Journal of the Optical Society of Korea, Vol. 8, No. 2, June 2004, pp. 65-71

Analysis of the Expressible Depth Range of Three-dimensional Integral Imaging System

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(Received May 12, 2004)

In this paper, we analyze the limitation on the expressible depth range that is one major problem of integral imaging. We provide the explanation that exactly predicts the experimental results and the way to evaluate the expressible depth range numerically. We also give the design method for the multi-central depth plane integral imaging system. Finally, we will show the experimental results for verification.

OCIS codes : 110.2990, 100.6890, 220.2740

I. INTRODUCTION

Integral imaging (II), that is also referred to as integral photography (IP), was first proposed by Lippmann in 1908 [1]. II has two steps, pickup and display process, to display three-dimensional (3D) images. In Fig. 1, a detailed description of these two steps is given. In the pickup process, each elemental lens of the lens array picks up two-dimensional (2D) scenes of the object from various directions. Then the pickup device such as a charge-coupled device (CCD) captures these images. A set of these captured 2D scenes is called the elemental image. In the display process, the elemental image is displayed by the display panel such as liquid crystal display (LCD), and the rays emitted from the elemental image retrace the original route to form the 3D integrated image.

II attracted a lot of researchers because of its various advantageous features such as full-parallax, full-color and real-time display of 3D image in a certain continuous viewing angle without any supplementary devices. In spite of such advantageous features, II has many drawbacks that make it difficult to apply II for practical 3D display systems [2-5]. The limitation on the expressible depth range of II is one of the major problems of II. Because of the fixed focal length of each elemental lens of the lens array, the quality of the integrated image is best at the central depth plane whose location is

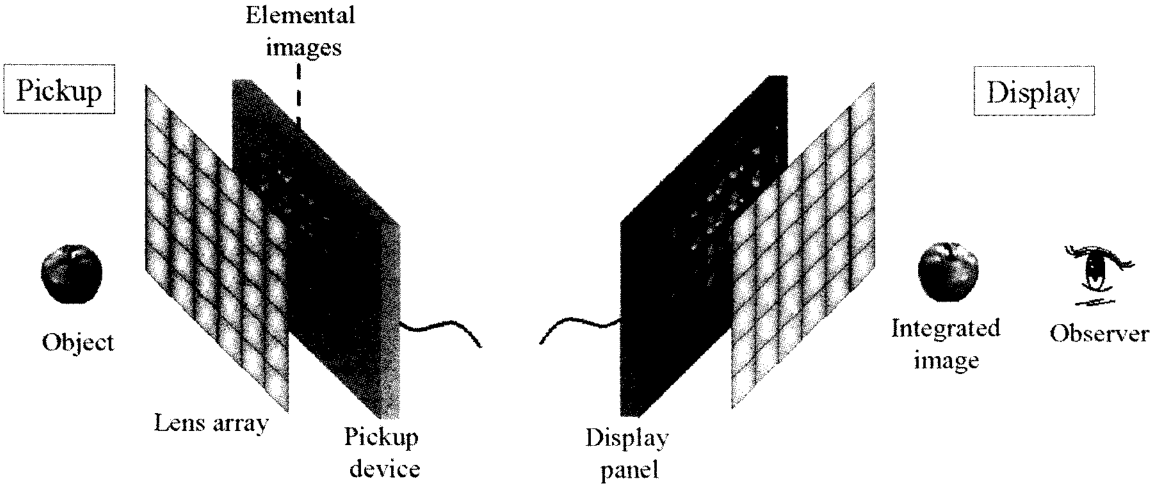


FIG. 1. The basic concept of integral imaging. Pickup and display processes.

