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ROBUST UNIT ROOT TESTS FOR SEASONAL AUTOREGRESSIVE PROCESS†

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ABSTRACT

The stationarity is one of the most important properties of a time series. We propose robust sign tests for seasonal autoregressive processes to determine whether or not a time series is stationary. The proposed tests are robust to the outliers and the heteroscedastic errors, and they have an exact binomial null distribution regardless of the period of seasonality and types of median adjustments. A Monte-Carlo simulation shows that the sign test is locally more powerful than the tests based on ordinary least squares estimator (OLSE) for heavy-tailed and/or heteroscedastic error distributions.

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1. INTRODUCTION

We consider the problem of testing of the random walk hypothesis for seasonal time series. There have been several researches in the literature on this subject such as Dickey *et al.* (1984), Hylleberg *et al.* (1990), So and Shin (1999a) and So (2001). However most of these tests are based on the OLSE and have complicated non-standard null distributions depending on the type of mean adjustment and the period of seasonality. Furthermore the usual OLSE-based tests suffer from size distortion and power loss by outliers for heavy-tailed errors and the normality and/or the finite variance assumptions for the innovations may be easily violated in practice. Thus we need new tests for seasonal unit root which have simple null distribution and are robust to possible outliers from heavy tailed errors. As a simple robust alternative to OLSE-based test, Campbell and Dufour (1995) first

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proposed sign-based test for a random walk hypothesis in the simple no mean model. Then So and Shin (2001) extended the sign test to a mean model and established several important properties of the test such as the exact binomial null distribution, the consistency, the robustness to heavy-tailed errors, and the invariance. In this paper we extend the sign test to a seasonal model. The sign tests follow a binomial null distribution so it leads to an exact test. The asymptotic distribution of the test statistics is normal regardless of the type of mean adjustment and the period of seasonality, thus separate tabulations of critical values are not required.

This paper is organized as follows. In Section 2, we introduce the sign tests for seasonal AR (autoregressive) processes and investigate the key properties of the tests such as finite sample null distribution, consistency, and invariance. In Section 3, we conduct the simulation study which shows that the sign tests are locally more powerful than the OLSE-based tests for heavy-tailed and/or ARCH (Autoregressive Conditional Heteroscedasticity) model. All proofs are given in Appendix.

2. Test Statistics and Properties

Consider the seasonal AR(1) process with mean

$$\begin{cases} y_t = \mu_t + u_t, \\ u_t = \rho u_{t-d} + e_t, \quad t = 1, \ldots, n \end{cases} \tag{2.1}$$

where y_t is an observation at time t, ρ is the unknown parameter, $y_{-d+1}, y_{-d+2}, \ldots, y_0$ are the initial conditions, d is the period of seasonality, and $\mu_t = \mu_{t+d}, t = 1, \ldots, n-d$. A time series is a quarterly data when $d = 4$, and it is a monthly data when $d = 12$. We also suppose e_t of model (2.1) satisfies the following Assumption 2.1. Let $\mathcal{F}_t = \sigma(y_t, y_{t-1}, \ldots, y_{-d+1})$ be a σ-field generated by $(y_t, y_{t-1}, \ldots, y_{-d+1})$.

Assumption 2.1. $\{e_t\}$ is a sequence of errors such that $P(e_t > 0|\mathcal{F}_{t-1}) = 1/2$ and $P(e_t = 0|\mathcal{F}_{t-1}) = 0$.

We have $E(\text{sign}(e_t)|\mathcal{F}_{t-1}) = 0$ directly from Assumption 2.1 where $\text{sign}(e_t) = 1$ for $e_t \geq 0$ and $\text{sign}(e_t) = -1$ for $e_t < 0$. Assumption 2.1 is satisfied for independently and identically distributed (*iid*) errors following arbitrary continuous symmetric distributions. The time series (2.1) is stationary when $|\rho| < 1$, but is nonstationary when $|\rho| \geq 1$. Especially, y_t is a seasonal random walk when $\rho = 1$, and is of particular interest in economic and financial time series data. Thus we are interested in testing the random walk hypothesis that $H_0 : \rho = 1$ against the stationary alternative that $H_1 : |\rho| < 1$. In order to motivate the sign test, we first take the signs of each components of the Student's t-statistic for $H_0 : \rho = 1$ and propose the test statistic,

