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A GENERALIZATION OF THE HYERS-ULAM-RASSIAS STABILITY OF A FUNCTIONAL EQUATION OF DAVISON

Kil-Woung Jun*, Soon-Mo Jung and Yang-Hi Lee†

Abstract. We prove the Hyers-Ulam-Rassias stability of the Davison functional equation $f(xy) + f(x + y) = f(xy + x) + f(y)$ for a class of functions from a ring into a Banach space and we also investigate the Davison equation of Pexider type.

1. Introduction

In 1940, S. M. Ulam [17] raised the following problem for the stability of Cauchy equations: given a group F, a metric group E with a metric $d(\cdot,\cdot)$ and an $\epsilon > 0$, find $\delta > 0$ such that, if $f : F \to E$ satisfies $d(f(xy), f(x)f(y)) \le \delta$ for all $x, y \in F$, then there exists a homomorphism $g : F \to E$ such that $d(f(x), g(x)) \le \epsilon$ for all $x \in F$. In 1941, D. H. Hyers [6] affirmatively answered this question when F and E are Banach spaces. In 1978, Th. M. Rassias [14] generalized the result of Hyers. This Phenomenon is called the Hyers-Ulam-Rassias stability of Cauchy equations. This terminology is also applied to other functional equations. The result of Rassias [14] was further generalized by Rassias [15], Rassias and Semrl [16], Găvruta [4] and many authors (see [8, 13]). The case of the stability of Pexider equations was investigated by J. Chmieliński and Tabor [2]. For more details of the Hyers-Ulam-Rassias

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stability of all kinds of functional equations, we refer the reader to [7, 11, 12].

During the 17th ISFE, T. M. K. Davison [3] introduced the functional equation

$$f(xy) + f(x + y) = f(xy + x) + f(y) \tag{1.1}$$

and asked for the general solution if the domain and range of f are assumed to be (commutative) fields. W. Benz [1] proved that every continuous solution $f : \mathbb{R} \to \mathbb{R}$ (the set of reals) of (1.1) is of the form $f(x) = ax + b$, where a and b are arbitrary constants. Recently, R. Girgensohn and K. Lajkó [5] obtained the general solution of the Davison functional equation (1.1) without any regularity assumptions on f. They proved that every solution $f : \mathbb{R} \to \mathbb{R}$ of (1.1) has the form

$$f(x) = A(x) + b$$

where $A : \mathbb{R} \to \mathbb{R}$ is an additive function and $b \in R$ is an arbitrary constant. S. -M. Jung and P. K. Sahoo [9, 10] proved the Hyers-Ulam-Rassias stability of the Davison functional equation (1.1). In this paper, using the direct method, we obtain some generalization of the above theorems and investigate the functional equation (1.1) of Pexider Type.

2. Stability of Davison equation

Throughout this paper, we denote by F a ring with the unit element 1 and by E a Banach space. Let $\varphi : F \times F \to [0, \infty)$ be a function such that

$$\sum_{n=1}^{\infty} 2^{-n} \varphi(2^{n-1}x, 2^{n-1}y + z) < \infty$$

for all $x, y, z \in F$.

THEOREM 1. *If a function $f : F \to E$ satisfies the inequality*

$$\|f(xy) + f(x + y) - f(xy + x) - f(y)\| \leq \varphi(x, y) \tag{2.1}$$

for all $x, y \in F$, then there exists a unique additive function $A : F \to E$ such that

$$\|f(6x) - A(x) - f(0)\| \leq \sum_{n=0}^{\infty} \frac{M(2^n x)}{2^n}, \tag{2.2}$$

for all $x \in F$, where $M(x) = \frac{1}{2}[\varphi(4x, -4x) + \varphi(4x, -4x+1) + \varphi(8x, -2x) + \varphi(3x, 0) + \varphi(3x, 1) + \varphi(6x, 0) + \varphi(7x, -x) + \varphi(7x, -4x+1) + \varphi(14x, -2x)]$.