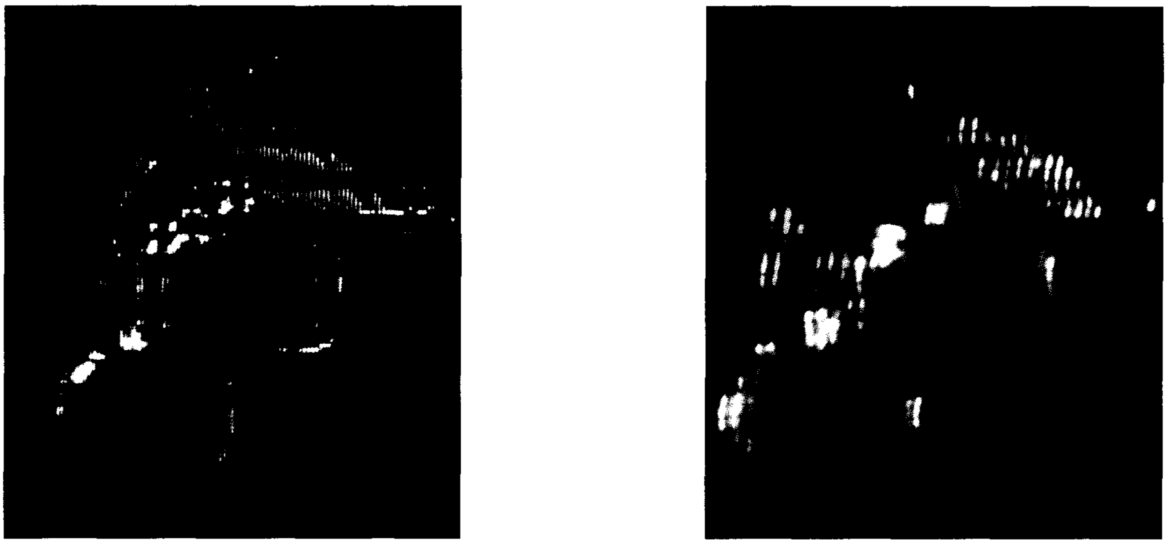
determined by the well-known lens law. And the quality degrades as the integrated image goes farther from the central depth plane. Therefore the expressible depth range maintaining good image quality is limited, and it is not large enough. A lot of research results analyzed this limitation based on the diffraction of the light wave, however the results do not match well with the experimental results [6-8].

This paper provides the explanation that predicts the experimental results exactly, and analyzes the limitation on the expressible depth range of II numerically. We will also discuss the application of this analysis for the design of parameters of multi-central depth plane II system.

II. MOTIVATION

The main objective of 3D display is to obtain one more degree of freedom in the displayed image than for the 2D display system. In other words, the display system should be capable of displaying the image with a certain thickness to be called a 3D display system. Therefore, the limited image depth of the displayed image has been recognized as a serious problem of the II 3D display system. Previous research results had focused on the diffraction of the light wave to explain the depth limitation of II [6-8]. They calculated the diffraction of light emitted from the elemental image at the integrated image plane that is different from the central depth plane. Obviously, the diffraction becomes larger as the integrated image plane goes farther from the central depth plane, and it was thought that this limits the expressible depth range of II. Simulation results based on the diffraction show the blurred integrated image located out of the central depth plane. That is a natural result considering the diffraction effect. However the leading effect that is shown in the experimental results of the integrated image located out of the central depth plane is not blurring. Rather, the integrated image is distorted as it goes farther from the central depth plane.

Figure 2 shows the experimental results that show the limited image depth of II. Two cherry images are displayed by an II 3D display system at different locations. The left is 90 mm distant from the lens array and the right is 180 mm distant from the lens array. In Fig. 2 (a), the central depth plane is located at the location of the left cherry. Then the right cherry is in front of the central depth plane. As shown in Fig. 2 (a), the parts of the left cherry are repeated and this is the main effect that leads the distortion of the integrated image. We named this effect the repeated image effect. In Fig. 2 (b), the central depth plane is located at the location of the right cherry, and the left cherry is in behind the central depth plane in this case. The parts of the right cherry are clipped as shown in Fig. 2 (b). We named this effect the clipped image effect. We introduced these terminologies in the conference [9]. As we can see from Fig. 2, the leading effect that limits the expressible depth range of II is not the diffraction of light, but the clipped or repeated image effect.



(a) (b)

FIG. 2. Integrated images located out of the central depth plane. The left cherry is 90mm distant from the lens array and the right cherry is 180mm distant from the lens array for both (a) and (b). (a) The central depth plane is set to the location of the left cherry and the repeated image effect occurs. (b) The central depth plane is set to the location of the right cherry and the clipped image effect occurs.

