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A Case Study of Procedural and Conceptual Knowledge Construction in the Computer Environments[1]

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This study investigated three preservice teachers' mathematical problem solving among hand-in-write-ups and final projects for each subject. All participants' activities and computer explorations were observed and video taped. If it was possible, an open-ended individual interview was performed before, during, and after each exploration. The method of data collection was observation, interviewing, field notes, students' written assignments, computer works, and audio and videotapes of preservice teachers' mathematical problem solving activities. At the beginning of the mathematical problem solving activities, all participants did not have strong procedural and conceptual knowledge of the graph, making a model by using data, and general concept of a sine function, but they bult strong procedural and conceptual knowledge and connected them appropriately through mathematical problem solving activities by using the computer technology.

Keywords: preservice teachers, mathematical problem solving, procedural knowledge, conceptual knowledge.
ZDM Classification: B59, D59
MSC2000 Classification: 97B50, 97D50

INTRODUCTION

In mathematics instruction, many controversies exist about whether computer technology can aid in teaching mathematics. Many studies about the effectiveness of using computer technology for teaching mathematics have appeared in recent years (Kang 2003; Hannafin et al. 2001; Jung 2002; Steipen et al. 2000). Among many attempts to encourage using computer technology in school mathematics, the NCTM

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Standards recommended the use of appropriate technology of all levels of mathematics. The emphasis on the use of computer technologies in the classroom and, in particular, the use of computer exploration as a context of mathematics instruction are reflected in the recommendations of the NCTM Standards (1989).

Interactive computer environments have the potential for offering immediate response during mathematical problem solving. The potential of interactive computer environments allows students to carry out actions and operations that can help students construct mathematical knowledge. Through the computer explorations students can make conjectures, test hypotheses based on empirical evidence, and create generalizations. Kaput (1992) and Wiske & Houde (1993) mentioned that computers equipped with appropriate software enable users to gather, manipulate, and represent mathematical ideas in ways that are impractical or impossible with traditional technologies like paper, pencil, and chalkboards.

Using computer technology in teaching school mathematics creates new instructional environments. The emphasis of the use of computer technology in the classrooms and in particular the use of computer-based exploration as a context of mathematics instruction have been reflected in the recommendation of the NCTM Standards (1989). Although the power of using computer technology in the exploration of mathematical problems has been recognized and stressed by many educators, we do not have many research studies on mathematics in computer-based explorations.

Especially research has failed to clarify how computer technology can contribute to the construction of procedural and conceptual knowledge of mathematics. Up to now most researches on procedural and conceptual knowledge in computer environments have only focused on classifying programming languages. Which program language has more random access and rich interrelationship characteristic in relation to conceptual knowledge in humans, and which computer language has more the characteristic flavor of procedural knowledge. How computer-based explorations affect the knowledge construction of mathematics, therefore, emerges as an issue of research on teacher education program for theoretical framework. This situation leads us to do research on the effectiveness of using computer-based exploration in pre-service teacher education in terms of procedural and conceptual knowledge construction.

PURPOSE OF THE STUDY

One of the major issues of mathematics instruction is students' explorations in computer-based environments. Recently many popular calls for instructional purpose, are not much based on research evidence concerning the potential effects of recommended changes. Hiebert & Lefevere's (1986) definition and description is used for the theoretical bases for mathematical knowledge construction that estimate the effectiveness of computer-based exploration that can provide a research evidence for recommended changes. The purpose of this study was to examine several aspects of pro-service teacher's knowledge construction in computer-based environments by investigating (1) procedural and conceptual knowledge on mathematics problems pre-service teachers hold, (2) how preservice teachers obtain procedural and conceptual knowledge of mathematics through computer-based explorations (3) how pre service teachers connect procedural and conceptual knowledge of mathematics during computer-based explorations. This study focused on the investigation of preservice teachers' knowledge on mathematics problems in computer-based explorations.

