(DUT) to the NFM equipment.

- (b) Perform the self-calibration of the NFM equipment using the hot standard noise source.
- (c) Keeping the hot noise source connected to the NFM equipment as in step (b), measure the noise power  $F_{H,dB}$  by taking the reading of the NFM equipment.
- (d) Connect the cold noise source to the NFM equipment and measure the noise power,  $F_{C,dB}$ .
- (e) Connect DUT to the NFM equipment and record the measured noise power,  $F_{X,dB}$ .

We used the HP 8970B/8971C NFM system and its special functions 9.3 and 9.4 enable us to measure the power density delivered from the DUT. It is noted that the quantities,  $F_{H,dB}$ ,  $F_{C,dB}$ , and  $F_{X,dB}$ , have the unit of dB referenced to  $T_0=290$  K (-174 dBm/Hz)<sup>[3]</sup>:

$$\text{Power displayed} = 10 \log \frac{\text{unknown power density}}{T_0} \text{ (dB)}.$$

The noise sources through the steps (b) to (e) should be powered by 28 V DC.

#### 3-2 Mathematical Model

Using the noise powers measured, we obtain the Y-factors  $Y_1$  and  $Y_2$  to eliminate the noise power of the NFM equipment,  $kT_{NFM}B$ .

$$Y_1 = \frac{F_H}{F_C} = \frac{k(T_H + T_{NFM})B}{k(T_C + T_{NFM})B} \quad (3)$$

$$Y_2 = \frac{F_X}{F_C} = \frac{k(T_X + T_{NFM})B}{k(T_C + T_{NFM})B} \quad (4)$$

where  $F_H$ ,  $F_C$  and  $F_X$  are the measured powers referenced to  $T_0$  in linear unit of standard hot noise source, standard cold noise source, and the DUT, respectively. Note  $T_H$  and  $T_C$  are standard noise temperatures of hot and cold standard noise sources, respectively, and  $T_{NFM}$  is the equivalent noise temperature of the NFM equipment. The standard noise temperature is calibrated using primary standard noise sources and a radiometer, which is a low-noise high-sensitivity rf receiver system. From (3) and (4), the noise temperature of the DUT  $T_X$  is given by

$$T_X = T_C + (T_H - T_C) \frac{Y_2 - 1}{Y_1 - 1}. \quad (5)$$

Note that (5) is valid only for the DUT with no adapter. In general, the input connector of an NFM

equipment and the connector of a noise source differ from each other, so an adapter should be used as illustrated in Fig. 2. Let  $T_X$  and  $T_{X'}$  be the noise temperature of a noise source and that of the noise source with an adapter. The temperature  $T_{X'}$  is given by [4]

$$T_{X'} = T_X \alpha_L + T_a(1 - \alpha_L) \quad (6)$$

where  $\alpha_L$  and  $T_a$  are the efficiency and physical temperature of the adapter. The adapter efficiency is discussed in the next section. Substituting (5) into (6) as  $T_{X'}$ , the noise temperature  $T_X$  and corresponding ENR<sub>dB</sub> are expressed by

$$T_X = \left\{ T_C + (T_H - T_C) \frac{Y_2 - 1}{Y_1 - 1} \right\} \frac{1}{\alpha_L} + T_a \left( 1 - \frac{1}{\alpha_L} \right) \quad (7)$$

$$\text{ENR}_{\text{dB}} = 10 \log \frac{T_X - T_0}{T_0} \quad (8)$$

It is worthy to mention that equations (7) and (8) are the mathematical model for the measurands  $T_X$  and ENR<sub>dB</sub>. In (8), the ENR is defined for noise sources of which  $T_X$  is greater than 290 K.

#### IV. Measurement Uncertainty

Since the mathematical model of the measurand is given in (7) and (8), the uncertainty is easily evaluated in accordance with the international or national documents [5], [6]. The combined uncertainty of the noise temperature of a noise source under test is given by

$$\Delta T_X = \left\{ \left[ \frac{\partial T_X}{\partial T_C} \Delta T_C \right]^2 + \left[ \frac{\partial T_X}{\partial T_H} \Delta T_H \right]^2 + \left[ \frac{\partial T_X}{\partial \alpha_L} \Delta \alpha_L \right]^2 + \left[ \frac{\partial T_X}{\partial T_a} \Delta T_a \right]^2 + \left[ \frac{\partial T_X}{\partial Y_2} \Delta Y_2 \right]^2 + \left[ \frac{\partial T_X}{\partial Y_1} \Delta Y_1 \right]^2 \right\}^{\frac{1}{2}} \quad (9)$$

