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Analysis of System Performance Degradation Using Sinusoidally Modulated Signal in Optical Fiber Communication Systems

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The response of a single-mode fiber to a sinusoidally modulated input has been studied to see its utility in measuring system performance in the presence of fiber nonlinearities. The sinusoidally modulated signal models an alternating bit sequence of ones and zeros in on-off keying. The sinusoidal response of normally dispersive fiber shows a strong correlation with eye-opening penalty (EOP) over a wide range of the nonlinearity parameter N ($0.1 < N^2 < 100$). This result implies that the measurement of the sinusoidal response can be an alternate way of measuring EOP without having a long sequence of randomly modulated input bits. But in the anomalous dispersion region, the sinusoidal response has a much more limited range of application to estimate system performance.

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I. INTRODUCTION

In a digital communication link, *bit error rate* (BER) is the most important parameter to measure the performance of the communication link between a transmitter and a receiver. In an optical fiber communication system, BER may often be measured only experimentally. This is because the high quality performance of a conventional optical fiber communication link (BER < 10^{-9}) requires an extremely large number of bits to evaluate BER, which makes numerical simulation of BER generally impractical. A simpler way of estimating performance is to observe eye-opening. The eye-opening is quantified by measuring the minimum value between the sampled values of marks (ones) and spaces (zeros) in the received bit sequence. The eye-opening is a useful system performance metric when signal distortion is a more limiting factor than noise, and *eye-opening penalty* (EOP) is often used to assess system performance degradation due to signal transmission through the fiber. Mathematically, EOP is defined as below [1].

$$\mathrm{EOP}[\mathrm{dB}] = -10\log\left(\frac{\text{eye - opening with fiber (after transmission)}}{\text{eye - opening without fiber (back - to - back)}}\right) \quad (1)$$

In this paper, the input optical signal is assumed to have a raised-cosine form which models an alternating bit sequence of ones and zeros. The sinusoidally modulated signal enables us to analyze the optical transmission impairments due to fiber nonlinearities, including the effects of intersymbol interference (ISI). Indeed, when ISI is predominantly caused by the neighboring pulses, it may be argued that the alternating pattern is the worst-case pattern. In the following section, the effect of fiber nonlinearities will be studied using a sinusoidally modulated signal. The sinusoidal analyses will be compared to more realistic cases by numerical simulations in which a *pseudo-random bit sequence* (PRBS) is used for the input bit sequence, and the range of its validity has been discussed.

II. SYSTEM PERFORMANCE DEGRADATION ANALYSIS USING SINUSOIDALLY MODULATED SIGNAL

1. Theoretical Background

The dispersion effect of the fiber is often expressed in terms of RMS pulse width (σ_t) which provides an estimate of the maximum allowable bit rate by a simple relationship, $\sigma_t R_b < \frac{1}{4}$ where σ_t is the RMS pulse width at the output of the fiber, and R_b is the bit rate [2]. The effect of dispersion on the system performance may also be approximated in the frequency domain by defining the transfer function of the fiber in power ($H_p(\omega)$). Under some conditions, the fiber can be modeled as a pseudo-linear system in power for digital communication purposes [3], in which case $H_p(\omega)$ is related to the optical input and output power by

$$P_{out}(\omega)=H_p(\omega)P_{in}(\omega) \quad (2)$$

The above relationship can be used in determining the bandwidth of the fiber. If we consider an optical input signal intensity-modulated by a constant amplitude sinusoidal wave, the amplitude of the output optical signal decreases as the modulation frequency increases, resulting in a low-pass system response because the dispersion becomes more significant as the modulation frequency (or bit rate) increases [4]. Therefore the effect of finite bandwidth on system performance is to limit the bit rate that can be transmitted over a given distance, and it is often quantified by the dispersion power penalty, which is the required input power increase to compensate for the decrease of output peak power (that is, decrease of signal to noise ratio) caused by dispersion. Similarly, the amplitude of the output optical signal will decrease at a given modulation frequency as the transmission distance increases because of fiber dispersion.

