Analysis of Students' Use of Metaphor: The Case of a RME-Based Differential Equations Course[*]

Ju, Mi-Kyung

Department of Mathematics Education, Ewha Womans University, 11-1 Daehyun-dong,
Seodaemun-gu, Seoul 120-750, Korea; Email: mkju11@ewha.ac.kr

Kwon, Oh Nam

Department of Mathematics Education, Seoul National University. San 56-1, Sillim-dong,
Gwanak-gu, Seoul 151-742, Korea; Email: onkwon@snu.ac.kr

This research applies the discursive approach to investigate the social transformation of students' conceptual model of differential equations. The analysis focuses on the students' use of metaphor in class in order to find kinds of metaphor used, their characteristics, and a pattern in the use of metaphor. Based on the analysis, it is concluded that the students' conceptual model of differential equations gradually becomes transformed with respect to the historical and cultural structure of the communal practice of mathematics. The findings suggest that through participating in the daily practice of mathematics as a historical and cultural product, a learner becomes socially transformed to a certain kind of a cultural being with historicity. This implies that mathematics education is concerned with the formation of historical and cultural identity at a fundamental level.

1. INTRODUCTION

What does a student learn in mathematics class? How does a student change as s/he participates in mathematical practice in class? This paper aims to address these questions

by applying the social practice theory to analyze mathematics classroom discourse. Specifically, the discourse analysis focuses on students' use of conceptual metaphor in order to investigate their conceptual model of key concepts in differential equations. The purpose of the analysis is to describe not only the characteristics of students' conceptual model of differential equations but also the social transformation of the conceptual through course participation.

The notion of social transformation is the core of social practice theory. In social practice theory, knowledge such as mathematics is the historical and cultural structuring of social existence in a mathematics community. Learning is considered as a process of legitimate peripheral participation through which a learner becomes transformed with respect to the cultural organization of a community (Lave & Wenger 1991; Wenger 1998). Although the notion of mathematics as an abstract, disembodied, universal, value-free and transcendental truth has traditionally been the most dominant discourse in shaping teaching and learning mathematics at schools, the newly developing perspective on social practice has influenced theory and practice in mathematics education in a fundamental way.

Indeed, recently, the notions of community and practice become increasingly popular in theoretical discourse of mathematics education and become basic units for analysis of classroom interaction. In the perspective, cultural and social processes are integral to mathematical activity. Mathematics class is considered of as a practice community where participants negotiate their mathematical meanings and ways of doing mathematics to create its own mathematical culture through daily practice of mathematics (*cf.* Cobb & Bauersfeld 1995; Voigt, Seeger & Waschescio 1998).

In this context, this research applies the theoretical perspectives of social practice to the analysis of mathematical practice in classroom to investigate the social transformation of students. Social practice theorists argue that the cultural epistemological standpoint historically constructed by a mathematics community is mediated through mathematics. As a consequence of the mediation, a learner's epistemological perspective gets socially transformed with respect to the communal epistemological standpoint through participation in daily practice of mathematics (Lave & Wenger, 1991). Then how does social transformation happen? How is the process like? This research is to seek to answer to these questions by analyzing students' discursive practice in mathematics class.

2. METAPHOR AS A CULTURAL MODEL

Contemporary cognitive scientists have shown that mathematical cognition is fundamentally metaphorical (Lakoff & Nunez 2000). It has been shown that human

beings are born with innate arithmetic ability such as subsidizing. In addition, recent brain research has discovered that some mathematical abilities are associated with certain areas of the brain. It is conceptual metaphor through which the primitive innate mathematical abilities become elaborated into a higher level of mathematical abilities. Metaphor as cross-domain mapping bridges between the source domain of physical sensorimotor experience and the target domain of an abstract concept so as to enable people to reason about an abstract concept following the inferential structure of a source domain of tangible sensorimotor experience (Lakoff & Nunez, 2000).