III. ANALYSIS OF DEPTH LIMITATION

1. Repeated and Clipped Image Effects

The analysis based on the diffraction assumes that the integrated image is shown at the integrated image plane. However, as described before, such analysis cannot give explanation that reflects the experimental results exactly. To explain the clipped and repeated image effects, we assume that what we really see is the elemental image focused at the central depth plane. In that assumption, the focused elemental image has directivity given by each elemental lens, so right and left eyes of the observer can see different images. Therefore, 3D perception is given by parallax in II 3D display system. In this case, the mismatch between the location of the plane where the observer's eye focuses and the integrated image plane leads to the distortion.

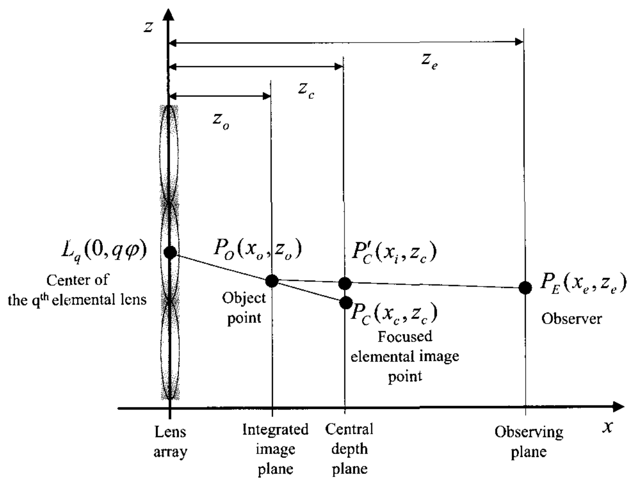


FIG. 3. Description of distortion of the integrated image located out of the central depth plane.

Figure 3 shows the situation. Pickup and display process for one point of the object P_O is described in Fig. 3. P_O is picked up through the center of the elemental lens L_q as the elemental image, and that elemental image is displayed through the center of the same elemental lens at the central depth plane instead of the integrated image plane. Therefore, the information of P_O is displayed at the point P_C. However, if the observer was located at the location of point P_E, we want to show the information of P_O in the direction of P'_C on the central depth plane. The location of P'_C is determined as the intersection of the central depth plane and the line $\overline{P_O P_E}$. The difference between P_C and P'_C can be thought as the amount of distortion. We will show detailed analysis on this difference in later sections. With the assumption that II 3D display system shows focused elemental images at the central depth plane, we can explain the clipped or repeated image effect.

In Fig. 4 (a), the situation where the integrated image plane is behind the central depth plane is described. In this case, a point on the object located at the integrated image plane is picked up through the center of each elemental lens, and displayed at the central depth plane. In the display process, each picked up elemental image point forms distinct ray tubes after passing through the lens array. Obviously, there exist spaces between ray tubes, and they are indicated as the hatched regions in Fig. 4 (a). If the observer was located inside such spaces just like the point P_O in Fig. 4 (a), the observer cannot observe the picked up information. Part of the entire object will not be shown to the observer like this, and this leads to the clipped image effect. In Fig. 4 (b), the integrated image plane is located in front of the central depth plane. In this case, ray tubes of each picked up elemental image point overlap. In Fig. 4 (b), such overlaps are indicated as