$$S_{d,n} = \sum_{t=1}^{n} \text{sign}(y_t - y_{t-d})\text{sign}(y_{t-d} - \widehat{\mu}_{t-d}), \tag{2.2}$$

where $\widehat{\mu}_t$ represents the estimate of the median of the t^{th} data point y_t. In estimating μ_t we consider two different types of median adjustments, namely common and seasonal median respectively. For the common median model with $\mu_t = \mu,\ t = 1, \ldots, n$, let $\widehat{\mu}_{c,t}$ be the common recursive median of y_t, namely $\widehat{\mu}_{c,t}$ is the median of $y_1, y_2, \ldots, y_t$. For the seasonal median model with $\mu_t = \mu_{t+d}, t = 1, \ldots, n-d$, let $\widehat{\mu}_{s,t}$ denote the seasonal recursive median of y_t, namely $\widehat{\mu}_{s,t}$ is the median of the observations in the same season until the t^{th} observation. So and Shin (2001) point out that the recursive mean or median adjustment improves the power of the unit root test. The reader is also referred to So and Shin (1999b) and Shin and So (2001) for more details of the merits of recursive mean adjustment. The statistical properties of the test statistics are summarized in the following Theorem 2.1. Here BIN(n, p) stands for the binomial distribution with n trials and success probability p.

THEOREM 2.1. *Consider model (2.1) with Assumption 2.1 and suppose $y_t - \widehat{\mu}_t$ has no atoms at zero. If $\rho = 1$, then we can represent the test statistic (2.2) as $S_{d,n} = \sum_{t=1}^{n} \text{sign}(e_t)\text{sign}(y_{t-d} - \widehat{\mu}_{t-d})$, and we have*

(a) $\{S_{d,t}, \mathcal{F}_t\}_{t=1}^{n}$ *is a martingale,*

(b) $(S_{d,n} + n)/2 \sim \text{BIN}(n, 0.5)$.

From Theorem 2.1, therefore, if $S_{d,n} \leq 2\,\text{BIN}_\alpha\,(n, 0.5) - n$, then we reject the seasonal random walk null hypothesis. Since we have $S_{d,n}/\sqrt{n} \xrightarrow{\mathcal{D}} N(0,1)$ due to the central limit theorem as $n \to \infty$, we reject the null hypothesis, when $S_{d,n}/\sqrt{n} \leq -z_\alpha$ where $-z_\alpha$ stands for a lower α^{th} quantile of the standard normal distribution. Now we examine some properties of the seasonal sign test statistic $S_{d,n}$: the invariance to monotone data transformation, and the consistency.

PROPOSITION 2.1. *Consider the seasonal AR model (2.1) with Assumption 2.1. Then the value of the seasonal sign test $S_{d,n}$ is invariant under any strictly monotone data transformations $y_t \to h(y_t)$, $t = 1, \ldots, n$.*

THEOREM 2.2. *Let $n = md$ and let the stationary processes $\{y_{i+td}\}_{t=0}^{m-1}$, $i = 1, \ldots, d$ be generated by the seasonal AR model (2.1) with $|\rho| < 1$ and the iid errors $\{e_{i+td}\}_{t=0}^{m-1}$, $i = 1, \ldots, d$ respectively. Let $F_t(\cdot)$ be the cumulative distribution function (cdf) of e_t such that $F_t(\cdot) = F_{t-d}(\cdot)$. If*

$$E\Big[\text{sign}(u_t)\big\{F_t\big((1-\rho)u_t\big) - F_t(0)\big\}\Big] > 0, \tag{2.3}$$

then the test is consistent as $n \to \infty$.

One simple sufficient condition for (2.3) is that $F(\cdot)$ is continuous and strict increasing function around zero. Specifically it means that $f_t(x_t) > 0$ for $x_t \in [-\epsilon, \epsilon]$ for some ϵ where $f_t(\cdot)$ is a probability density function of e_t. We note that above conditions for consistency is satisfied for a wide class of heavy-tailed error distribution with zero median when $|\rho| < 1$.