Based on the theory of metaphor, Lakoff and Nunez (2000) argue that mathematics as we know it is brain-and-mind-based. That is, mathematics is a product of the function of the brain and of the body, and our experience in the world. Primitive innate mathematical abilities become elaborated into a more sophisticated kind of mathematical knowledge through experience. In the process, it is conceptual metaphor that mediates the experiential world to mathematical knowledge. On one hand, the existence of innate arithmetic ability explains the universality of mathematical concepts such as numbers. On the other hand, the universal innate arithmetic ability becomes transformed to a culturally specific kind of mathematics by conceptual metaphor through which projects the culturally specific inferential structure of lived-in experience to the domain of mathematics. In this regard, mathematical metaphor represents a cultural model of mathematical concepts.

Cultural models are defined as presupposed, taken-for-granted models of the world that are widely shared by the members of a community and that play an enormous role in their understanding of that world and their behavior in it (Holland & Quinn 1987). When considering that mathematics is a product of metaphor reflecting a specific cultural structure shared in a society, conceptual models of mathematics are cultural models. Cultural knowledge such as mathematics is organized into a culturally structured model that provides a culturally shared basis for seeing, reasoning, and in general, doing. Metaphor plays an important role in the construction of the cultural model since it allows for knowledge to be mapped from known domains of the culturally structured lived-in world onto conceptualizations in the social and psychological abstract domains. For instance, Ju (2001) found that while professional mathematicians creatively apply metaphor in mathematical arguments, the use of metaphor should be legitimized by peer mathematicians.

The notions of legitimacy and taken-to-be-shared in the use of metaphor imply that mathematical metaphor is related to the conceptual model culturally shared in a mathematics community.

3. RESEARCH DESIGN

3.1. Developmental Research Project of an Inquiry-Oriented Mathematics Class

This research is part of developmental research project of a university-level inquiry-oriented differential equations course. In traditional differential equations classes, students were instructed a set of algorithms for solving specific types of differential equations. Although differential equations were invented to model natural phenomena of continuous change such as planet motion, traditional courses of differential equations rarely contributed to develop students abilities for modeling and problem solving in the discipline. This critical reflection on traditional differential equations class triggered reform movements since the 1980s, which was initiated by Artigue and Gautheron (1983).

In the reformed setting, diverse mathematical methods such as numerical and graphical as well as quantitative were introduced to enhance students' understanding and problem solving in differential equations. Employed with technology, numerical methods are powerful to construct reliable approximate solutions. Graphical methods provide overall information about solution by analyzing differential equations geometrically. The emphasis on graphical and numerical methods reflects the recently evolving interest of the subject in dynamical system (Rasmussen 2001).

However, it has been criticized that the traditional reform movement of differential equations employed the diverse methods only in the introductory stage of class. More importantly, students' participation was most often ignored. In this context, this developmental research shares but extends the educational vision presented by the traditional reform movement by emphasizing students' active participation in the construction of mathematics. For the purpose, this developmental research takes the theory of Realistic Mathematics Education (RME) as the principle for the course development. Based on the didactical phenomenology of Freudenthal (1973; 1983), RME recommends that mathematics be learned through progressive mathematization out of everyday experience that is experientially and mathematically realistic to a learner (Treffers 1987).

Thus, context problems take an essential role in the project class in order to provide a ground for progressive mathematization. A set of context problems was developed by the research team to reflect mathematical phenomena experientially realistic to the students. The context problems were also of historical significance. For instance, one of the worksheet problems was concerned with spring-mass motions, which historically motivated the invention of differential equations (Kline 1972). The context problems were sequenced in order to lead the students to go through a series of horizontal and vertical mathematization for the mathematical reinvention.

In the project class, the students formed small groups of three or four to work on the context problems. While the students worked in a small group setting, the instructor interacted with the students. After the small group discussion, the students joined together for a whole class discussion. The students came to the front to share their results and the instructor facilitated the sharing of their mathematical ideas. The students do not merely seek for solutions for specific mathematical tasks but also negotiate and renegotiate their mathematical meaning. Through the process of progressive mathematization, a taken-as-shared mathematical reality emerges gradually among the students.