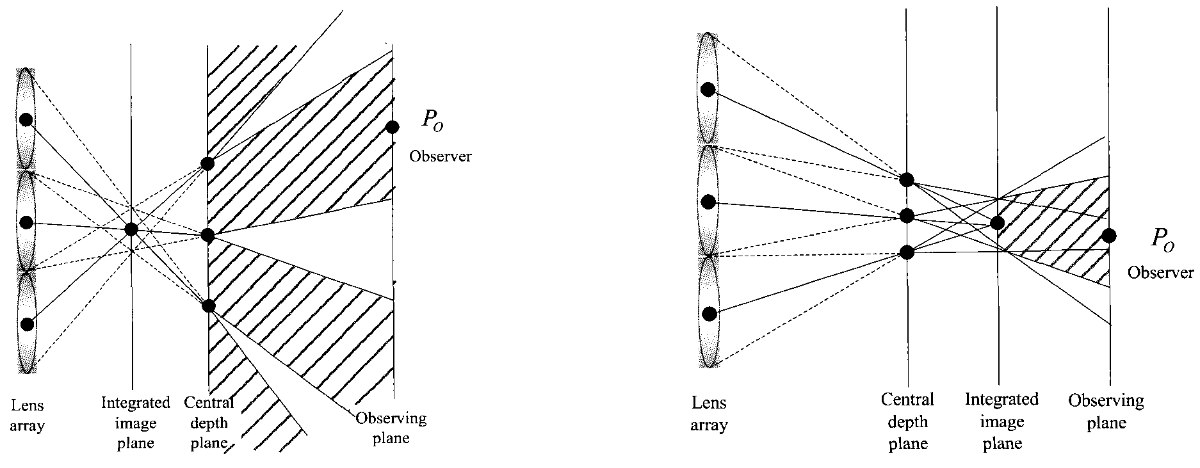


FIG. 4. (a) Clipped image effect and (b) repeated image effect.

hatched region. The observer that is located inside that overlaps like the point P_O in Fig. 4 (b) will observe the information of picked up point of the object from various locations on the central depth plane. Therefore, part of the entire object is shown to the observer repeatedly, and this is the reason of the repeated image effect. As a result, the clipped image effect occurs for the integrated image behind the central depth plane, and the repeated image effect occurs for the integrated image in front of the central depth plane.

2. Evaluation Method for the Distortion of the Integrated Image

For the numerical analysis of the amount of distortion, we will use the notations and coordinates that are shown in Fig. 3. Without any loss of generality, we assume one-dimensional lens array for the analysis. The one-dimensional lens array is laid on the x-axis and the center of entire lens array fits to the origin of the coordinate system. The center of the qth elemental lens is named as L_q, and its coordinate is (0, $q\varphi$) where φ is the pitch of each elemental lens. As described in the section before, to analyze the amount of the distortion of the integrated image located out of the central depth plane, we should calculate the difference between P_C and P'_C, $|x_c - x_i|$. At first, we will express x_i in terms of x_o. Considering the fact that P'_C is the intersection of the central depth plane and the line $\overline{P_O P_E}$, we can obtain the result as

$$x_i = \frac{x_e - x_o}{z_e - z_o}(z_c - z_e) + x_e \quad (1)$$

Then, if we obtain the relationship between x_o and x_c, x_i will be expressed in terms of x_c. P_O is the intersection of the integrated image plane and the line $\overline{L_q P_C}$. Therefore we can obtain the equation

$$x_o = \frac{z_o}{z_c}(x_c - q\varphi) + q\varphi, \quad (2)$$

where q is the index of the elemental lens. In (2), the value of q cannot be determined arbitrarily. The observer should be inside the ray tube that is emitted from the qth elemental lens and focused at the point P_C. This condition can be satisfied if the observer is located between two lines that connect P_C and two boundaries of the qth elemental lens, $\left(0, \left(q + \frac{1}{2}\right)\varphi\right)$ and $\left(0, \left(q - \frac{1}{2}\right)\varphi\right)$. This situation can be achieved if q satisfies the condition given as

$$\left[\frac{x_c - x_e}{z_c - z_e} - \frac{x_c - (q - 1/2)\varphi}{z_c}\right]\left[\frac{x_c - x_e}{z_c - z_e} - \frac{x_c - (q + 1/2)\varphi}{z_c}\right] < 0 \quad (3)$$

The notation for q that satisfies such condition is $q_0(x_c)$ where the fact that q_0 is the function of x_c is indicated. Then, (2) can be modified as

$$x_o = \frac{z_o}{z_c}(x_c - q_0\varphi) + q_0\varphi \quad (4)$$

The amount of distortion at the point x_c can be calculated as

$$\delta = (x_c - x_i)^2. \quad (5)$$

If we substitute (1) and (2) to (5) and arrange the symbols well, we can obtain

$$\delta = \left[x_c - \frac{z_c - z_e}{z_e - z_o}x_e + \frac{z_o(z_c - z_e)}{z_c(z_e - z_o)}(x_c - q_0\varphi) + \frac{q_0\varphi(z_c - z_e)}{z_e - z_o} - x_e\right]^2 \quad (6)$$

or

$$\delta = \left[\frac{(z_c - z_o)(z_c - z_e)}{z_c(z_e - z_o)}q_0\varphi + \frac{z_e(z_c - z_o)}{z_c(z_e - z_o)}x_c - \frac{z_c - z_o}{z_e - z_o}x_e\right]^2. \quad (7)$$