or in the form of the standard uncertainty

$$u_{T_X} = \left\{ \left[ \frac{\partial T_X}{\partial T_C} u_{T_C} \right]^2 + \left[ \frac{\partial T_X}{\partial T_H} u_{T_H} \right]^2 + \left[ \frac{\partial T_X}{\partial \alpha_L} u_{\alpha_L} \right]^2 + \left[ \frac{\partial T_X}{\partial T_a} u_{T_a} \right]^2 + \left[ \frac{\partial T_X}{\partial Y_2} u_{Y_2} \right]^2 + \left[ \frac{\partial T_X}{\partial Y_1} u_{Y_1} \right]^2 \right\}^{\frac{1}{2}} \quad (10)$$

where the sensitivity coefficients of each quantity are expressed by

$$s_{T_C} \equiv \frac{\partial T_X}{\partial T_C} = \left( 1 - \frac{Y_2 - 1}{Y_1 - 1} \right) \frac{1}{\alpha_L} \quad (11)$$

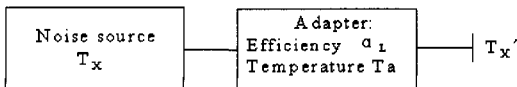


Fig. 2. A noise source with an adapter.

$$s_{T_H} \equiv \frac{\partial T_X}{\partial T_H} = \left( \frac{Y_2 - 1}{Y_1 - 1} \right) \frac{1}{\alpha_L} \quad (12)$$

$$s_{\alpha_L} \equiv \frac{\partial T_X}{\partial \alpha_L} = -\frac{1}{\alpha_L^2} \left( T_C + (T_H - T_C) \frac{Y_2 - 1}{Y_1 - 1} \right) + T_a \frac{1}{\alpha_L^2} \quad (13)$$

$$s_{T_a} \equiv \frac{\partial T_X}{\partial T_a} = 1 - \frac{1}{\alpha_L} \quad (14)$$

$$s_{Y_2} \equiv \frac{\partial T_X}{\partial Y_2} = \frac{T_H - T_C}{Y_1 - 1} \frac{1}{\alpha_L} \quad (15)$$

$$s_{Y_1} \equiv \frac{\partial T_X}{\partial Y_1} = (T_H - T_C) \frac{1 - Y_2}{(Y_1 - 1)^2} \frac{1}{\alpha_L} \quad (16)$$

For the sake of convenience, the uncertainty  $\Delta T_X$  or  $u_{T_X}$  is firstly evaluated. The uncertainty  $\Delta \text{ENR}$  or  $u_{\text{ENR}}$  is then obtained by

$$u_{\text{ENR}} = 10 \log \left\{ \frac{(T_X + \Delta T_X) - T_0}{T_X - T_0} \right\} = 10 \log \left( 1 + \frac{\Delta T_X}{T_X - T_0} \right) \quad (17)$$

As previously mentioned, we will not attempt to compensate the mismatch effect of the noise source during its calibration but consider the effect as a contribution to the measurement uncertainty.

##### 4-1 Type A Evaluation of Standard Uncertainty

In general, we do not make measurements repeatedly to obtain the ENR values of a noise source under test. Nevertheless, a sequence of measurements was performed to obtain ENR values. The resultant Type A standard uncertainty of ENR values is to be 0.044 dB ( $n=14$ ). Since the ENR calibration data have been obtained under statistical control, the Type A uncertainty was obtained using the equation for a pooled sample standard variation<sup>[5]</sup>

$$s_p^2 = \frac{\sum_{i=1}^N \nu_i s_i^2}{\sum_{i=1}^N \nu_i} \quad (18)$$

where  $s_i^2$  is the estimate of the variance of the  $i$ th series of  $n_i$  observations and  $\nu_i = n_i - 1$  is the degree of freedom for  $i$ th series. Since we obtained a set ( $N=1$ ) of data with  $n_i=14$ , (18) reduces to  $s_p^2 = s_1^2$ . Therefore, Type A uncertainty is obtained by  $u_A = s_p / \sqrt{N} = s_1$ .