One way of estimating the dispersion power penalty at a given bit rate is to consider the alternating sequence, i.e., ...,1,0,1,0,1,0,..., because that sequence has the highest possible freqency component $\omega/2\pi = R_b/2$. The alternating sequence has also been considered in [5] to study how coding may be used to counter the effect of dispersion.

In practice, the situation is much more complicated when nonlinearities are present, but measuring the sinusoidal response at $\omega/2\pi = R_b/2$ may still give an indication of the system performance degradation. We wish to test this supposition, and to determine the extent to which sinusoidal response may be used to measure performance. The measurement of sinusoidal response may give a better system performance estimate than the measurement of the rms width of an isolated pulse. Also, it should be a much simpler test scheme compared to a BER measurement which requires a very long pseudo-random bit sequence.

In modern optical communication systems, fiber nonlinearities can not be simply ignored because the input peak power is increasing and the nonlinearities are accumulating with the advent of optical amplifiers. Therefore it is of interest to study how the fiber nonlinearities affect the sinusoidal response, and to see whether the sinusoidal response is still valid to measure the worst case ISI effect on the system performance when the fiber nonlinearities are not negligible. In the following, the sinusoidal response of dispersion alone case will be studied first, and the result will then be extended to include the effect of fiber nonlinearity.

2. Sinusoidal Response of the nonlinear Schrödinger equation (NLSE)

First consider the dispersion alone case with a periodic input signal, for which we can solve the NLSE analytically. The resulting analytical expression can be used to estimate the sinusoidal response in a dispersion dominant system, and it will also be useful to check numerical simulation results.

The normalized NLSE without nonlinear terms is given below [6].

$$\frac{\partial U}{\partial \xi} = -\frac{1}{2} j \cdot \mathrm{sgn}(\beta_2) \frac{\partial^2 U}{\partial \tau^2} \quad (3)$$

where $\mathrm{sgn}(\beta_2) = +1$ if $\beta_2 > 0$, $\mathrm{sgn}(\beta_2) = -1$ if $\beta_2 < 0$ (β_2 is the dispersion coefficient), $U(\xi, \tau)$ is the slowly varying envelope of the field normalized by the path averaged power, P_{avg}, and τ is the local time normalized by the bit period T_b such that $\tau = t/T_b$ since we are interested in a periodic input signal in this work. ξ is the normalized distance by the dispersion distance $L_D = \frac{T_b^2}{|\beta_2|}$.

If the input pulse sequence is periodic, we can express it in a Fourier series as below.

$$U(0,\tau) = \sum_{n=-\infty}^{\infty} C_n(0) e^{jn\omega_p \tau} \quad (4)$$

where ω_p is the fundamental angular frequency of $U(0, \tau)$.

Since the linear response of a periodic signal will also result in a periodic signal, the output signal can also be expressed in a Fourier series.

$$U(\xi,\tau) = \sum_{n=-\infty}^{\infty} C_n(\xi) e^{jn\omega_p \tau} \quad (5)$$

Now the Fourier series coefficients, $C_n(\xi)$, which are functions of transmission distance, can be derived by substituting Eq.(5) into Eq.(3), and the output may be written as

$$U(\xi,\tau) = \sum_{n=-\infty}^{\infty} C_n(0) \exp\left[\frac{j}{2} \mathrm{sgn}(\beta_2) \omega_p^2 n^2 \xi\right] e^{jn\omega_p \tau} \quad (6)$$

For example, if we model the alternating bit sequence as a raised-cosine wave with its period $T_p = 2 \times T_b$ (T_b = bit period), the normalized input signal can be written as below.

$$U(0,\tau)=\frac{1}{2}+\frac{1}{2}\cos\omega_p\tau=\frac{1}{2}+\frac{1}{4}f(\omega_p) \tag{7}$$

where $f(\omega_p)=e^{j\omega_p\tau}+e^{-j\omega_p\tau}$ and $\frac{\omega_p}{2\pi}=\frac{1}{2}$.