The research questions guiding the investigation were:

1. What procedural and conceptual knowledge of mathematics problems do pre-service teachers have?
2. What procedural and conceptual knowledge of mathematics problems are constructed through computer-based exploration?
3. What connections do pre-service teachers make between their procedural and conceptual knowledge of mathematics problems through computer-based exploration?
4. What conditions influence pre-service teachers' connection of procedural and conceptual knowledge of mathematics problems during computer-based exploration?

This research has theoretical framework, such as procedural and conceptual knowledge. Procedural knowledge is knowledge of the skills needed to carry out mathematical tasks and problems. There are two parts in procedural knowledge. One consists of knowledge of written symbols as representing some concepts. The other consists of the set of rules, formulas, and algorithms that are used to solve mathematics problems. A key feature of procedures is that they are executed in a predetermined linear sequence (Hiebert & Lefevre 1986). A significant feature of the procedural knowledge is that it is structured. Procedural knowledge can be constructed by rote learning.

Conceptual knowledge is defined as knowledge of facts, properties, and relations in mathematics. Conceptual knowledge can be thought of as a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information (Hiebert & Lefevre 1986, p. 3). Conceptual knowledge could not be generated by rote learning. Facts and propositions learned by rote are stored in memory as isolated bits of information, not linked with any conceptual network (Hiebert & Lefevre 1986, p. 8).

Linkage between procedural and conceptual knowledge can not only develop problem-solving ability and mathematical understanding but also promote many advantages on each other. According to Hiebert & Legevre (1986) procedural knowledge that is informed by conceptual knowledge results in symbols that have meaning and procedures that can be remembered that procedural knowledge provides a formal language and action sequences that raise the level and applicability of conceptual knowledge. There are several reasons to believe that connecting procedures with their conceptual underpinnings is the key in producing procedures that are stored and retrieved more successfully. If procedures are linked with conceptual knowledge, they become stored as part of a network of information, glued together is less likely to deteriorate than an isolated piece of information, because memory is especially good for relationships that are meaningful and highly organized (Baddeley 1976; Anderson 1983).

If procedural and conceptual knowledge are connected each other, retrieval is enhanced because the knowledge structure, or network, of which the procedure is a part comes equipped with numerous links that enable access to the procedure. The conceptual links increase the chances that the procedure will be retrieved when needed, because they serve as alternate access route for recall. If conceptual knowledge is linked to procedures it can enhance problem representations and simplify procedural demands and promote transfer and reduce the number of procedures required (Hievert & Legevre 1986). Linkage between procedural and conceptual knowledge benefits not only procedural knowledge but also conceptual knowledge.

Some benefits for conceptual knowledge arise from the formal language system and syntax conventions, whereas other emerges from the use of rules and procedures. Symbols help to organize and operate on conceptual knowledge. The symbol system can also construct conceptual knowledge. Procedures are able to facilitate the application of conceptual knowledge because highly routinized procedures can reduce the mental effort required in solving a problem and thereby make possible the solution of complex tasks (Hiebert & Leferve 1986).

According to Bruner (1973) and Tulbing (1986), there are three main factors that inhibit the creation and recognition of relationships between procedural and conceptual knowledge. First, the failure to establish relationships between procedural and conceptual knowledge comes from deficits in the knowledge base. Deficiencies in procedures or concepts, although sometimes hidden, can be the source of weak or missing connections. The second, the failure of encoding and constructing relationships between units of information is that due to a tendency to overlook to encode relationships. The third factor to impede the construction of relationships between units of knowledge is that knowledge just acquired open is context bound.

When students could not appropriately connect between procedural and conceptual knowledge, they may have some understanding of the mathematical concept but not solve the problem, or they may be able to perform some tasks but could not understand what they are doing. The connection between the procedural and conceptual knowledge is very important for students to solve mathematics problems.