Fig. 3 illustrates the Type A uncertainty of the ENR determination of a 15 dB noise source. Although we evaluated the Type A uncertainty for only a 15 dB noise source, we can expect the same Type A uncertainty for 5 dB and 21 dB noise sources.

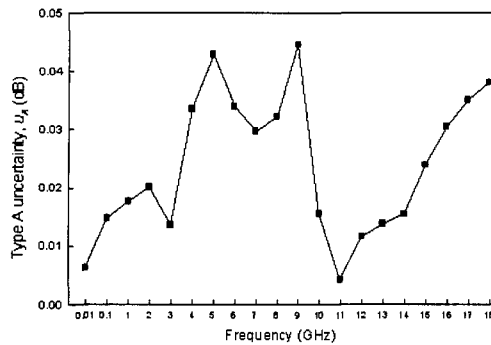


Fig. 3. Type A uncertainty of the ENR determination for a 15 dB ENR noise source.

#### 4-2 Type B Evaluation of Standard Uncertainty

##### 4-2-1 Uncertainty in the Calibration of the Standard Noise Source

A noise source needs the calibration according to an appropriate calibration procedure. If the noise source is used to calibrate another noise source under test, then the former is referred to as 'a standard noise source'. Normally the standard noise source is calibrated at a national metrology institute(NMI), the Korea Research Institute of Standards and Science(KRISS) in our country. A calibration report issued by the NMI usually contains the expanded uncertainty(coverage factor  $k=2$ , which means the confidence level is 95.45 %) [5],[6].

The effective noise temperature,  $T_X$ , of a DUT is given by

$$T_X = T_{avail} (1 - |\Gamma_S|^2) \tag{19}$$

where  $T_X$ , the effective noise temperature, is proportional to the power emanating from the output port of the noise source when it is connected to a non-reflecting load. The noise temperature  $T_{avail}$  is proportional to the available noise power from the source and is obtained when reflection coefficients of source and load are complex conjugates of each other.  $\Gamma_S$  is the reflection coefficient of the noise source.

A calibration report issued by an NMI, National Institute of Standards and Technology(NIST) of U.S.A. contains typically the ENR data of a noise source under test as shown in Table 1. Dividing the expanded uncertainty  $U_{TC}$  by the coverage factor of 2, we obtain the standard uncertainty of the cold standard noise source. For a hot noise source, we can obtain the standard uncertainty in a similar way. We quoted the uncertainty from a calibration report issued by the

Table 1.  $U_{ENR,dB}$  and  $U_{TC}$  of a standard cold noise source.

| Frequency (GHz) | ENR(dB) | $U_{ENR}(dB)$ | $T_C(K)$ | $U_{TC}(K)$ |
|-----------------|---------|---------------|----------|-------------|
| 0.03            | 5.18    | 0.07          | 1,247    | 15.5        |
| 2               | 4.83    | 0.07          | 1,171    | 14.3        |
| 4               | 4.67    | 0.07          | 1,140    | 13.8        |
| 6               | 4.76    | 0.08          | 1,157    | 16.1        |
| 8               | 4.83    | 0.08          | 1,172    | 16.4        |
| 10              | 4.96    | 0.09          | 1,198    | 19.1        |
| 12              | 4.89    | 0.10          | 1,183    | 20.8        |

NIST. A reference coaxial noise source, which is one of key components for establishing the measurement standards of the noise temperature, is now under development by the technical cooperation with the KRISS and the All-Russian Scientific Research Institute of Physical-Technical and Radiotechnical Measurements (VNIIFTRI), Russian Federation. After completion of the reference noise source, the national standard of RF noise will be established in conjunction with the total power coaxial radiometer developed in 1989 [7].

##### 4-2-2 Uncertainty in the Measurement of an Adaptor Loss

A noise source might have virtually any types of connector. Therefore, we might have to use an adaptor in order to connect the noise source to an NFM equipment. In such case, the loss of the adaptor has to be measured to compensate its effect on the ENR of the noise source. From the equivalent circuit shown in Fig. 4, the quantity  $\alpha_L$ , the ratio of the available power at the output to that at the input, is given by [8]

$$\alpha_L = \frac{Z_M}{Z_N} \frac{|S_{21}|^2}{|1 - S_{11}\Gamma_S|^2} \frac{1 - |\Gamma_S|^2}{1 - |\Gamma_2|^2} \tag{20}$$