3. SIMULATION

In this chapter, we conduct a set of Monte Carlo experiments to investigate the finite sample performance of the seasonal sign tests, $S_{d,n}$, for testing the seasonal unit root null hypothesis against the stationary alternative.

We consider the seasonal AR(1) process with mean

$$\begin{cases} y_t = \mu_t + u_t, \\ u_t = \rho u_{t-d} + e_t, \quad t = 1, \ldots, n \end{cases} \tag{3.1}$$

where y_t is an observation at time t, and the initial values y_t, $t = -d+1, -d+2, \ldots, 0$, are set to zero. We use $\widehat{\mu}_{c,t}$ and $\widehat{\mu}_{s,t}$ to estimate μ_t, and $S_{d,n}^c$ and $S_{d,n}^s$ denote the corresponding test statistics.

In addition to the standard normal distribution we consider for e_t the double exponential distribution setting the location parameter to 0 and the scale parameter to 1, the Pareto distribution setting the scale parameter to 1 and the shape parameter to 1/2, and the standard Cauchy distribution to examine the effects of heavy-tailed errors on both tests. The double exponential distribution has a finite variance, but the standard Cauchy distribution has an infinite variance. The Pareto distribution has both infinite mean and variance. We also examine the performances of the sign test under ARCH(1) errors that in model (3.1) $e_t = (1 + 0.9e_{t-d}^2)^{1/2}\epsilon_t$ where $\epsilon_t \sim N(0, 1)$. We leave the detailed study of the condition for the consistency of the sign test for ARCH(1) errors for future work.

TABLE 3.1 *Empirical sizes(%) and size-adjusted powers(%) of the mean-adjusted tests for model $y_t = \rho y_{t-d} + e_t$ (The number of replications is 10,000)*

					Common		Seasonal	
d	n	*size*	e_t	ρ	DHF^c	$S^c_{d,n}$	DHF^s	$S^s_{d,n}$
4	120	4.12	$N(0,1)$	1.00	4.99	4.12	5.07	4.10
				0.99	5.70	5.73	5.09	5.54
				0.95	20.28	15.70	8.53	12.03
				0.90	53.39	30.77	16.23	20.88
			DE	1.00	5.16	4.07	6.01	4.18
				0.99	6.99	7.56	5.52	6.62
				0.95	22.59	30.86	8.01	20.03
				0.90	57.99	58.00	14.14	36.06
			Cauchy	1.00	3.06	4.14	15.02	4.08
				0.99	5.85	49.08	4.19	20.87
				0.95	22.43	94.46	0.73	44.03
				0.90	78.08	98.78	1.16	56.83
			Pareto	1.00	0.45	4.07	15.81	4.12
				0.99	10.65	94.90	4.26	59.42
				0.95	77.45	98.12	0.58	52.05
				0.90	94.36	95.95	1.22	47.71

NOTE : "size" *represents the exact size of* $S^c_{d,n}$ *and* $S^s_{d,n}$. DHF^c *and* DHF^s *denote common and seasonal mean adjusted tests, respectively. DE represents the double exponential distribution, Cauchy is the standard Cauchy distribution, and Pareto denotes the Pareto distribution with the zero median and the shape parameter 0.5.*

Because the OLSE-based test of Dickey *et al.* (1984; DHF hereafter) is most widely used in practice, we investigate its size and power and compare them with those of the seasonal sign test. We set the nominal level as 5% for every set of experiments, but sometimes the size of $S_{d,n}$ does not meet 5% because of the discreteness of binomial distribution. The exact size depends on the sample size n by Theorem 2.1. For example, when $n = 120$, the nominal level is 4.12%. We exhibit Table 3.1 for heavy-tailed disturbances and Table 3.2 for ARCH disturbances for the case of $d = 4$, and $n = 120$. The common characteristic of Table 3.1 and Table 3.2 is that in contrast to DHF the size of $S_{d,n}$ is quite close to its exact level for all error types and median adjustments. In Table 3.1 the powers of $S_{d,n}$ are locally higher than those of DHF, when ρ is close to 1. Therefore $S_{d,n}$ seems to be locally more powerful than DHF under the heavy-tailed disturbances. In the seasonal model with Cauchy and Pareto distributions, the powers of the