The Fourier series coefficients of the input signal are $C_0(0)=1/2$, $C_1(0)=C_{-1}(0)=1/4$, and $C_n(0)=0$ (for all n $\neq 0, \pm1$). The output signal by assuming $\beta_2>0$ is then

$$U(\xi,\tau)=\frac{1}{2}+\frac{1}{4}e^{\frac{j}{2}\omega_p^2\xi}f(\omega_p)=\frac{1}{2}+\frac{1}{2}e^{\frac{j}{2}\omega_p^2\xi}\cos\omega_p\tau \tag{8}$$

Therefore when the input field signal is given by Eq.(7), the input and output optical power signals are,

$$P(0,\tau)=|U(0,\tau)|^2=\frac{3}{8}+\frac{1}{8}\cos 2\omega_p\tau+\frac{1}{2}\cos\omega_p\tau \tag{9}$$

$$P(\xi,\tau)=|U(\xi,\tau)|^2=\frac{3}{8}+\frac{1}{8}\cos 2\omega_p\tau+\frac{1}{2}\cos\frac{\omega_p^2\xi}{2}\cos\omega_p\tau \tag{10}$$

From the output power signal, $P(\xi,\tau)$, we can see that the fundamental frequency component (ω_p), C_1, is periodic as a function of distance while the DC and the second harmonic components remain constant. Because of its periodicity with respect to the distance parameter, ξ, the magnitude of C_1 will have its first null at $\xi_o=z_o/L_D=\pi/\omega_p^2=1/\pi\cong 0.3183$, where the fundamental frequency component will die out completely. Since we may not get any further information from the magnitude of C_1 after the first null, the magnitude of C_1 should be measured before the first null occurs at $\xi_o=0.3183$ in the case of dispersion alone. In physical units, ξ_o corresponds to $0.3183\times L_D=0.3183\times T_b^2/|\beta_2|$. For example, in 10 Gb/s systems, the first null distance (z_o) will occur around 160 km for a typical dispersion coefficient of conventional single mode fiber, $|\beta_2|=20$ [ps^2/km]. At the same bit rate, the first null distance (z_o) is around 1060 km for a typical dispersion coefficient of dispersion-shifted fiber, $|\beta_2|=3$ [ps^2/km].

Figure 1 (a) shows the magnitude of the Fourier series coefficients of the optical power signal with dispersion alone at a few fixed distances, while Figure 1 (b) shows their evolution as a function of distance. Both figures are generated numerically by the split-step Fourier method [6] with the input optical field given by Eq.(7), and the results agree well with the analytical expression of Eq.(10).

Figure 2 (a) and (b) show the evolution of the Fourier series coefficients in the normal dispersion region ($\beta_2>0$) when fiber nonlinearity is non-negligible, specifically N = 2. N is the nonlinear parameter which is defined as $N^2=\frac{\gamma P_{avg}T_b^2}{|\beta_2|}$ (γ = fiber nonlinearity, P_{avg} = path averaged power). Unlike the dispersion alone case, it is observed that new frequency components, mainly at $\omega=3\omega_p$, are generated, which indeed shows that the fiber acts as a nonlinear system. Figures 1 (b) and 2 (b) show that the magnitude of the fundamental frequency component behaves like a low-pass filter as a function of propagation distance before the first null. Since the difference between the two figures depends on whether or not fiber nonlinearity is present, the curves may reveal how the nonlinearity affects the system performance.

Figure 3 ((a) normal dispersion, (b) anomalous dispersion) shows the evolution of the magnitude of the fundamental frequency component, $|C_1|$, at a few different N values. While $|C_1|$ decreases as N increases in the normal dispersion region at a given normalized distance, z/L_D, before the first null, the opposite occurs in the anomalous region. This is because the anomalous dispersion region supports solitons. The input pulse will