Therefore, the amount of distortion of the entire integrated image can be evaluated by integrating and normalizing this function on the central depth plane as

$$\sigma = \frac{\sqrt{\int_{x_o \in \Omega} \delta(x_c)dx_c}}{|\Omega_{\max} - \Omega_{\min}|}, \quad (8)$$

where the set $\Omega = (\Omega_{\min}, \Omega_{\max})$ is the range of the object on the integrated image plane. With this function, we can estimate the quality of the image located out of the central depth plane quantitatively.

3. Design optimization of the integral imaging system with multiple central depth planes

To overcome the restriction on the expressible depth range of II, some successful research results proposed various methods to increase the number of the central depth plane of an II system [8,10,11]. With this multi-central depth plane II, the expressible depth range of II can be enhanced by locating each central depth plane at a different location. However, these research results only provide the way to generate multiple central depth planes. The optimum design parameters to locate each central depth plane at the appropriate location are necessary to apply the multi-central depth plane II for practical use. In this section, we provide the way to design parameters of the multi-central depth plane II system based on the analysis in the section before.

For most cases, the observer is located sufficiently far away from the lens array compared with the central depth plane and the integrated image plane. So the conditions $z_e >> z_c$ and $z_e >> z_o$ are satisfied. In this case, (7) can be approximated as

$$\delta \cong \left[-\frac{z_c - z_o}{z_c} q_0 \varphi + \frac{z_c - z_o}{z_c} x_c \right]^2 . \qquad (9)$$

In addition, if the condition ze >> φ is also satisfied, the lines that connect P_E and two boundaries of the q_0^{th} elemental lens are approximately parallel to the x-axis, then q_0 also can be simplified as

$$q_0 \cong \left\lfloor \frac{x_c}{\varphi} + \frac{1}{2} \right\rfloor, \qquad (10)$$

where $\lfloor\cdot\rfloor$ means omission of fractions. In this situation, the function δ in the range $-\frac{\varphi}{2} < x_c < \frac{\varphi}{2}$ is repeated periodically with the period of φ. Therefore, σ for the entire integrated image can be represented by the value of σ in the range $-\frac{\varphi}{2} < x_c < \frac{\varphi}{2}$. In this range, q_0 equals to zero, and (9) becomes

$$\delta \cong \left(\frac{z_c - z_o}{z_c} \right)^2 x_c^2 . \qquad (11)$$

By substituting (11) into (8), the amount of distortion can be approximated as

$$\sigma \cong \frac{\sqrt{\int_{-\varphi/2}^{\varphi/2} \left[\left(\frac{z_c - z_o}{z_c} \right)^2 x_c^2 \right] dx_c}}{\varphi} = \frac{1}{2\sqrt{3}} \left| 1 - \frac{z_o}{z_c} \right| \sqrt{\varphi} . \qquad (12)$$

This equation means that the amount of distortion increases approximately linearly as the integrated image goes farther from the central depth plane. And the magnitude of the tangent of this function decreases as the value of z_c decreases and the value of φ increases. Therefore we can infer that the expressible depth range of II can be increased by using a smaller pitch of the elemental lens or by locating the central depth plane farther from the lens array. This result can be applied to the design of system parameters of the multi-central depth plane II. If we determine the tolerable level of σ as σ_M, we can define the expressible depth range of II as the range of z_o where σ does not exceed σ_M. By using (12), we can obtain the expressible depth range as

$$z_c - \frac{2\sigma_M z_c \sqrt{3}}{\sqrt{\varphi}} \le z_o \le z_c + \frac{2\sigma_M z_c \sqrt{3}}{\sqrt{\varphi}} . \qquad (13)$$