METHODOLOGY

Three cases were studied to investigate a grounded theory of the knowledge construction of preservice teachers through computer-based explorations. An observation of classroom activity method with clinical interview (Ginsburg 1981; Opper 1977) was used to gain insight into the subject knowledge of mathematics and knowledge construction through computer-based explorations. Researcher introspection was used to identify the sequence of mental processes as the subjects performed computer-based explorations. Typological strategy (LeCompte & Preissle 1993) was used to analyze the data. The analysis of data consisted of category-based grouping for investigating how subjects constructed their mathematical knowledge. The category-based grouping was identified on the bases of Hiebert and Lefever's (1986) definition and description on procedural and conceptual knowledge.

A classroom in mathematics education department at the Dongguk University was selected for this study. The class was designed for mathematical investigation using computers for preservice teachers. The class emphasized computer exploration of various computer software applications. The class was scheduled for two hours from 1:00 pm to 3:00 pm. The classroom has 20 computers that connected internet, 1 scanner, 1 laser printer, and an overhead projector with a screen. There were 19 students (7 males and 12 females) in the classroom. The lesson was mainly focused on how the students used software to introduce mathematical concepts, and to explore mathematical problems by using computer software-Algebra Xpresser, Geometer's Sketchpad (GSP), Excel, and Math-view.

Three preservice teachers, Kim, Lee and Jung, participated in the study by random-pick-up method with volunteer basis. However, Lee, one of the participants, withdrew his willingness to involve this study during the first exploration due to his personal reasons. The data of this study mainly based on the two subjects who completed all three explorations. All subjects took EGC405 course that was a prerequisite for this course at the Dongguk University. All subjects had more than average background in mathematics, and they showed positive attitude with enthusiasm toward this study.

This study observed three explorations among hand-in-write-ups and final projects for each subject. All participants' classroom activities and computer explorations were observed and video taped. If it was possible, an open-ended individual interview was performed before, during, and after each exploration. Open-ended clinical interviews along with thinking about procedures were used to observe and collect data on participants' knowledge process during the computer-based explorations. Finally an overall interview was held at the end of the explorations.

The method of data collection was observation, interviewing, field notes, students' written assignments, computer works, and audio and video records of students' computer explorations. The main sources of data were observation, interviewing and subjects' written assignments. If it was not enough to collect information of subjects' mental process by just observation and interview, this study used subjects' written assignments and written projects for data collection.

Throughout the period of the research, data collection and analysis were processed together. Typological strategy (Lecompte & Preissle 1993) was used for data analysis. Hiebert & Lefevre's (1986) definition and description of procedural and conceptual knowledge was used to develop guidelines to analyze the data. For the supplement of the guidelines, enumerative coding categories were developed by analytic induction and constant comparison (Lecompte & Preissle 1993).

SUMMARY OF FINDINGS

Kim's case study

At the beginning of the first exploration, Kim already had a previous procedural and conceptual knowledge of the graph $X^2+Y^2=1$. But he did not have procedural and conceptual knowledge for the graph of $X^n+Y^n=1$ for every *n*.

Through a step-by-step method by carrying out linear sequences as n increases. And visual information from computer screens. He becomes aware of exterior features of the graphs $X^n+Y^n=1$ that representing some concepts and found a pattern to predict a general case for *n*. But still he did not show enough evidence of exhibition of conceptual knowledge.

After exploring odd *n* and even *n* number separately, he could generalize his process and extended his generalization. Eventually during the computer exploration, he showed procedural and conceptual knowledge and a connection of two types of knowledge. When procedural and conceptual knowledge were constructed, the two types of knowledge were linked together. Because of his lack of previous knowledge of Algebra, he could not fully connect his exploration results to new mathematical knowledge. But the visual information from the computer exploration was very effective on his knowledge connection.

Restricted time in the computer explorations was a critical factor to constructing procedure and conceptual knowledge and their connection in computer exploration for mathematics. Computer exploration for mathematics could give him a wide view of mathematical knowledge and could help him discover open-ended mathematical fields. All three factors positively influenced on the linkage of procedural and conceptual knowledge. At the beginning of the second exploration Kim did not have any experience on making a model by using data, and he did not show any procedural knowledge for the question. Group discussion obviously helped his procedural knowledge construction.