Proof. If we replace y by $y+1$ in (2.1), we have

$$\|f(xy+x)+f(x+y+1)-f(xy+2x)-f(y+1)\| \leq \varphi(x,y+1) \tag{2.3}$$

for any $x, y \in F$. Thus it follows from (2.1) and (2.3) that

$$\begin{aligned}
&\|f(xy)+f(x+y)+f(x+y+1)-f(y)-f(xy+2x)-f(y+1)\| \\
\leq\ &\|f(xy)+f(x+y)-f(xy+x)-f(y)\| \\
&\|f(xy+x)+f(x+y+1)-f(xy+2x)-f(y+1)\| \\
\leq\ &\varphi(x,y)+\varphi(x,y+1)
\end{aligned}$$

for all $x, y \in F$. Replacing y by 4y in the last inequality, we obtain

$$\begin{aligned}
&\|f(4xy)+f(x+4y)+f(x+4y+1)-f(4y) \\
&-f(4xy+2x)-f(4y+1)\| \\
\leq\ &\varphi(x,4y)+\varphi(x,4y+1)
\end{aligned}$$

for all $x, y \in F$. From (2.1) and the last relation we get

$$\begin{aligned}
&\|f(x+4y)+f(x+4y+1)-f(2x+2y)-f(4y) \\
&-f(4y+1)+f(2y)\| \\
\leq\ &\|f(4xy)+f(x+4y)+f(x+4y+1)-f(4y) \\
&-f(4xy+2x)-f(4y+1)\| \\
&+\|f(4xy)+f(2x+2y)-f(4xy+2x)-f(2y)\| \\
\leq\ &\varphi(x,4y)+\varphi(x,4y+1)+\varphi(2x,2y)
\end{aligned}$$

for all $x, y \in F$. If we replace x by $x-y$ in the above inequality, then we get

$$\begin{aligned}
&\|f(x+3y)+f(x+3y+1)-f(2x)-f(4y)-f(4y+1)+f(2y)\| \\
\leq\ &\varphi(x-y,4y)+\varphi(x-y,4y+1)+\varphi(2x-2y,2y)
\end{aligned}$$

for every $x, y \in F$. If we substitute $3x$ for x in the last inequality, we have

$$\begin{aligned}
&\|f(3x+3y)+f(3x+3y+1)-f(6x)-f(4y) \\
&-f(4y+1)+f(2y)\| \\
\leq\ &\varphi(3x-y,4y)+\varphi(3x-y,4y+1)+\varphi(6x-2y,2y)
\end{aligned} \tag{2.4}$$

for any $x, y \in F$. If we replace y by $-x$ in (2.4), we have

$$\begin{aligned}(2.5)\quad &\|f(0)+f(1)-f(6x)-f(-4x)-f(-4x+1)+f(-2x)\| \\ &\le \varphi(4x,-4x)+\varphi(4x,-4x+1)+\varphi(8x,-2x)\end{aligned}$$

for every $x \in F$. If we replace y by 0 in (2.4), we have

$$(2.6)\quad \|f(3x)+f(3x+1)-f(6x)-f(1)\| \le \varphi(3x,0)+\varphi(3x,1)+\varphi(6x,0)$$

for every $x \in F$. If we replace x, y by $2x, -x$, respectively in (2.4), we have

$$\begin{aligned}(2.7)\quad &\|f(3x)+f(3x+1)-f(12x)-f(-4x)-f(-4x+1)+f(-2x)\| \\ &\le \varphi(7x,-x)+\varphi(7x,-4x+1)+\varphi(14x,-2x)\end{aligned}$$

for every $x \in F$. From (2.5), (2.6) and (2.7),

$$\begin{aligned}(2.8)\quad &\left\|\frac{f(12x)-f(0)}{2}-(f(6x)-f(0))\right\| \\ &= \frac{1}{2}\|2f(6x)-f(12x)-f(0)\| \\ &\le \frac{1}{2}[\|f(0)+f(1)-f(6x)-f(-4x)-f(-4x+1)+f(-2x)\| \\ &\quad + \|f(3x)+f(3x+1)-f(6x)-f(1)\| \\ &\quad + \|f(3x)+f(3x+1)-f(12x)-f(-4x)-f(-4x+1)+f(-2x)\|] \\ &\le \frac{1}{2}[\varphi(4x,-4x)+\varphi(4x,-4x+1)+\varphi(8x,-2x)+\varphi(3x,0) \\ &\quad +\varphi(3x,1)+\varphi(6x,0)+\varphi(7x,-x)+\varphi(7x,-4x+1)+\varphi(14x,-2x)] \\ &= M(x)\end{aligned}$$

for every $x \in F$. If we substitute $2^{n-1}x$ for x in (2.8) and divide by 2^{n-1} the resulting inequality, then

$$\left\|\frac{f(2^n\cdot 6x)-f(0)}{2^n}-\frac{f(2^{n-1}\cdot 6x)-f(0)}{2^{n-1}}\right\| \le \frac{M(2^{n-1}x)}{2^{n-1}}$$

for all $n \in N$ and $x \in F$. Hence

$$\|f(6x) - f(0) - \frac{f(2^n \cdot 6x) - f(0)}{2^n}\| \leq \sum_{i=1}^{n} \frac{M(2^{i-1}x)}{2^{i-1}} \tag{2.9}$$

for all $n \in N$ and $x \in F$ and

$$\|\frac{f(2^n \cdot 6x) - f(0)}{2^n} - \frac{f(2^{m+n} \cdot 6x) - f(0)}{2^{m+n}}\| \leq \sum_{i=n+1}^{n+m} \frac{M(2^{i-1}x)}{2^{i-1}}$$

for all $m, n \in N$ and $x \in F$. The hypothesis just before this theorem implies that $\{\frac{f(2^n \cdot 6x) - f(0)}{2^n}\}$ is a Cauchy sequence for $x \in F$ and thus converges.