In the multi-central depth plane II, if the location of the first central depth plane is determined, we can cascade the expressible depth range of the second central depth plane in front of or behind the expressible depth range of the first central depth plane. For the case that the second expressible depth range is in front of the first expressible depth range, the second central depth plane can be calculated by using (12) and (13) as

$$z_c' = \frac{\sqrt{\varphi} + 2\sigma_M \sqrt{3}}{\sqrt{\varphi} - 2\sigma_M \sqrt{3}} z_c , \qquad (14)$$

where z_c' is the distance between the second central depth plane and the lens array. On the other hand, if the second expressible depth range is cascaded behind the first expressible depth range, the location of the second central depth plane is,

$$z_c' = \frac{\sqrt{\varphi} - 2\sigma_M \sqrt{3}}{\sqrt{\varphi} + 2\sigma_M \sqrt{3}} z_c . \qquad (15)$$

Then the enhanced expressible depth range of multi-central depth plane II can be calculated as

$$\frac{4\sqrt{3}\sigma_M}{\sqrt{\varphi}} (z_c + z_c') . \qquad (16)$$

By substituting (14) into (16), this enhanced expressible depth region can be obtained as

$$\frac{8\sqrt{3}\sigma_M}{\sqrt{\varphi} - 2\sigma_M \sqrt{3}} z_c \qquad (17)$$

for the case that the second expressible depth range is cascaded in front of the first expressible depth range. For the opposite case, the enhanced expressible depth range is obtained by substituting (15) into (16) as

$$\frac{8\sqrt{3}\sigma_M}{\sqrt{\varphi} + 2\sigma_M \sqrt{3}} z_c . \qquad (18)$$

Comparing (17) and (18), we can notice that the enhanced expressible depth range is larger for the case that the second expressible depth range is cascaded in front of the first expressible depth range. However, this result considers only the distortion of the integrated image to evaluate the quality of image. In fact, the resolution of the integrated image is also a significant factor that affects the quality of image. The pixel pitch of the display panel is magnified at the central depth plane as

$$p_c = \frac{z_c}{g} p_d , \qquad (19)$$

where p_d is the pixel pitch of the display panel, p_d is the pixel pitch at the central depth plane and g is the gap between the display panel and the lens array. Therefore the resolution of the integrated image degrades as the central depth plane goes farther from the lens array. Hence, for the case that the resolution of the integrated image is not important relatively, it is good to cascade the second expressible depth range in front of the first expressible depth range. However, if the resolution of the integrated image is important, the second expressible depth range should be cascaded behind the first expressible depth range to avoid severe degradation of the resolution of the image.

IV. SIMULATION AND EXPERIMENTAL RESULTS

To confirm the validity of the analysis above, we compared the expected integrated image that is obtained