During exploring parameter A and B he tried to connect his previous knowledge on parameter A with parameter B, but he still could not grasp the whole concept of the problem. Even though he somewhat had procedural and conceptual knowledge, he could not strongly connect the two types of knowledge. After finishing the exploration of parameter D, he connected knowledge of each parameter and applied the knowledge to make a model function. He achieves the construction of relationships between pieces of information and he could exploration. His knowledge construction developed from procedural knowledge to conceptual knowledge and they connected two types of knowledge.

His computer exploration provided plenty of sub-information about the role of parameters in the graphing the function. The sub-information affected positively on the linked of procedural and conceptual knowledge. He constructed a kind of procedural and conceptual knowledge and linked the two types of knowledge. But he could not extend or create new solution. His restricted solution can be explained by his lack of previous knowledge on functions, researcher's quick-hints at the beginning of the computer exploration.

At the beginning of third exploration, he did not show any knowledge of the graph of the equation $xy = ax + by + c$. He just had basic procedural knowledge without concept-tual knowledge on the problem. He explored the equation $xy = ax + by + c$ with various substitution of coefficients a, b, and c with real numbers. His conception on this mathematical syntax such as, intercept, led to a correct answer resulting from a strong conceptual knowledge construction. Even though he did not try to verify the asymptotes by algebraic proof, he could find the properties of asymptotes only by the visual verification from the computer exploration. His previous knowledge on basic mathematical concepts verified by visual information induced strong procedural and conceptual knowledge construction.

In the middle of the exploration of the graph of $xy = 3x + 3y + c$ for $c = 0, 1, 2$ he could not explain the intercepts $0, -1/3$, and -2.3 for $c = 0, 1$ and 2 respectively. During the exploration he had procedural and conceptual knowledge. He could not fully link them together. After exploring $xy = 3x + 2y + 0$ for $c = 0, 1, 2, 3$ he can confirmed his discovery by using algebraic calculation. His analysis and conclusion on the asymptotes came from algebraic proof and visual information through computer exploration. Multiple approaches to and issue could help to create, extend, and generalize the result. These multiple approaches helped his mathematical understanding on the asymptotes by linking procedural and conceptual knowledge through computer exploration. For the exploration of $X^2Y^2 = ax^2 + by^2 + c$ for $c = 0, 1, 2$ he remarkably generalized the properties of the asymptotes such as if $a = 5$ and $b = 7$ the asymptotes would occur at x = square root of 7 and y = square root of 5. He linked procedural and conceptual knowledge for the asymptotes. But he could not explore various characteristic of the graph such as the differences of negative and positive value of the parameter, the role of constant, etc. He could not find any specific relationship between the intercepts and the values of a, b and c. He reported on the occurrence of the intercepts for the case of $c < 0$.

He used propositions for shifting curves that were stored in his memory. He had strong conceptual knowledge of shifting functions. He finished his exploration with a satisfactory result, and he could try to do further mathematical exploration on his problem mainly was performed by standard and routine processes. He exhibited procedural and conceptual knowledge, and he could link them appropriately.

Lee's case study

In the beginning of the first exploration, he showed that he had of the meaning of tangent. And he had weak procedural and conceptual knowledge on the problem. During computer exploration, his trial and error methods and group work helped his procedural and conceptual knowledge construction. After the first exploration Lee withdrew his willingness to participate in this study for personal reasons.

Jung's case study

When Jung explored exercise #5 of assignment #1, she already had basic procedural and conceptual knowledge of a simple sine function. But she did not have extended knowledge of a general sine function $y = a\sin(bx + c)$. After she explored the function with changing parameters a, b and c, she realized a affects amplitude. She also achieved the construction of relationships between different a values. She constructed procedural and conceptual knowledge by systematic way in which achieved a linkage between two types of knowledge on parameter a.