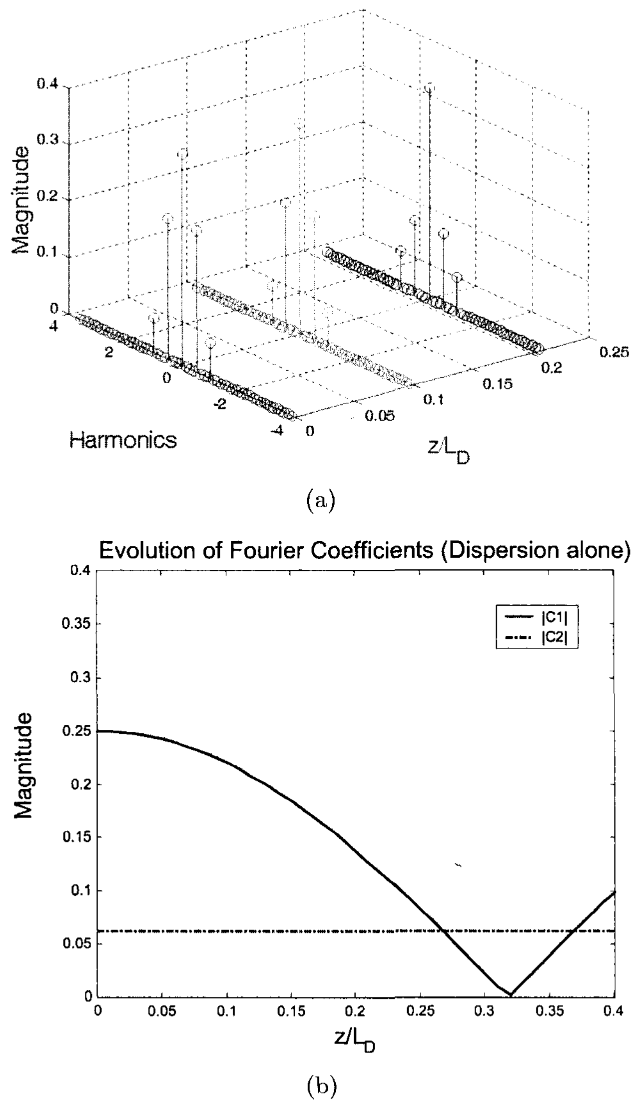
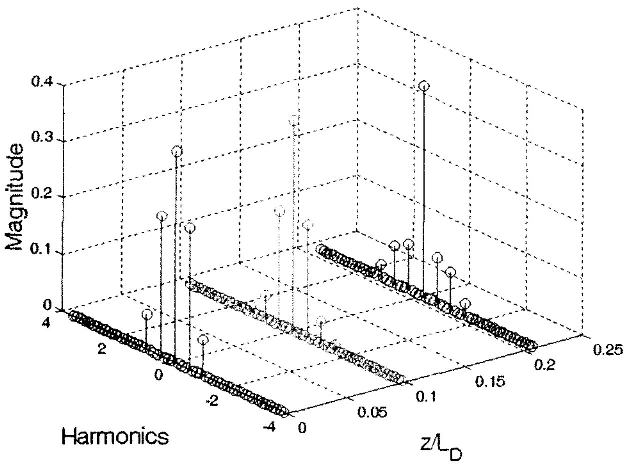
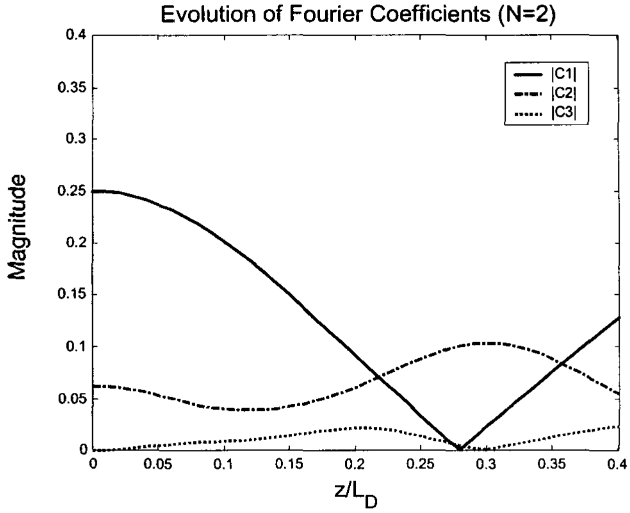


FIG. 1. Fourier series coefficients evolution with dispersion (normal) alone. (a) at three different distances (b) $|C_1|$ and $|C_2|$ as a function of transmission distance.