Therefore we can define $A : F \to E$ by

$$A(x) = \lim_{n \to \infty} \frac{f(2^n \cdot 6x) - f(0)}{2^n}$$

for $x \in F$. From (2.9), we obtain (2.2). If we replace x, y by $x + y, -x$, respectively in (2.4), we have

$$\begin{aligned} &\|f(3y) + f(3y+1) - f(6x+6y) - f(-4x) \\ &- f(-4x+1) + f(-2x)\| \\ &\leq \varphi(4x+3y, -4x) + \varphi(4x+3y, -4x+1) \\ &+ \varphi(8x+6y, -2x) \end{aligned} \tag{2.10}$$

for every $x, y \in F$. If we replace x, y by $x + y, -y$ in (2.4), we have

$$\begin{aligned} &\|f(3x) + f(3x+1) - f(6x+6y) - f(-4y) \\ &- f(-4y+1) + f(-2y)\| \\ &\leq \varphi(3x+4y, -4y) + \varphi(3x+4y, -4y+1) \\ &+ \varphi(6x+8y, -2y) \end{aligned} \tag{2.11}$$

for every $x, y \in F$. If we replace x, y by $2y, -y$, respectively in (2.4), we have

$$\begin{aligned} &\|f(3y) + f(3y+1) - f(12y) - f(-4y) \\ &- f(-4y+1) + f(-2y)\| \\ &\leq \varphi(7y, -4y) + \varphi(7y, -4y+1) + \varphi(14y, -2y) \end{aligned} \tag{2.12}$$

for every $y \in F$. If we replace x, y by $2x, -x$, respectively in (2.4), we have

$$\begin{aligned}&\|f(3x) + f(3x+1) - f(12x) - f(-4x)\\&- f(-4x+1) + f(-2x)\|\\&\le \varphi(7x, -4x) + \varphi(7x, -4x+1) + \varphi(14x, -2x)\end{aligned} \tag{2.13}$$

for every $x \in F$. From (2.10), (2.11), (2.12) and (2.13),

(2.14)

$$\begin{aligned}&\|2f(6x+6y) - f(12x) - f(12y)\|\\&\le \|f(3y) + f(3y+1) - f(6x+6y) - f(-4x) - f(-4x+1) + f(-2x)\|\\&\quad + \|f(3x) + f(3x+1) - f(6x+6y) - f(-4y)\\&\quad - f(-4y+1) + f(-2y)\|\\&\quad + \|f(3y) + f(3y+1) - f(12y) - f(-4y) - f(-4y+1) + f(-2y)\|\\&\quad + \|f(3x) + f(3x+1) - f(12x) - f(-4x) - f(-4x+1) + f(-2x)\|\\&\le \varphi(4x+3y, -4x) + \varphi(4x+3y, -4x+1) + \varphi(8x+6y, -2x)\\&\quad + \varphi(3x+4y, -4y) + \varphi(3x+4y, -4y+1) + \varphi(6x+8y, -2y)\\&\quad + \varphi(7y, -4y) + \varphi(7y, -4y+1) + \varphi(14y, -2y)\\&\quad + \varphi(7x, -4x) + \varphi(7x, -4x+1) + \varphi(14x, -2x) =: M'(x, y)\end{aligned}$$

for every $x, y \in F$.

Replacing x, y by $2^n x, 2^n y$ and dividing the resulting inequality by 2^n in (2.14), we have

$$\left\|\frac{f(2^n \cdot 6x + 2^n \cdot 6y)}{2^{n-1}} - \frac{f(2^n \cdot 12x)}{2^n} - \frac{f(2^n \cdot 12y)}{2^n}\right\| \le \frac{M'(2^n x, 2^n y)}{2^n}$$

for every $x, y \in F$. From the hypothesis just above this theorem and the definition of A, we obtain

$$2A(x+y) - A(2x) - A(2y) = 0$$

for every $x, y \in F$. Since $A(2x) = 2A(x)$, we obtain

$$A(x+y) = A(x) + A(y)$$

for every $x, y \in F$. It remains to show that A is uniquely determined. Let $B : F \to E$ be another additive mapping satisfying (2.2). Then