DHF test show unusual pattern because of extreme values. Table 3.2 displays the simulation result under the ARCH errors, and the sizes of DHF are 9.84% and 9.66% for common and seasonal median, respectively. They are much distorted from their nominal level, 5%. However, the sizes of $S_{d,n}$ are 4.17% and 4.13%, and are close to their exact nominal level 4.12%. We also conducted the simulation study for other cases of $d = 2$, 4, and 12, and $n = 40$, 120, 180, and 240, and the general shapes of simulation results were similar to the previous case and thus omitted. Thus in contrast to the OLSE-based tests which suffer from severe size distortion and power loss for heteroscedastic and/or heavy-tailed errors, the sign test $S_{d,n}$ seems to be robust in the sense that it has not only stable size but also reasonable power in most cases.

TABLE 3.2 *Empirical sizes(%) and size-adjusted powers(%) of the mean-adjusted tests for seasonal AR(1) mean model with ARCH error (The number of replications is 10,000)*

					Common		*Seasonal*	
d	n	$size$	e_t	ρ	DHF^c	$S^c_{d,n}$	DHF^s	$S^s_{d,n}$
4	120	4.12	ARCH	1.00	9.84	4.17	9.66	4.13
				0.99	5.97	6.17	5.02	6.17
				0.95	15.45	18.43	7.81	13.09
				0.90	38.00	35.62	13.84	21.49

NOTE : Model $y_t = \rho y_{t-d} + e_t, e_t = (1 + 0.9e^2_{t-d})^{1/2}\epsilon_t$, and $\epsilon_t \sim N(0,1)$. See the note to Table 3.1.

4. CONCLUSION

The sign tests follow an exact binomial null distribution, regardless of the types of mean adjustment and the period of seasonality. Simulation results under the heavy-tailed and ARCH errors show that the seasonal sign tests are robust and locally more powerful than the standard OLSE-based test. Thus the sign tests seem to provide useful robust alternative to OLSE-based tests not only for random walk hypothesis, as shown in Campbell and Dufour (1995) and So and Shin (2001), but also for the seasonal AR models.

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APPENDIX

PROOF OF THEOREM 2.1. For convenience, let $v_t = y_t - \widehat{\mu}_t$.

(a) $$E(S_{d,n}|\mathcal{F}_{n-1}) = E\left(\sum_{t=1}^{n}\text{sign}(e_t)\text{sign}(v_{t-d})\middle|\mathcal{F}_{n-1}\right) = \text{sign}(v_{n-d})E\big(\text{sign}(e_n)|\mathcal{F}_{n-1}\big) + \sum_{t=1}^{n-1}\text{sign}(e_t)\text{sign}(v_{t-d}).$$

The first part of the right hand side is equal to zero because we have $E(\text{sign}(e_n)|\mathcal{F}_{n-1}) = 0$ from Assumption 2.1, and the second part is $S_{d,n-1}$ from (2.2).

(b) Let $s_{d,t} = \text{sign}(e_t)\text{sign}(v_{t-d}) = \text{sign}(e_t v_{t-d})$. From Assumption 2.1 and the assumption that $y_t - \widehat{\mu}_t$ has no atoms at zero, we have that $P(s_{d,t} = -1|\mathcal{F}_{t-1}) = P(s_{d,t} = 1|\mathcal{F}_{t-1}) = 1/2$ and $S_{d,n} = \sum_{t=1}^{n} s_{d,t}$. Let $M(\tau)$ be the moment generating function (MGF) of $S_{d,n}$, then