$$\Gamma_2 = S_{22} + \frac{S_{21}S_{12}\Gamma_S}{1 - S_{11}\Gamma_S} \tag{21}$$

The quantity  $\alpha_L$  is a direct measure of how much noise the two-port or the adaptor will contribute to the output and  $\alpha_L$  is independent of  $|\Gamma_L|$ . One can readily measure S-parameters of the adaptor using a network analyzer. It is assumed that  $S_{22}$  is negligibly small and that the characteristic impedances  $Z_M$  and  $Z_N$  are 50Ω. Furthermore, if we assume  $|\Gamma_S| \approx 0$ ,  $|\Gamma_2| \approx 0$ , then

$$\alpha_L \approx |S_{21}|^2. \tag{22}$$

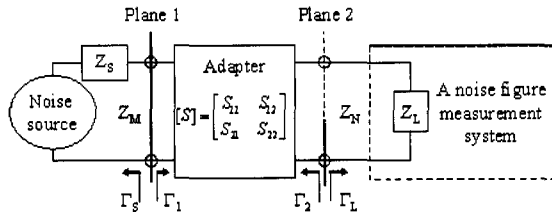


Fig. 4. Schematic circuit to depict adapter effect in the noise source calibration.

From the measured  $S_{21, \text{dB}} (= 20 \log |S_{21}|)$  of the adapter, the quantity  $a_L$  in (6) is given by

$$a_L = 10^{\frac{S_{21, \text{dB}}}{10}} \quad (23)$$

It is emphasized that  $S_{21, \text{dB}}$  is divided by a factor 10 because the factor is relevant to the power.

The first contribution to  $u_{a_L}$ ,  $u_{a_L, \text{approx}}$  comes from the approximation of (22). The measured VSWR of the noise sources are illustrated in Fig. 5 and the maximum VSWR corresponding to  $|\Gamma_s|$  in (20) is found to be 1.12:1, 1.15:1, and 1.40:1 for 5 dB, 15 dB, and 21 dB ENR noise sources, respectively. For an Type N-to-7 mm adapter with a return loss of >28 dB, the uncertainty  $u_{a_L, \text{approx}}$  was evaluated to be 0.0058, 0.0069, and 0.0150 in a linear quantity with the rectangular distribution for 5 dB, 15 dB, and 21 dB ENR noise sources, respectively.

The second contribution to  $u_{a_L}$ ,  $u_{a_L, S_{21} \text{ meas}}$ , is the uncertainty of S-parameter measurement using a network analyzer. We used HP 8720D network analyzer with the  $S_{21}$  uncertainty of 0.020 dB from 0.05 GHz to 0.5 GHz, 0.022 dB from 0.5 GHz to 8 GHz, and 0.027 dB from 8 GHz to 18 GHz. Division of the uncer-

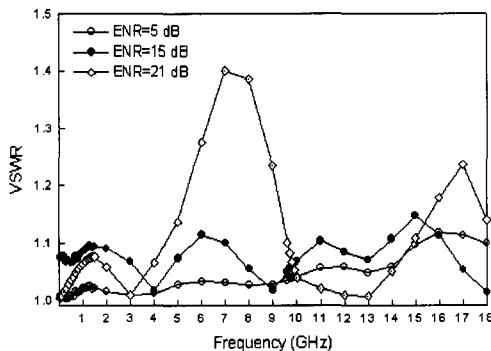


Fig. 5. Measured VSWR of the noise sources with different ENR values. The 28 V DC voltage was applied to each noise source so that the noise sources is in 'on' state.

tainty by the factor of  $\sqrt{3}$  (rectangular distribution) gives the standard uncertainty  $u_{S_{21}}$ . For instance, the measured  $S_{21}$  of a 3.5 mm-to-7 mm coaxial adapter at 1 GHz frequency is  $-0.0229$  dB and the standard uncertainty of the  $S_{21}$ ,  $u_{S_{21}}$ , is  $0.022 \text{ dB}/\sqrt{3} = 0.013 \text{ dB}$ . The standard uncertainty of the measurement of the adapter loss due to the  $S_{21}$  measurement is obtained by

$$u_{a_L, S_{21} \text{ meas}} = \begin{cases} (\text{positive}) 10^{\frac{u_{S_{21}}}{10}} - 1 = 0.0030 \\ (\text{negative}) 1 - 10^{-\frac{u_{S_{21}}}{10}} = -0.0030 \end{cases}$$

$$\rightarrow u_{a_L, S_{21} \text{ meas}} = \pm 0.0030.$$

Consequently, the uncertainty in the determination of  $a_L$  is given by

$$u_{a_L} = \sqrt{u_{a_L, \text{approx}}^2 + u_{a_L, S_{21} \text{ meas}}^2} \quad (24)$$

The uncertainty of  $a_L$  for another frequency can be obtained in a similar way.