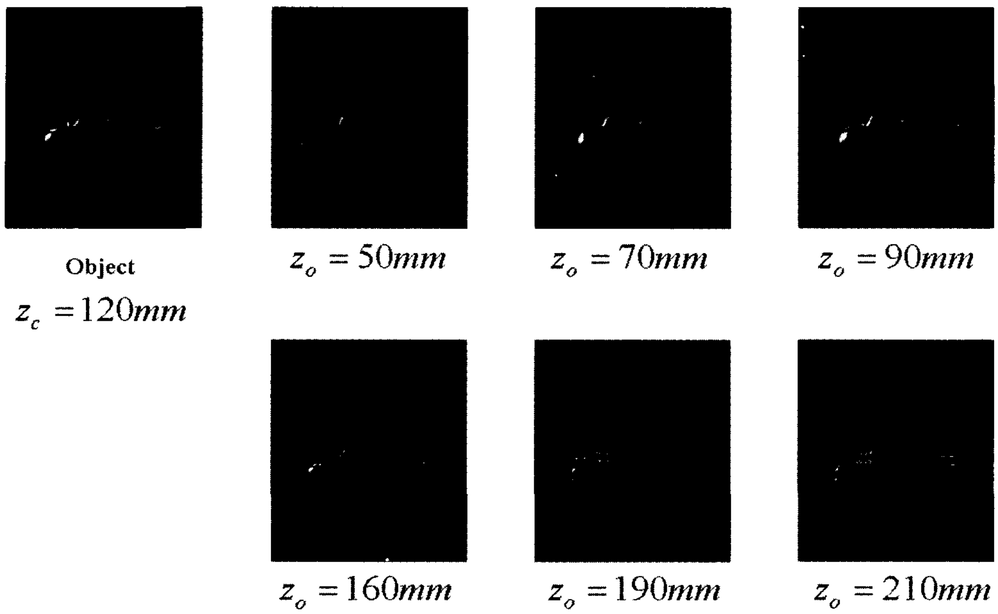


FIG. 5. Simulation results of integrated images with the central depth plane at the location of 120 mm in front of the lens array.

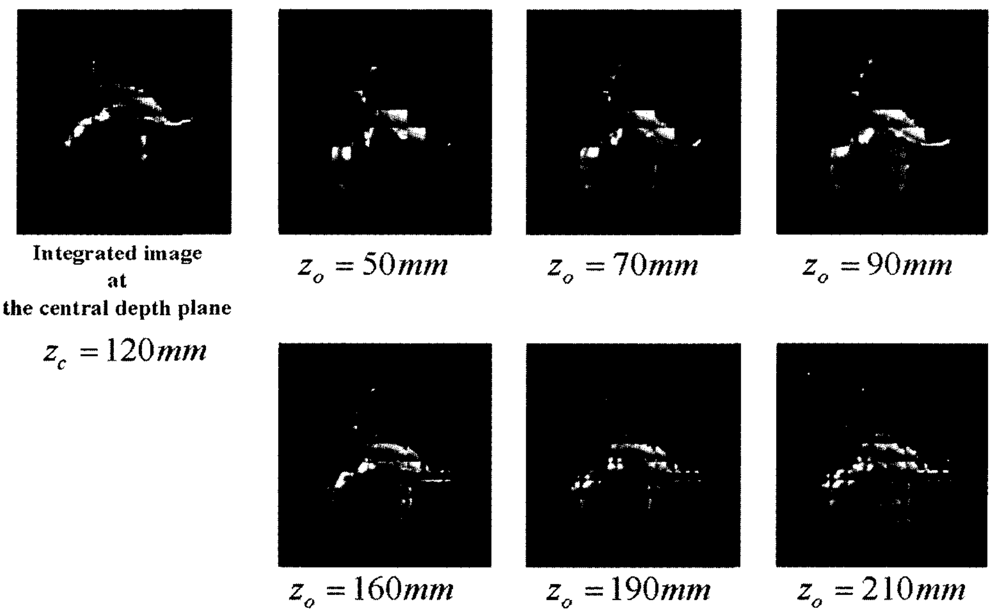


FIG. 6. Experimental results of integrated images with the central depth plane at the location of 120 mm in front of the lens array.

by a computer simulation based on the analysis above with the integrated image that is obtained by a real II 3D display system. The lens array is composed of 13 by 13 elemental lenses with the focal length of 22 mm and pitch of 10 mm. The central depth plane is located at 120 mm in front of the lens array. The locations of the image planes to be used in the experiment is 50 mm, 70 mm, 90 mm, 160 mm, 190 mm and 210 mm in front of the lens array. In the simulation, the analysis discussed above considering only one-dimensional lens array has been expanded to consider a two-dimensional lens array.

Figures 5 and 6 show the simulation results and experimental results, respectively. Slight mismatch in the alignment and the size of image of the experiment made little difference in the results. However the simulation results predict the experimental results almost exactly considering the errors in the experiment, therefore we can say that the proposed analysis provides the correct understanding of the depth limitation in II 3D display system.

V. CONCLUSION

In this paper, we proposed a new assumption on the way that II provides a 3D image to the observer. With this assumption, the repeated and clipped image effects have been explained. We also analyzed the limited image depth of II, and proposed a function that can evaluate the distortion of the integrated image. By using this result, we proposed the way to design the parameters of the multi-central depth plane II system. Then we verified the assumption experimentally.

ACKNOWLEDGEMENT

This work was supported by the Next-Generation Information Display R&D Center, one of the 21st Century Frontier R&D Programs funded by the Ministry of Science and Technology of Korea.

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