$$\begin{aligned} M(\tau) &= E\left(e^{\tau S_{d,n}}\right) = E\left(\prod_{t=1}^{n} e^{\tau s_{d,t}}\right) = E\left\{E\left(\prod_{t=1}^{n} e^{\tau s_{d,t}}\middle|\mathcal{F}_{n-1}\right)\right\} \\ &= E\left\{E\left(e^{\tau s_{d,n}}\prod_{t=1}^{n-1} e^{\tau s_{d,t}}\middle|\mathcal{F}_{n-1}\right)\right\} = E\left\{E\left(e^{\tau s_{d,n}}\middle|\mathcal{F}_{n-1}\right)\prod_{t=1}^{n-1} e^{\tau s_{d,t}}\right\} \\ &= \frac{1}{2}\left(e^{-\tau}+e^{\tau}\right)E\left(\prod_{t=1}^{n-1} e^{\tau s_{d,t}}\right) = \left\{\frac{1}{2}\left(e^{-\tau}+e^{\tau}\right)\right\}^2 E\left(\prod_{t=1}^{n-2} e^{\tau s_{d,t}}\right) \\ &= \cdots = \left\{\frac{1}{2}(e^{-\tau}+e^{\tau})\right\}^n. \end{aligned}$$

The MGF of $(S_{d,n}+n)/2$ is

$$e^{\frac{n}{2}\tau}M\left(\frac{\tau}{2}\right) = e^{\frac{\tau}{2}n}\left\{\frac{1}{2}(e^{-\frac{\tau}{2}}+e^{\frac{\tau}{2}})\right\}^n = \left\{\frac{1}{2}(e^{\tau}+1)\right\}^n.$$

This is also the MGF of the binomial distribution with number of trials n and the probability of success $1/2$. □

PROOF OF PROPOSITION 2.1. Let $\tilde{\mu}_t$ be the median of $h(y_1), \ldots, h(y_t)$. Since $\text{sign}(y_t - y_{t-d})\text{sign}(y_t - \widehat{\mu}_{t-d}) = \text{sign}(h(y_t) - h(y_{t-d}))\text{sign}(h(y_t) - \tilde{\mu}_{t-d})$ for all $t = 1, \ldots, n$ from the strict monotonicity of $h(\cdot)$, we get the result. □

PROOF OF THEOREM 2.2.

$$\begin{aligned}\frac{S_{d,n}}{n} &= \frac{1}{n}\sum_{t=1}^{n}\text{sign}(y_t - y_{t-d})\text{sign}(y_{t-d} - \widehat{\mu}_{t-d})\\ &= \frac{1}{n}\sum_{t=1}^{n}\text{sign}(e_t - (1-\rho)(y_{t-d} - \mu_{t-d}))\text{sign}(y_{t-d} - \widehat{\mu}_{t-d}).\end{aligned}$$

Let $g(y_{t-d}) = (1-\rho)(y_{t-d}-\mu_{t-d})$ and $S_{d,n}/n = A_n+B_n$ where $A_n = n^{-1}S_{d,n}-B_n$, and

$$B_n = n^{-1}\sum_{t=1}^{n} E\left\{\text{sign}(e_t - g(y_{t-d}))\text{sign}(y_{t-d} - \widehat{\mu}_{t-d})|F_{t-1}\right\}.$$

Since nA_n is martingale, we have $nE|A_n| < \infty$, and for any $\delta > 0$ as $n \to \infty$,

$$\begin{aligned}P\left[|A_n| \geq \delta\right] &= P\left[n|A_n| \geq n\,\delta\right]\\ &= P\left[|n\,A_n| \geq n\,\delta\right]\\ &\leq (n\delta)^{-1}E|nA_n| \longrightarrow 0.\end{aligned}$$

And we can represent $B_n = n^{-1}\sum_{t=1}^{n}\text{sign}(y_{t-d} - \widehat{\mu}_{t-d})\{1 - 2\,F_{t-d}(g(y_{t-d}))\}$, because

$$\begin{aligned}&E\left\{\text{sign}(e_t - g(y_{t-d}))|\mathcal{F}_{t-1}\right\}\\ &\quad= P\big\{\text{sign}(e_t - g(y_{t-d})) = 1|\mathcal{F}_{t-1}\big\} - P\big\{\text{sign}(e_t - g(y_{t-d})) = -1|\mathcal{F}_{t-1}\big\}\\ &\quad= P\big\{e_t \geq g(y_{t-d})|\mathcal{F}_{t-1}\big\} - P\big\{e_t < g(y_{t-d})|\mathcal{F}_{t-1}\big\}\\ &\quad= 1 - F_{t-d}(g(y_{t-d})) - F_{t-d}(g(y_{t-d})).\end{aligned}$$