She successfully approached to find period of the sine function having changing the b parameter. But she could not find a pattern and failed to generalize a formula. She had knowledge of skills needed to carry out the exploration and she connected to a web of knowledge about the period. She could not, however, yet fully extend her solution. Somehow she linked between procedural and conceptual knowledge on parameter and period of sine function. At each exploration she predicted, explained, and represented the mathematical concepts for sine function in terms of c values. But she could not generalize shifting of sine function for all cases. Her knowledge construction was followed in the order of procedural, conceptual, and linkage of procedural and conceptual knowledge.

She was mainly dependent on visual information for measuring of a numerical position of a graph. Her lack of prerequisite knowledge on graphing was a factor for hindering a perfect knowledge construction. She already had procedural and conceptual knowledge for the shifting graphs. She just tested and verified her conjecture for shifting by using computer exploration. The process of verification by using computer exploration helped her build connection of her precious knowledge of shifting the general function to new knowledge of shifting sine function.

She did not notice b parameter influence shifting magnitude of the graph. So she could not construct strong linkage of procedural and conceptual knowledge on shifting. Various attempts with different variables were important to view the whole picture of the sine function. A narrow perception on the mathematical concept could prohibit to construct procedural and conceptual and their linkage. Without any confusion on the shifting concept, she strongly connected procedural and conceptual knowledge through the computer exploration.

For the second exploration she also, like Kim, understood what the problem means, but did not have strong procedural and conceptual knowledge on the problem. She used step-by-step and structured method to collect a set of graphs and compared the shape of the graphs to fine d pattern. Through the investigation process, she achieved a construction of relationships of how different parameters affect the shape of graph. Her knowledge construction obviously came from visual information through computer exploration.

She showed knowledge of properties and relations in graphing $xy = ax + by + c$ in terms of b that were related to other existing knowledge that she had in terms of a parameter. She achieved a construction of relationships between a and b parameters. She could compare a family of curves with appropriately changing parameters. These procedures could help her understanding of what features of the graph were controlled by parameters. She could extend and generalize how a and b parameters control the function $xy = ax + by + c$. She showed strong linkage between procedural and conceptual knowledge on how the graph was controlled by a, b parameters through visual information from the computer exploration.

She was aware of exterior features of a set of curves with two different values of a and

b respectively, and get a knowledge of meaning on the dependence of a and b parameter. By rote learning, it is not possible to generate this kind of knowledge. With carrying out linear sequences of the exploration for different parameters a, b and c she acquired meaningful relationships between each parameter and connected previously discovered knowledge to the new knowledge. Her knowledge construction came in the order of procedural, conceptual, and linkage of two types of knowledge. She showed strong linkage of procedural and conceptual knowledge acquired from the computer exploration.

For the third exploration, she already had background knowledge of the problems. Through computer exploration with Excel and with a joint exploration on the graphing calculator, she reinforced a construction of procedural and conceptual knowledge. She used graphing calculator to help her exploration. The use of graphing calculator obviously helped her construct knowledge that could not be generated by rote learning. It was a kind of triangulation approach to a problem. Even though she used non-standard processes, the processes had been varied and extended to fit this problem.

With the cubic equation $y = 4x^3 - 80x^2 + 375x$ already found, she easily carried out the exploration with Algebra Xpresser. Through the Algebra Xpresser she observed the maximum value to be somewhere around 500 inch^3 occurs when the length of the cut-out is 3.036 inch. In this exploration she mainly depended on visual information with using trace function. She showed maximum value propositions that were stored in her memory and connected the proposition to the knowledge which was discovered during the computer exploration. Her procedural knowledge was quite depended on the knowledge of how to use the computer software. But her mathematical conceptual knowledge was independent on the knowledge of computer software. Her conceptual knowledge on this problem relied on the visual information that was one of the special features that computer technology could provide.

To find a maximum volume of a box by using GSP, she used basic geometry construction that she already had and followed step-by-step method. She did not use a set of formula or algorithm but create an animation drawing to generalize her solution. This was non-routine processes but was as efficient as standard process. Through three different approaches she created extended, and generalized her solution. She showed positive attitudes to the problem, and felt comfortable to using computer technology for mathematical exploration.