#### 4-2-3 Uncertainty of the Measurement of Ambient Temperature

In (7),  $T_a$  is the physical temperature of the adapter used at a laboratory. It is assumed that the calibration laboratory has been thermally stabilized prior to the calibration so the temperature of the adapter to be the ambient temperature of the laboratory. A thermometer, TR-72S, T&D Corporation, Japan, has been calibrated at the Temperature and Humidity Group of the KRISS. The expanded uncertainty ( $k=2$ ) is  $\pm 1.0$  K and the standard uncertainty of the ambient temperature measurement is  $\Delta T_a = 1.0 \text{ K}/2 = 0.5 \text{ K}$ .

#### 4-2-4 Uncertainty in the Measurement of Y-factors

From (3) and (4), the uncertainty in the measurement of Y-factors is expressed by

$$\Delta Y_1 = \left\{ \left[ \frac{\partial Y_1}{\partial F_H} \Delta F_H \right]^2 + \left[ \frac{\partial Y_1}{\partial F_C} \Delta F_C \right]^2 \right\}^{\frac{1}{2}}$$

$$= \left\{ \left[ \frac{1}{F_C} \Delta F_H \right]^2 + \left[ \frac{-F_H}{F_C^2} \Delta F_C \right]^2 \right\}^{\frac{1}{2}} \quad (25)$$

$$\Delta Y_2 = \left\{ \left[ \frac{\partial Y_2}{\partial F_X} \Delta F_X \right]^2 + \left[ \frac{\partial Y_2}{\partial F_C} \Delta F_C \right]^2 \right\}^{\frac{1}{2}}$$

$$= \left\{ \left[ \frac{1}{F_C} \Delta F_X \right]^2 + \left[ \frac{-F_X}{F_C^2} \Delta F_C \right]^2 \right\}^{\frac{1}{2}} \quad (26)$$

Equations (25) and (26) indicate the uncertainty of the  $Y$ -factor determination due to the uncertainty of the noise power measurements using the NFM equipment. To obtain a stable reading, the smoothing factor of the NFM equipment was set to 16. The smoothing was performed in an arithmetically averaging mode.

For the determination of the  $Y$ -factors, one has to measure the noise powers,  $F_H$ ,  $F_C$ , and  $F_X$ . From the operation manual of HP 8970B/8971C NFM system, the instrumentation uncertainty  $\Delta F_i (i=H, C, \text{ and } X)$  is  $0.1 \text{ dB}/\sqrt{3}$  from a rectangular probability distribution. The instrumentation uncertainty in a linear quantity  $u_{i, \text{inst}}$  is  $0.023/\sqrt{3}$ .

The second contribution to the uncertainty in the noise power measurements comes from mismatch. The mismatch uncertainty  $u_{i, \text{mis}} (i=H, C, \text{ and } X)$  in dB is expressed by [8]

$$u_{i, \text{mis}} (\text{dB}) = \frac{20 \log(1 \pm |\Gamma_{i,2}| |\Gamma_{i,L}|)}{\sqrt{2}}, \quad i = H, C, \text{ or } X \quad (27)$$

and its probability distribution is U-shaped one. In (27), if the adapter is used,  $|\Gamma_{i,2}|$  has to be calculated using (21) with  $\Gamma_{i,S}$  and  $S$ -parameters of the adapter used; otherwise,  $|\Gamma_{i,2}| = |\Gamma_{i,S}|$ . And  $|\Gamma_{i,L}|$  is the input reflection coefficient of the NFM equipment. The mismatch uncertainty in a linear quantity is  $u_{i, \text{mis}} = \{[1 \pm |\Gamma_{i,2}| |\Gamma_{i,L}|]^2 - 1\} / \sqrt{2}$ . In general, manufacturers specify the matching performance of an equipment in terms of the voltage standing wave ratio (VSWR). The input VSWR of the HP 8970B NFM equipment specified in the operation manual is less than 1.7. The maximum VSWR of 5 dB and 15 dB noise sources are 1.3 from 10 MHz to 30 MHz, 1.15 from 30 MHz to 5000 MHz, and 1.20 from 5 GHz to 18 GHz frequency ranges. A 21 dB noise source shows higher VSWR than the noise source of lower ENR does as presented in Fig. 5. Therefore, the relative uncertainty of the noise power measurement using an NFM equipment is given by

$$u_{F_i} = \Delta F_i = \sqrt{u_{i, \text{inst}}^2 + u_{i, \text{mis}}^2}, \quad i = H, C, \text{ or } X. \quad (28)$$