(a)



(b)

FIG. 2. Fourier series Coefficients evolution with nonlinearity (N=2). (a) at three different distances (b) $|C_1|$, $|C_2|$, and $|C_3|$ as a function of distance.

evolve into a fundamental soliton if (1/2) < N < (3/2), and a second-order soliton if (3/2) < N < (5/2), and so forth. Second and higher order solitons break up into spiked pulses and reassemble periodically while they propagate. Therefore, in the anomalous region, the sinusoidal test to see the worst case ISI effect may not be appropriate because other effects like modulation instability or optical amplifier noise can be more limiting factors on system performance [1,2]. However, when the N value is sufficiently small (N < 1/2) such that the input pulse does not evolve into a fundamental soliton, the sinusoidal method may still be useful to assess the worst case system performance even in the anomalous dispersion region. In the following section, the sinusoidal analysis will be compared with EOP to determine the extent to which these are correlated.

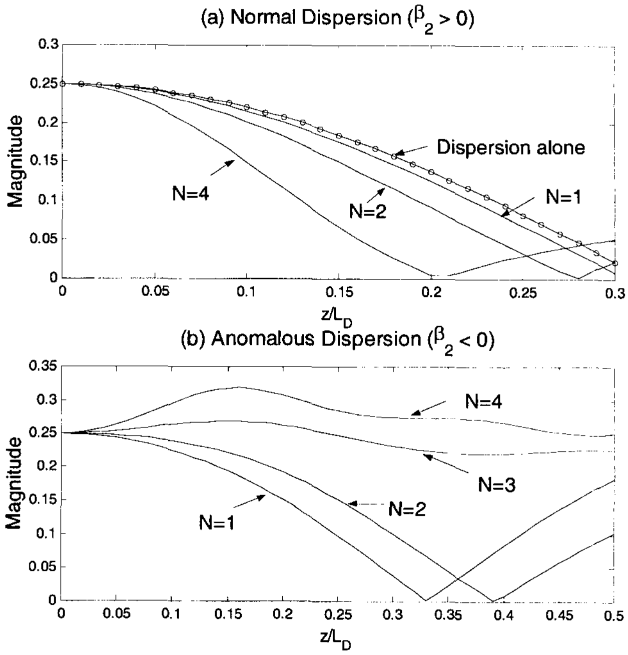


FIG. 3. Evolution of the fundamental Fourier series coefficient magnitude ($|C_1|$) as a function of transmission distance. (a) Normal dispersion region ($\beta_2 > 0$) (b) Anomalous dispersion region ($\beta_2 < 0$)

3. Eye-Opening Penalty and Sinusoidal Response

In the previous section, it is observed that the magnitude of the fundamental Fourier series coefficient, $|C_1(\xi)|$, may be a good indicator of system performance degradation even in the presence of fiber nonlinearity. In this section, $|C_1(\xi)|$ will be compared with a more general measure of system performance, namely the EOP.

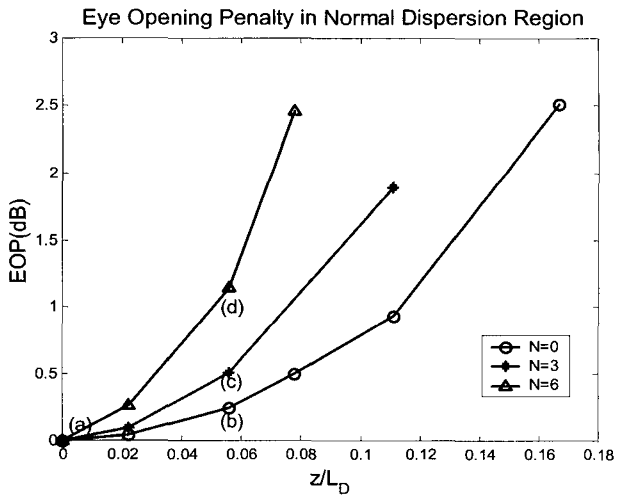


FIG. 4. Eye-opening penalties in the normal dispersion region. (Eye patterns, corresponding to conditions (a), (b), (c), (d), are contained in Figure 5.)