All of these factors were very helpful for her construct strong procedural and conceptual knowledge and their linkage in computer exploration. If students had some previous procedural and conceptual knowledge in non-computer environments, it was very helpful for students construct strong procedural and conceptual knowledge and their linkage in computer exploration.

DISCUSSION

For a preservice teacher who has not enough graphing and trigonometry experience, computer technology can show how certain parameters control different aspects of the graphing, stretching, shrinking, shifting, and amplitudes. Much like the inspection Jung did, preservice teachers can explore the tendencies of the parameters. Through exploration, preservice teachers could be guided to the desired goal. By the computer exploration, preservice teachers will have a concrete foundation of the concept of parameters since they explored the aspects of them for themselves.

For preservice teachers who have been exposed to trigonometry, technology can serve several purposes including a review of existing concepts, and a facilitator of expansions of the previously learned concepts. Computer technology speeds up the routine of plotting points and allows the student to explore more complex situations using the basic concept.

Using computer to explore functions is a good opportunity for preservice teachers because it allows them to study the parts of behavior of a function and it's relationship with parameters. Also, they can easily explore how the composition of two linear functions or many is predicted. There are many different possible approaches in computer exploration that students can work in groups to see how many solutions that they can come up with. Group cooperation has plenty of advantage in computer-based exploration. At the end of class each group could explain what they have found and discuss what they have produced results. In explaining and listening to fellow classmates, students will learn more. By working with computer environments, students will get a better feel for what is happening when they factor a quadratic equation.

Like Kim's first exploration, a discussion to get students thinking of how to go about solving the problem could start with what types of combination of linear functions is necessary to get a parabola tangent two both at the same time. This would involve a discussion of slope since one function needs to have a positive slope and the other needs to have a negative slope. With this being the case then the discussion could lead to what direction the parabola would have. By using this type of discussion students would be able to make some connections between linear and parabola functions as well as what controls the slope and direction of the functions. Using Algebra Xpresser makes this type of exploration possible because the students would be able to experiment without being slowed down by having to graph all of the different functions. Also, once they found one that looked like a solution then they would need to prove it.

In computer-based exploration students learn mathematics by using concepts that aid them in understanding and directing the visual information. The fundamental basis for this claim is that appropriate computer-based environments can help students elaborate on, and become cognizant of, the mathematics implicit in certain kinds of intuitive thinking. When "intuition is translated into a program it becomes more obstructive and more accessible to reflection (Papert 1980, p. 145)." Verbal information supplemented by visual helps adults learn new concepts better than verbal information alone (Canth & Herron 1978; Dwyer 1982; Holliday 1975; Rigney & Lutz 1976). This research result also supports their arguments. Obviously computer-based exploration tremendously helped students connect procedural and conceptual knowledge that is an indication of mathematical understanding.

Krutetskii (1976) mentioned that a solution to a mathematical problem can be obtained in several ways, adding that one can not discover much about mathematical thinking by analyzing test results. Too mach emphasis in the classroom on the results instead of on the mathematical thinking that generated the results through computer-based explorations will give students a false conception of mathematics. This argument illustrates the importance of the mathematical thinking that can be generated by computer-based exploration. Computer based exploration can help students learn to think mathematically. The exploration focused on mathematical thinking can "develop a mathematical point of view valuing the processes of mathematization, abstraction which having the predilection to apply them. They can develop competence with the tools of the trade in which using those tools in the service of good of understanding structure-mathematical sense making (Schoenfeld 1992)."

In conclusion, computer-based exploration for mathematics is a powerful tool to assist preservice teachers' mathematical understanding. Thus, computer-based exploration for mathematics can enhance students' mathematical competence. In solving mathematical problems in which the basic facts, propositions, or relations of the different attribute functions, sequences, finding roots of a function become a part of students' conceptual knowledge only if these basic facts, propositions, and relations are related to some other existing knowledge need by the students. This meant that procedural knowledge proceeded with conceptual knowledge, or procedural knowledge overlapped with conceptual knowledge in some aspect.

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