(2.15)
$$\begin{aligned}
&\|A(x) - B(x)\| \\
&\le \|\frac{f(6 \cdot 2^n x) - f(0)}{2^n} - A(x)\| + \|\frac{f(6 \cdot 2^n x) - f(0)}{2^n} - B(x)\| \\
&= \|\frac{f(6 \cdot 2^n x) - f(0) - A(2^n x)}{2^n}\| + \|\frac{f(6 \cdot 2^n x) - f(0) - B(2^n x)}{2^n}\| \\
&\le \frac{1}{2^n} \sum_{i=0}^{\infty} \frac{M(2^i x)}{2^{i-1}}
\end{aligned}$$

for every $x \in F$. Taking the limit (2.15) as $n \to \infty$ we obtain

$$A(x) = B(x) \quad \text{for all } x \in F.$$

This completes the proof of Theorem 1. □

In the following corollary, the Hyers-Ulam stability of the Davison functional equation is proven.

COROLLARY 1. *If a function $f : F \to E$ satisfies the inequality*

$$\|f(xy) + f(x + y) - f(xy + x) - f(y)\| \le \epsilon$$

for all $x, y \in F$, then there exists a unique additive function $A : F \to E$ such that

$$\|f(6x) - A(x) - f(0)\| \le 9\epsilon$$

for all $x \in F$.

The following corollary reminds us of the famous theorem of Rassias [13].

COROLLARY 2. *Let F be a normed algebra and let $0 \le p < 1$. If a function $f : F \to E$ satisfies the inequality*

$$\|f(xy) + f(x + y) - f(xy + x) - f(y)\| \le \theta(\|x\|^p + \|y\|^p)$$

for all $x, y \in F$, then there exists a unique additive function $A : F \to E$ such that

$$\begin{aligned}
\|f(6x) - A(x) - f(0)\| \le \frac{1}{2 - 2^p}\theta[(2 \cdot 2^p + 2 \cdot 3^p + 5 \cdot 4^p + 6^p \\
+ 2 \cdot 7^p + 8^p + 14^p + 1)\|x\|^p] + 3 \cdot \|1\|^p
\end{aligned}$$

for all $x \in F$.

3. Stability of Davison equation of Pexider type

Now we prove the stability of Davison functional equation of Pexider type.

Let $\varphi : F \times F \to [0, \infty)$ be a function and let $\psi : F \times F \to [0, \infty)$ be a function defined by

$$\psi(x, y) = \varphi(x, y) + \varphi(xy + x, 0) + \varphi(0, y) + \varphi(1, xy) + \varphi(xy + 1, 0) + \varphi(0, xy)$$

for all $x, y \in F$.

THEOREM 2. *If the functions $f, g, h, k : F \to E$ satisfy the inequality*

$$\|f(xy) + g(x + y) - h(xy + x) - k(y)\| \leq \varphi(x, y) \tag{2.16}$$

for all $x, y \in F$ and ψ is a function such that

$$\sum_{n=1}^{\infty} 2^{-n} \psi(2^{n-1}x, 2^{n-1}y + z) < \infty$$

for all $x, y, z \in F$, then there exists a unique additive function $A : F \to E$ such that

$$\begin{aligned} \|f(6x) - A(x) - f(0)\| \leq & \sum_{n=0}^{\infty} \frac{M(2^n x)}{2^n} + \varphi(1, 6x) \\ & + \varphi(6x + 1, 0) + \varphi(0, 6x) + \varphi(0, 0) \end{aligned} \tag{2.17}$$

$$\|g(6x) - A(x) - g(0)\| \leq \sum_{n=0}^{\infty} \frac{M(2^n x)}{2^n} \tag{2.18}$$

$$\|h(6x) - A(x) - h(0)\| \leq \sum_{n=0}^{\infty} \frac{M(2^n x)}{2^n} + \varphi(0, 0) + \varphi(6x, 0) \tag{2.19}$$

$$\|k(6x) - A(x) - k(0)\| \leq \sum_{n=0}^{\infty} \frac{M(2^n x)}{2^n} + \varphi(0, 0) + \varphi(0, 6x) \tag{2.20}$$