#### 4-3 The Combined Standard Uncertainty and Expanded Uncertainty

The combined standard uncertainty of  $T_X$  is obtained using (9). The effective degree of freedom was calculated using the well-known Welch-Satterthwaite formula<sup>[5],[6]</sup> and the value was much greater than 100.

Resultantly, the coverage factor  $k$  is 2 for a confidence level of 95.45 %. The expanded uncertainty of  $T_X$  is given by

$$U_{T_X} = k \cdot \Delta T_X = k \cdot u_{T_X}. \quad (29)$$

The corresponding expanded uncertainty of the ENR is obtained by

$$U_{\text{ENR}} (\text{dB}) = 10 \log \left( 1 + \frac{U_{T_X}}{T_X - T_0} \right). \quad (30)$$

### V. Results and Discussion

The combined uncertainty in the calibration of noise

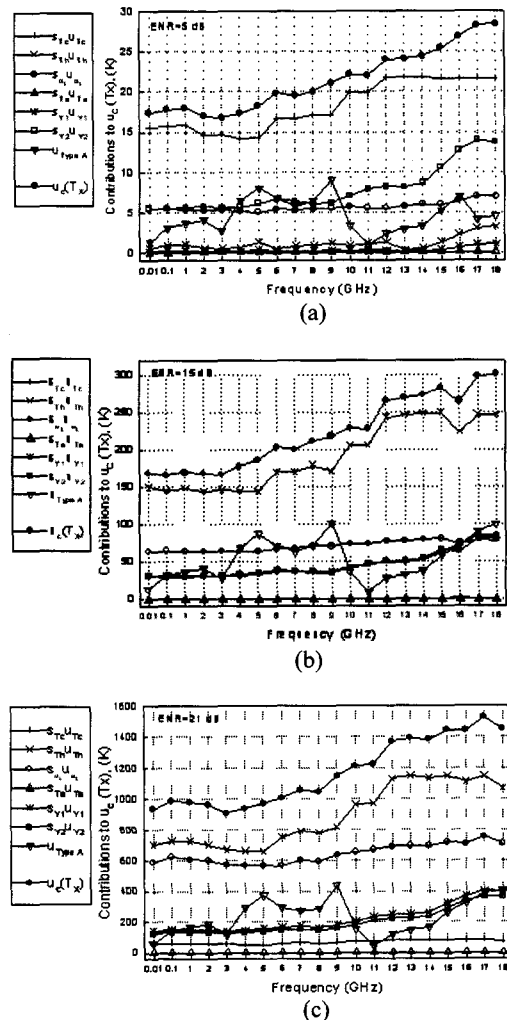


Fig. 6. Contributions to the combined uncertainty in the calibration of the effective noise temperature of thermal noise sources. For the noise sources with the nominal ENR of (a) 5 dB, (b) 15 dB, and (c) 21 dB the uncertainty was evaluated.

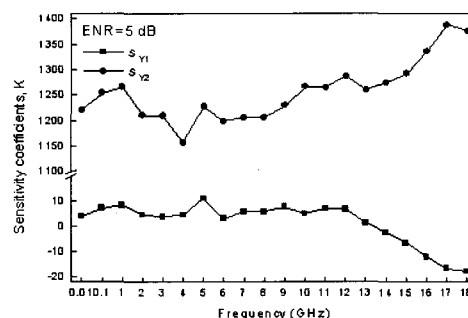
sources and each contribution are presented in Fig. 6. It is observed that the uncertainty of the cold standard noise source,  $\Delta T_C$ , contributes predominantly for a 5 dB ENR noise source while for 15 dB and 21 dB ENR noise sources the uncertainty of the hot standard noise source  $\Delta T_H$  dominates. Therefore, in order to reduce the uncertainty of the noise source calibration, the national standards of the noise temperature should be established.