3.2. Data Collection and Data Analysis

Data were collected through participatory observation in a differential equations course at a Korean university in Fall 2002. All class sessions were video recorded and transcribed for later detailed analysis. Interviews were conducted systematically to probe the students' conceptual understanding and problem solving of differential equations. All the interviews were video recorded. In addition, students' works such as exams, journals and worksheets were collected for supplement the analysis of data from class observation and interview. This paper presents the findings from the analysis that primarily focuses on the students' mathematical discourse in the project class. In the analysis, language is considered as an important indicator to depict the students' mathematical reasoning under the assumption that "Facets of cultural values and beliefs, social institutions and forms, roles and personalities, history and ecology of a community may have to be examined in their bearing on communicative events and patterns" (Hymes 1974, p. 4).

Metaphor is one of the important communicative patterns shared in a mathematics community (Ju 2001; Lakoff & Nunez 2000; English 1997; Pimm 1987). Indeed, the students were encouraged to participate in mathematical communication to construct mathematical knowledge meaningful for themselves in the project class. Thus, there were abundant opportunities to observe the students' discursive practice of mathematics. In particular, the students used conceptual metaphor to deliver their mathematical reasoning to their peers.

When considering that metaphor is not merely representing but rather integral to the students' mathematical reasoning, metaphor can be seen as a significant indicator in describing the model of a mathematical concept held by a speaker. Therefore, by investigating the patterns in students' use of metaphor, especially, how it changes through students' participation in mathematics practice in class, this paper aims to describe how students' conceptual model of differential equations — particularly differential equations in this research — becomes transformed and in what way the transformation is social.

The discourse analysis specifically focuses on the following questions: what kind of metaphor did students use in representing their mathematical reasoning? What are the characteristics of the metaphors used? How does the students' use of metaphor change through the semester?

4. FINDINGS

4.1. Kinds of Metaphor Used and Characteristics

Mathematically, solutions of differential equations have dual aspects. On one hand, they are treated as objects: something satisfying a given equation. On the other hand, differential equation solutions are functions representing phenomena changing with respect to the independent variables assigned. These dual aspects of differential equation solutions reflected in the students' use of metaphor, which can be largely categorized into two types: machine metaphor and fictive motion metaphor.

"A Function Is A Machine" is one of the grounding metaphors for functions (Lakoff & Nunez 2000). In the machine metaphor, functions are regarded as machine processing inputs to produce outputs. The notion of algorithm is based on the machine metaphor in the sense that an algorithm as a metaphorical machine operates on input to yield output. Followings are examples from the class:

> "Since the time interval decreased to 0.5, I mean as Δt decreased to 1/2, the coefficient of the rate of the change decrease.so we <u>made</u> it 0.15P. Then what we computed last time was P.uh P0 begins with 3 P.P1 is 3.P1 is 1 there but 1. Because (it) means 0.5, 3+ we add there what <u>we put 3 into P</u> (in the equation). As a result, <u>0.342 came out</u>. We add it to get 3.342."

In this example, the students worked on a context problem concerning prediction of population. In the previous session, the students predicted an animal population for each year with a given differential equation $dP/dt = 0.3P(1-P/12.5)$. In this session, the problem asked to predict the animal population for every six months. In the above transcript, the student presented her solution of how to make a prediction from the differential equation. In her explanation, she treated the given differential equation as a machine. With the changed time interval, she changed the coefficient as if she modified the machine to fix the new purpose. Then she put a number to operate the machine out of which a number springs.

With the machine metaphor, the students regarded differential equations as a device for calculation. So they need to put a number to acquire a numeral value. As a result, their approaches were quantitative and their reasoning focused on the product of change identified by differential equations. Due to the focus on the product, motion metaphor

did not highlight the process of change, in particular, rate of change in differential equation. Also, since they had to input definite numbers in differential equations, their approaches were discrete. In the above example, she constructed the approximate solution graph of a differential equation by computing values at discrete points.