for all $x \in F$, *where*

$$\begin{aligned}M(x) =& \frac{1}{2}[4\varphi(0,0) + \varphi(0,1) + \varphi(0,-x) + \varphi(0,-2x) + \varphi(0,-2x)\\&+ \varphi(0,3x) + \varphi(0,-4x) + 2\varphi(0,-4x+1) + \varphi(0,-7x^2)\\&+ 2\varphi(0,-16x^2) + \varphi(0,-16x^2+4x) + \varphi(0,-28x^2)\\&+ \varphi(0,-28x^2+7x) + 4\varphi(1,0) + \varphi(1,3x) + \varphi(1,-7x^2)\\&+ 2\varphi(1,-16x^2) + \varphi(1,-16x^2+4x) + \varphi(1,-28x^2)\\&+ \varphi(1,-28x^2+7x) + 2\varphi(3x,0) + \varphi(3x,1) + \varphi(3x+1,0)\\&+ \varphi(4x,-4x) + \varphi(4x,-4x+1) + 3\varphi(6x,0) + \varphi(7x,-x)\\&+ \varphi(7x,-4x+1) + \varphi(8x,-2x) + \varphi(14x,-2x)\\&+ \varphi(-7x^2+1,0) + \varphi(-7x^2+7x,0) + 2\varphi(-16x^2+1,0)\\&+ \varphi(-16x^2+4x,0) + \varphi(-16x^2+4x+1,0)\\&+ 2\varphi(-16x^2+8x,0) + \varphi(-28x^2+1,0)\\&+ \varphi(-28x^2+7x+1,0) + 2\varphi(-28x^2+14x,0)].\end{aligned}$$

Proof. If the functions $f, g, h, k : F \to E$ satisfy (2.16), then by setting $y = 0, x = 0$ and $x = 1$ separately in (2.16), we obtain

$$\begin{aligned}&\|f(0) + g(x) - h(x) - k(0)\| \le \varphi(x,0),\\ &\|f(0) + g(y) - h(0) - k(y)\| \le \varphi(0,y) \text{ and}\\ &\|f(y) + g(y+1) - h(y+1) - k(y)\| \le \varphi(1,y).\end{aligned} \tag{2.21}$$

From (2.21), we have

$$\begin{aligned}&\|f(y) - g(y) + h(0) + k(0) - 2f(0)\|\\ \le& \|f(y) + g(y+1) - h(y+1) - k(y)\|\\ &+ \|-g(y+1) + h(y+1) + k(0) - f(0)\|\\ &+ \|-g(y) + k(y) + h(0) - f(0)\|\\ \le& \varphi(1,y) + \varphi(y+1,0) + \varphi(0,y).\end{aligned} \tag{2.22}$$

From (2.16), (2.21) and (2.22), we get

$$\begin{aligned}&\|g(xy) + g(x+y) - g(xy+x) - g(y)\|\\ \le& \|f(xy) + g(x+y) - h(xy+x) - k(y)\|\end{aligned}$$

$$
\begin{aligned}
&+ \| - f(0) - g(xy + x) + h(xy + x) + k(0)\| \\
&+ \| - f(0) - g(y) + h(0) + k(y)\| \\
&+ \| - f(xy) + g(xy) - h(0) - k(0) + 2f(0)\| \\
\le\ & \varphi(x, y) + \varphi(xy + x, 0) + \varphi(0, y) + \varphi(1, xy) \\
&+ \varphi(xy + 1, 0) + \varphi(0, xy).
\end{aligned}
$$

By Theorem 1, we easily obtain (2.18). From (2.18) and (2.21), we have

$$
\begin{aligned}
&\|h(6x) - h(0) - A(x)\| \le \|h(6x) - g(6x) - f(0) + k(0)\| \\
&+ \|g(6x) - A(x) - g(0)\| + \|f(0) + g(0) - h(0) - k(0)\| \\
&\le \sum_{n=0}^{\infty} \frac{M'(2^n x)}{2^n} + \varphi(0, 0) + \varphi(6x, 0).
\end{aligned}
$$

Now using (2.21) again, we also get the assertion for g, k and f. □

COROLLARY 3. *If a function $f : F \to E$ satisfies the inequality*

$$\|f(xy) + g(x + y) - h(xy + x) - k(y)\| \le \varepsilon$$

for all $x, y \in F$, then there exists a unique additive function $A : F \to E$ such that

$$\|f(6x) - A(x) - f(0)\| \le 57\varepsilon$$

for all $x \in F$.

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Kil-Woung Jun
Department of Mathematics
Chungnam National University
Taejon 305-764, Korea
E-mail: kwjun@math.chungnam.ac.kr

Soon-Mo Jung
Mathematics Section
College of Science and Technology
Hong-Ik University
Chochiwon 339-701, Korea
E-mail: smjung@wow.hingik.ac.kr

Yang-Hi Lee
Department of Mathematics Education
Kongju National University of Education
Kongju 314-060, Korea
E-mail: lyhmzi@kongjuw2.kongju-e.ac.kr