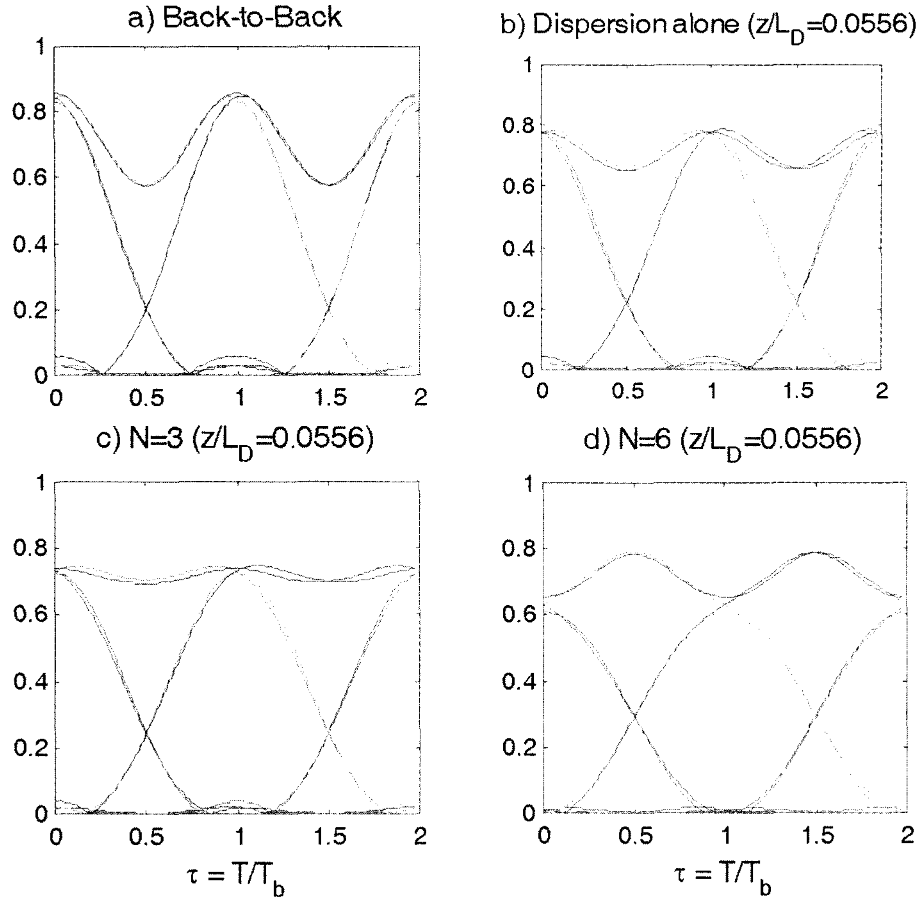


FIG. 5. Eye patterns in the normal dispersion region; (a) Back-to-back, (b) Dispersion alone at z/L_D=0.0556, (c) N=3 at z/L_D=0.0556, and (d) N=6 at z/L_D=0.0556

Figure 4 illustrates how EOP increases with transmission distance in the normal dispersion region. Figure 5 shows eye-diagrams of the received signals for the conditions indicated in Figure 4. In the calculation of EOP, a 32 bit pseudo-random sequence, $\sum_{k=1}^{N_b} b_k U(\tilde{\tau} - k\tilde{\tau}_b)$, is used as the input. N_b = 32 and b_k = the information bit sequence ('01011000101111011010100000101110'), and a Gaussian pulse shape $U(\tilde{\tau} = \frac{t}{t_o}) = \exp\left(-\frac{1}{2}\left(\frac{t}{t_o}\right)^2\right) = \exp\left(-\frac{1}{2}\tilde{\tau}^2\right)$ is assumed. The bit period in normalized unit, $\tilde{\tau}_b$, is taken to be 3, which corresponds to $T_b = 3\times t_o$ in physical units where t_o is the initial half width at half maximum of the Gaussian pulse.