"Fictive motion metaphor" is another dominant kind of metaphor that the students used, in which a solution of a differential equation is represented as a trajectory of a moving point. In general, in the fictive motion metaphor, variables are travelers moving along axes and the function relating the variables are conceptualized as a traveler moving across the plane or space. Following are examples of the fiction motion metaphor from the class:

> "Whatever P is, if it <u>becomes</u> 12.5, it keeps <u>going</u> just like this"
> "When P is smaller than 12.5, the rate of change is positive so it <u>keeps increasing</u>. Then as it <u>gets closer to</u> 12.5, uh to the <u>continuously changing</u> increment.uh.if it <u>becomes</u> 12.5, it does not <u>increase</u> anymore."

In these examples, the students were dealing with the same population problem discussed in the above. However, the students approached the context in a different way. In the analysis, the students thought of a solution in terms of a trajectory of a point continuously moving along. As a result, the students viewed problem situations as continuously changing. For instance, when the student used the verb "become" instead of "be" in the first example to represent a continuous change. Because the fictive motion metaphor represents a solution as a trajectory of a moving point, the students focused on the entire pattern of a solution curve. So the students qualitatively reasoned about differential equations based on the holistic pattern of solutions instead of precise values.

4.2. Social Transformation in the Use of Metaphor

So far, kinds of metaphor used and their characteristics have been identified. Then, what is the pattern in the students' use of metaphor in the class? In what aspect is the pattern social? In other words, does the pattern indicate the path of the students' social transformation with respect to the cultural organization of communal practice of mathematics? In this regard, one of the salient patterns in the use of metaphor is concerned with the switch between the two types of metaphor. Generally, there was an observed tendency for students to rely more often on fictive motion metaphor than on machine metaphor throughout the semester.

In the beginning of the semester, the students were more likely to interpret their mathematical tasks as algorithmic and discrete and as a result, they applied the machine metaphor more often. This is partly due to the nature of mathematical tasks. For instance, in the beginning of the semester, the course provided context problems to lead the students to reinvent Euler's method. In this way, the course designer intended discrete

and algorithmic approaches. However, the tendency was also the product of the students' own conception of "what is mathematics?" In the beginning of the semester, the students often expressed that law-like-principles or algorithms were the only "real mathematics". This kind of mathematical belief is the product of their previous mathematical practice such as grade making in mathematics class. However, machine metaphor rarely matched a given mathematical situation of change, as it was not efficient in revealing the changing aspects of the phenomenon under inquiry. Through meaning negotiation, the students realized the weakness of the machine metaphor and switched to the fictive motion metaphor, which is more appropriate to highlight the continuity of a changing situation. In this part, a sequence of examples will be presented to illustrate the way that the students proceeded from the machine metaphor to the fictive motion metaphor. The examples were from a session in which the students investigate a context of population growth to determine by which the change in the population is determined, initial population or time.

The students initiated their discussion with the machine metaphor, as illustrated in the following example:

> Student A: Before, in high school we learned Fibonacci sequence. Like there are two rabbits and they reproduce once a year and the babies reproduce in a year. I thought that this (problem) is similar to that.... When I thought about this problem in terms of Fibonacci sequence, we have to multiply by the number of fish to get this number so the equation is concerned with P. When we make a Fibonacci sequence, the degree is t. So t seems to be involved, too.

This student A began her discussion with a discrete model of population growth. She applied the notion of sequence to the situation and thought only about the population at each generation without taking account for the process of population growth between generations. She tried to explain how to get population at a certain moment. Thus, she developed an algorithmic expression to determine which variables were involved with population change.

However, the algorithmic discrete model was not quite persuasive and another student B disputed:

> Student B: I still believe that only P is involved with the population growth. In the beginning, I thought that time might be involved. (Drawing a graph). For instance, starting from an initial value it seemed to be related to time, if it starts from here, then it will merge from this point. So the population can be expressed in terms of only P regardless time.
> Student A: But then, the parents and the babies reproduce together.
> Other students: Should they reproduce just once?

In this argumentation, Student B introduced fictive motion metaphor by representing the population growth as a process changing along a curve. With the metaphor, she highlighted the continuity in the population growth. As a result, she could see the context

as an integrated whole and the insight enabled her to make a more meaningful claim.