If we let $B_n = C_n + D_n$ where $C_n = B_n - D_n$, and $D_n = n^{-1}\sum_{t=1}^{n}\text{sign}(y_{t-d} - \mu_{t-d})\{1 - 2\,F_{t-d}(g(y_{t-d}))\}$, then

$$\begin{aligned}|C_n| &= \left|\frac{1}{n}\sum_{t=1}^{n}\{1 - 2\,F_{t-d}(g(y_{t-d}))\}\{\text{sign}(y_{t-d} - \widehat{\mu}_{t-d}) - \text{sign}(y_{t-d} - \mu_{t-d})\}\right|\\ &\leq \left|\frac{1}{n}\sum_{t=1}^{n}\{\text{sign}(y_{t-d} - \widehat{\mu}_{t-d}) - \text{sign}(y_{t-d} - \mu_{t-d})\}\right|,\end{aligned}$$

since $-1 \leq 1 - 2\,F_{t-d}(g(y_{t-d})) \leq 1$. Therefore

$$E(|C_n|) \leq \frac{1}{n}\sum_{t=1}^{n} E\Big(\big|\text{sign}(y_{t-d} - \widehat{\mu}_{t-d}) - \text{sign}(y_{t-d} - \mu_{t-d})\big|\Big)$$

$$\leq \frac{2}{n}\sum_{t=1}^{n} P\big(|y_{t-d}-\mu_{t-d}| \leq |\widehat{\mu}_{t-d}-\mu_{t-d}|\big)$$

$$\leq \frac{2}{n}\sum_{t=1}^{n} P\big(|\widehat{\mu}_{t-d}-\mu_{t-d}| > \epsilon\big) + \frac{2}{n}\sum P\big(|y_{t-d}-\mu_{t-d}| \leq \epsilon\big) \longrightarrow 0$$

for any $\epsilon > 0$, since $\widehat{\mu}_t = \mu_t + o_p(1)$ and $y_{t-d} - \mu_{t-d}$ has no atoms at zero from the assumption. From this formula and the stationarity of y_t we have

$$\begin{aligned}\frac{S_{d,n}}{n} &= o_p(1) + o_p(1) + \frac{1}{n}\sum_{t=1}^{n} \operatorname{sign}(y_{t-d}-\mu_{t-d})\{1 - 2F_{t-d}(g(y_{t-d}))\} \\ &= E\big[\operatorname{sign}(y_{t-d}-\mu_{t-d})\{1 - 2F_{t-d}(g(y_{t-d}))\}\big] + o_p(1)\end{aligned}$$

by the weak law of large numbers. Let $\xi = E\left[\operatorname{sign}(y_t - \mu_t)\{1 - 2F_t(g(y_t))\}\right]$. If $\xi < 0$, then

$$P\left\{\frac{S_{d,n}+n}{2} \leq \mathrm{BIN}_\alpha(n, \frac{1}{2})\right\} = P\left\{\frac{S_{d,n}}{n} \leq \frac{2\,\mathrm{BIN}_\alpha(n, \frac{1}{2}) - n}{n}\right\}$$

$$= P\left\{\xi + o_p(1) \leq -\frac{z_\alpha}{\sqrt{n}} + O\Big(\frac{1}{\sqrt{n}}\Big)\right\} \longrightarrow 1 \text{ as } n \to \infty.$$

The condition $\xi < 0$ is equivalent to $E\left[\operatorname{sign}(u_t)\{F_t((1-\rho)u_t) - F_t(0)\}\right] > 0$, since $g(y_t) = (1-\rho)u_t$ and $F_t(0) = 0.5$. This completes the proof. □

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