To compare the EOP using PRBS with the sinusoidal response, *sinusoidal response penalty* (SRP) is defined as below.

$$\mathrm{SRP\,[dB]} = -10\log\left(\frac{|C_1(\xi)|}{|C_1(0)|}\right) \tag{11}$$

where $|C_1(\xi)|$ is the magnitude of the fundamental Fourier series coefficient of the received signal at ξ. For example, 1 dB penalty of SRP corresponds to $|C_1(\xi)|$ = 0.1986 since $|C_1(0)|$ = 0.25 in Eq.(7).

Figure 6 compares the critical transmission distances, $\xi_c = z_c/L_D$, when EOP and SRP reach 1 dB respectively as a function of N^2. In the normal dispersion region, the two curves agree very well over a wide range of N values. This result strongly indicates that the sinusoidal analysis can be used either experimentally or computationally as an alternate way of EOP measurement. Figure 6 also shows that the transmission distance for a 1 dB penalty remains almost constant when the N value is less than 1. This suggests that the fiber can be considered as a linear device as long as $N < 1$. However, when $N > 1$, the 1 dB penalty distance decreases as N increases. Physically, this can be interpreted as the maximum transmission distance for an allowable 1 dB penalty decreases as the signal power increases when other fiber parameter values are fixed. If we use $\gamma = 2.43\times10^{-3}$[1/(km × mW)], and $\beta_2 = 3$ [ps^2/km], and bit rate R_b = 10 Gb/s (T_b = 100 ps), N = 1 corresponds to a signal power 0.12 mW (path averaged). The 1 dB penalty distance with N = 1 is approximately 0.1 in normalized units, and it corresponds

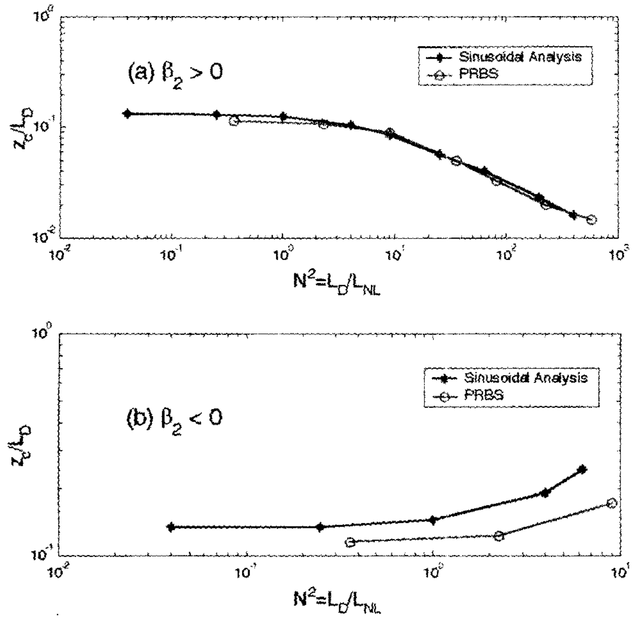


FIG. 6. 1 dB power penalty distances as a function of N^2; (a) in the normal dispersion region ($\beta_2 > 0$), (b) in the anomalous dispersion region ($\beta_2 < 0$)

to $z_c \approx 0.1L_D = 0.1T_b^2/|\beta_2| = 333$ km in physical units.

Figure 6(b) compares the 1 dB penalty distances of EOP and SRP in the anomalous dispersion region. Unlike the normal dispersion case, the 1 dB penalty distance increases as N increases. As we observed in Figure 3(b), the sinusoidal analysis may not be appropriate to assess system performance because $|C_1(\xi)|$ behaves irregularly and doesn't drop below its initial value when the N value is greater than 3.

III. CONCLUSIONS

In this paper, the transmission impairments due to fiber nonlinearities have been analyzed using a sinusoidally modulated signal which models an alternating bit sequence of ones and zeros in on-off keying. The sinusoidal response of nonlinear fiber shows a strong correlation with EOP in the normal dispersion region over a wide range of values of the normalized nonlinearity parameter N ($0.1 < N^2 < 100$). This result strongly indicates that the measurement of the sinusoidal response can be an alternate way of measuring EOP without having a long sequence of randomly modulated input bits. However, in the anomalous dispersion region where soliton formation is possible, the sinusoidal response has a much more limited range of application to estimate system performance.

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