Following Student B's dispute, Student A refuted back by trying to perpetuating her discrete model based on the machine metaphor. As listening their arguments, several students began to question the appropriateness of the model presented by Student B. Through further meaning negotiation, the class ended with more elaborated argument-tation based on fictive motion metaphor. In particularly, after the discussion, the class introduced software producing slope fields of differential equations. The technology visualizes slope fields in a very dynamic way, which enables the students to manipulate slope marks and see the continuity of the change. Through the experience, the students became to appreciate the appropriateness of fictive motion metaphor as language to represent contexts that differential equations concerned. Moreover, their use of fictive motion metaphor gets more sophisticated:

> Student C: In the cases that the number of fish is 3 and 5, respectively, if we look at the relation on the graphs, the slope was steeper when an initial value is larger since they reproduce to the same degree, so if you put the cursor here, the curve is identical to that curve starting from here. Like this if it starts from 2, doesn't it become 3 at a certain moment? Right? Because the same species reproduces to the same degree.

In the above example, the student C represented the population growth as a curve made by a traveling point (*e.g.*, "starting from here", "doesn't it become 3"). Moreover, by saying "to the same degree", she emphasized the notion of rate of change in differential equations. Also, she explains the change in the slope in terms of the change in the steepness of the travel route. These entailments of fictive motion metaphor extend it from an ordinary metaphor for function to a metaphor specific to represent differential equation revealing the notion of "rate of change" in differential equations.

The above examples were from a session in the early stage of the semester and the emergence of the fictive motion metaphor out of the machine metaphor had taken throughout an entire session. However, the students applied fictive motion metaphor increasingly and more efficiently throughout the semester. In the end of the semester, it was observed that the students promptly switched between the two metaphors depending on their appropriateness for a given mathematical task. This implies that the machine metaphor is not redundant. Rather, the machine metaphor provided an intermediary stage for the students to transgress to the model of differential equations based on fictive motion metaphor.

The change in the use and in the efficiency is interpreted as evidence showing that the students' conceptual model becomes transformed according to the historical and cultural organization of the practice in the mathematics community. In this regard, the transformation can be considered as social. It is social transformation in the sense that it reflects the instructional design of the course as communal practice. For instance, in the

first session, the professor contrasted two notions of differential equations: its algorithmic aspect and its meaning as language for describing changing contexts. The contrast is related to the historical and cultural meaning of differential equations grasped by him/her at a fundamental level. The subject of differential equations were historically invented as a language describing motion, that is, continuously changing situations such as two-body problem in astronomy, spring-mass motion, and elasticity (Kline 1972). In this aspect, fictive motion metaphor is the historically and culturally legitimate model of differential equations shared within the mathematics community. This suggests that change in the use of metaphor indicates the renegotiation of the students' conceptual model with respect to historical and cultural organization of the mathematics community.

5. CONCLUSION

The purpose of this research was to describe the social transformation of students in a differential equations course based on the analysis of mathematical metaphor that students used in class. The analysis shows that machine metaphor and fictive motion metaphor were the most dominant kinds of metaphor used by the students. Through meaning negotiation, the students realized their weakness and strength as a language for differential equations. Throughout the semester, the students gradually came to apply fictive motion metaphor more frequently and efficiently. This tendency in the use of metaphor is interpreted as indicating the social transformation of students' conceptual model of differential equations in the sense that the students come to grasp gradually the cultural legitimacy of fictive motion metaphor.

Although metaphor used in the class was classified into two types, each participant used metaphor creatively based on one's own understanding of a problem context. Through meaning negotiation, diverse perspectives evolved and emerged to form shared mathematical meaning in the class. As a consequence, the participants developed their own culture of mathematics in the class. In that regard, the mathematics class can be seen as a community of practice. It has its own unique culture of mathematics, which is distinct from the culture of the larger mathematics community but not entirely. This suggests that the meaning negotiation in mathematics class is not an arbitrary process but firmly grounded to the history and culture of a larger mathematics community. Therefore, on one hand, an individual learner plays a critical role as active agency to produce mathematics in practice. On the other hand, through participating in the practice of mathematics mediating a historical and cultural structure of a mathematics community, a learner becomes transformed to a certain kind of a cultural being with historicity.