Next contribution is the uncertainty in determining  $Y$ -factors. It should be mentioned that the contribution due to determination of  $Y_1$  for a 5 dB ENR noise source is negligently small because the sensitivity coefficient  $s_{Y_1}$  is smaller by an order of 2 to 3 compared to the coefficient  $s_{Y_2}$  as shown in Fig. 7(a). For a 15 dB ENR noise source, the two sensitivity coefficients have comparable values as shown in Fig. 7(b), which give almost the same contributions to combined uncertainty as shown in Fig. 6(b). Finally, for a 21 dB ENR noise source,  $s_{Y_1}$  is much greater than  $s_{Y_2}$  in the sense of magnitude. This means that  $s_{Y_1}u_{Y_1}$  is somewhat greater than  $s_{Y_2}u_{Y_2}$  as can be observed in Fig. 6(c). The rest contributions such as uncertainty in measuring the adapter efficiency, the ambient temperature, Type A uncertainty are quite small compared to the major contributions discussed earlier.

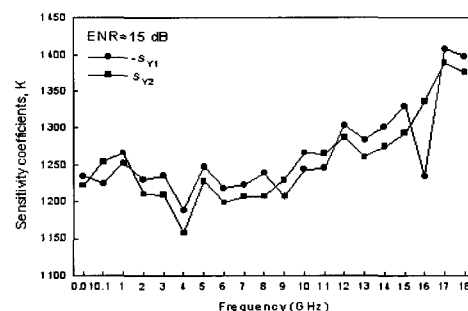
Fig. 8 summarizes the expanded uncertainty ( $k=2$ , confidence level of 95.45 %) in calibrating the noise sources of various ENR values. The expanded uncertainty ranges from 0.16 dB to 0.23 dB for 5 dB and 15 dB noise sources, and from 0.20 dB to 0.27 dB for a 21 dB ENR source in the frequency range of 10 MHz to 18 GHz. It is observed that the uncertainty has the tendency of increasing with respect to the frequency.

### VI. Conclusions

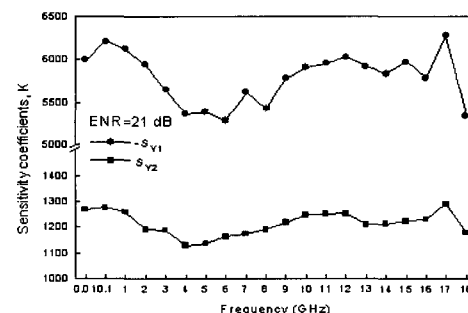
The KRISS has been disseminated the national standards of the noise temperature of coaxial thermal noise sources since early 1990s. To identify major contributions to measurement uncertainty in the calibration of the ENR of a noise source, and to let associated researchers of calibration laboratories in the country be able to evaluate the calibration uncertainty of their own system, we thoroughly evaluated the uncertainty in the frequency range of 10 MHz to 18 GHz. The contributions considered are the calibration uncertainty of the standard noise source, mismatch in determining  $Y$ -factors, measurement of adapter efficiency, ambient temperature variation, and repeatability.



(a) 5 dB ENR noise sources



(b) 15 dB ENR noise sources



(c) 21 dB ENR noise sources

Fig. 7. The sensitivity coefficients  $S_{Y_1}$  and  $S_{Y_2}$  for  $Y$ -factor determination.

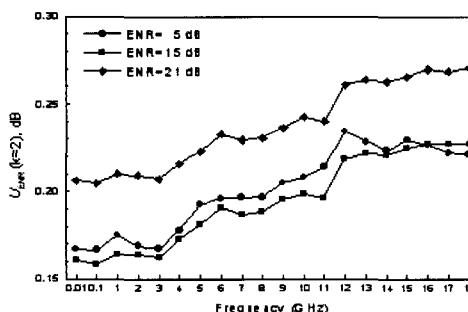


Fig. 8. The expanded uncertainty ( $k=2$ ) in the calibration of the ENR of thermal noise sources.

ency, ambient temperature variation, and repeatability.

The evaluated expanded uncertainty( $k=2$ ) is 0.23 dB for 5 dB and 15 dB noise sources, and 0.27 dB for a 21 dB ENR noise source. The establishment of the measurement standard of noise temperature which follows subsequently after the completion of a reference coaxial noise source should further reduce the uncertainty, which will be one of further research subjects.

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