Then, what creates a link between the two communities? Although the culture of a

mathematics class is the product of co-engagement among the participants, it is important to note that a mathematics teacher plays a critical role in the process. Even though the professor did not instruct in the class, she insinuated her mathematical perspective by her use of metaphor and in terms of her instructional design. For instance, context problems played an important role in the class in the sense they affected the students' choice of metaphor. Also, it was observed that technology contributed to the formation of cultural model of differential equations in the class by affecting the process of metaphorical switch. In general, it is fundamentally a mathematics teacher who selects context problems and technologies in order to infuse one's own mathematical perspective of legitimate practice of mathematics. This refutes the notion of mathematics as definite, self-contained and independent of who we are. This suggests that mathematics is intersubjective meaning deeply situated among people who practice it. Thus, a teacher's intervention is critical when it lays a foundation of a future possibility instead of confining students' potentiality.

This description of social transformation in mathematics class ultimately reveals the intricateness of learning mathematics. Learning mathematics is experience of liberation but at the same time of enculturation with respect to a specific history and culture. How do these two poles of experience lead to the creation of a learner as an integrated whole? What is the role of a mathematics teacher in the process of social transformation? What is the resource that a mathematics teacher can rely on to support the process? It is considered that further investigation of social transformation will provide a clue to these questions based on the historicity and culture of mathematics.

REFERENCES

Artigue, M. & Gautheron, V. (1983): *Systemes differentials, etude graphique*. Paris: CEDIC/F. Nathan.

Cobb, P. & Bauersfeld, H. (Eds.) (1995): *The Emergence of Mathematical Meaning: Interaction in Classroom Cultures*. Hillsdale, NJ: Lawrence Erlbaum Associates.   MATHDI **1995e**.03449

English, L. (1997): *Mathematical Reasoning: Analogies, Metaphors, and Images*. Mahwah, NJ: Lawrence Erlbaum Association.   MATHDI **1997f**.04354

Freudenthal, H. (1983): *Didactical Phenomenology of Mathematical Structure*. Dordrecht: Reidel Ser. Title: Mathematics education library.   MATHDI **1984c**.02648

Freudenthal, H. (1973): *Mathematics as an Educational Task*. Dordrecht: D. Reidel.

Hymes, D. (1974): Toward ethnographies of communication. In: *Foundations in Sociolinguistics: An Ethnographic Approach* (pp. 3–28). Philadelphia: University of Pennsylvania Press.

Ju, M. -K. (2001): *Being a Mathematician: An Ethnographic Account of the Cultural Production of a Mathematician at a University*. Unpublished Doctoral Dissertation: University of

California, Davis.

Kline, M. (1972): Ordinary Differential Equations in the Eighteenth Century. In: *Mathematical Thought from Ancient of Modern Times* (pp. 400–435). New York: Oxford University Press.

Lakoff, G. & Nunez, R. (2000): *Where Mathematics Comes From*. New York, NY: Basic Books. MATHDI **2002f.**05757

Lave, J. & Wenger, E. (1991): *Situated Learning: Legitimate Peripheral Participation*. Cambridge, MA: Cambridge University Press.

Pimm, D. (1987): *Speaking Mathematically*. New York: Routledge and Kegan.

Rasmussen, C. L. (2001): New Directions in Differential Equations: A Framework for Interpreting Students' Understandings and Difficulties. *The Journal of Mathematical Behavior* **20**, 55–87. MATHDI **2003a.**00371

Holland, D. & Quinn, N. (Eds.) (1987): *Cultural Models in Language & Thought*. New York, NY: Cambridge University Press.

Treffers, A. (1987): *Three Dimensions: A Model of Goal and Theory Description in Mathematics Instruction — The Wiskobas Project*. Dordrecht, Holland.

Voigt, J.; Seeger, F. & Waschescio, U. (Eds.) (1998): *Culture of Mathematics Classroom*. New York: Cambridge University Press.

Wenger, E. (1998): *Communities of Practice: Learning, Meaning, and Identity*. New York: Cambridge University Press.