Position Control of Chained Multiple Mass-Spring-Damper Systems – Adaptive Output Feedback Control Approaches

S. S. Ge, L. Huang, and T. H. Lee

Abstract: This paper addresses the issue of position control of a chain of multiple mass-spring-damper (CMMSD) units which can be found in many physical systems. The dynamic model of a CMMSD system with any degrees of freedom is expressed in a closed-form for the convenience of the controller design. Backstepping and model reference adaptive control (MRAC) approaches are then used to develop two adaptive output feedback controllers to control the position of a CMMSD system. The proposed controllers rely on the measurements of the input (force) and the output (position of the mass unit at the end of the chain) of the system without the knowledge of its parameters and internal states. Simulations are used to verify the effectiveness of the controllers

Keywords: Adaptive control, output feedback, position control, robotics.

1. INTRODUCTION

Chained multiple mass-spring-damper (CMMSD) units are found in many physical systems such as hyper-redundant mechanical systems [1], flexible link Robots [2-5] and multi-mass systems for vibration absorbers [6], to name a few. The position control of a CMMSD system is challenging due to the difficulties in measuring its system parameters and internal states which are "hidden" in a chain of mass-spring-damper units. One typical example is the control of a robot constrained by a flexible constraint where the measurement of the constraint states are almost impossible [4]. It was proved that the model free linear controller such as PID control is not effective to control a CMMSD system [2,7,8]. Though a CMMSD system model can be transformed into a group of decoupled integrators and the controller design can be simplified through the feedback linearization, it needs undesirable high order derivatives of the system states and introduces unstable internal dynamics into the controlled system [2,7]. Though singular perturbation is a powerful tool to make the controller design simpler and effective for CMMSD systems [7], the requirement of very large stiffnesses inside the CMMSD system limited its applications.

To control an uncertain CMMSD system when only the input (the force acting at the end of the chain) and the output (position of the mass unit at another end of the chain) are measurable, adaptive output feedback control should be investigated. The dynamic models of the CMMSD systems do not provide the matching condition, thus a lengthy state transformation is needed before the traditional non-recursive adaptive output feedback control approaches, such as model reference adaptive control (MRAC) can be applied [15]. On the other hand, CMMSD systems are recursive in their physical structures and their dynamic models can be expressed in the parametric strict feedback forms suitable for backstepping design [9]. Though CMMSD systems of low degrees of freedom are fairly simple in structure, control system design for the CMMSD system of high degrees of freedom is still worth investigation owing to the difficulties explained. As one of the contributions of the paper, a closed-form description is given for a CMMSD system of arbitrary degrees of freedom. To the best of our knowledge, this is the first time in the literature a closed form expression of the system model is reported for a general CMMSD system. In fact, the closed-form description lays the foundation for the design of two neat and concise adaptive output feedback controls. The beauty of the controllers presented lies in their clarity and elegant applications of existing theories in solving practical problems,

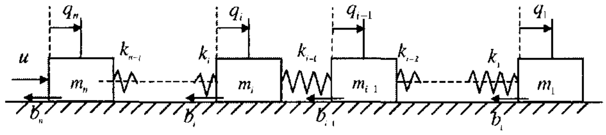

Fig. 1. A general chained multiple mass spring system (CMMSD).

Manuscript received September 15, 2003; revised April 30, 2004; accepted May 12, 2004. Recommended by Editor Chung Choo Chung.

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rather than fundamental contributions in theory. Both backstepping and MRAC methods are used to develop two adaptive output feedback controllers respectively to control the position of CMMSD systems without special requirements on the system parameters (unlike singular perturbation approach which requires sufficiently large stiffnesses of the springs in the CMMSD system), without the need for undesirable high derivative of system states and avoid unstable internal dynamics. The asymptotical stability of the controlled system is guaranteed under the controllers proposed. The main contributions of the paper are as follows:

- the establishment of a closed form description for a general CMMSD system with arbitrary any number of mass-spring-damper units with any stiffness of the spring, and
- the development of two elegant adaptive output feedback position controllers – backstepping based and MRAC based, that are independent of the system parameters and undesirable high derivative of system states.

The rest of paper is organized as follows. In Section 2, the dynamic model of the system is given. In Section 3, two adaptive output feedback controllers are developed with backstepping and MRAC approaches respectively. In section 4, the simulation study is done to verify and compare the effectiveness of the approaches proposed. The conclusion is given in Section 5.

2. DYNAMIC MODELING AND PROBLEM FORMULATION

A chained multiple mass spring system with n mass units is schematically shown in Fig. 1, where m_i is the mass, b_i is the viscous coefficient and q_i is the displacement measured from the equilibrium position along the X axis of the i th unit ($i = 1,2\ldots n$). There are $n-1$ springs connecting the mass units with k_i being the linear spring constant ($i = 1,2\ldots n-1$). It is understood that only q_1, the output of the system, and u, the input force, are measurable.

According to Newton's second law, we have the following system dynamics:

$$\begin{aligned} m_1\ddot{q}_1 &= -b_1\dot{q}_1 + k_1(q_2 - q_1) = -b_1\dot{q}_1 - k_1q_1 + k_1q_2, \\ m_2\ddot{q}_2 &= -b_2\dot{q}_2 - (k_1+k_2)q_2 + k_1q_1 + k_2q_3, \\ m_i\ddot{q}_i &= -b_i\dot{q}_i - (k_{i-1}+k_i)q_i + k_{i-1}q_{i-1} + k_iq_i + 1, \quad (1) \\ &\quad (i = 3,4,\cdots,n-1), \\ m_n\ddot{q}_n &= -b_n\dot{q}_n - k_{n-1}q_n + k_{n-1}q_{n-1} + u. \end{aligned}$$

Defining $x_1 = q_1$, $x_2 = \dot{q}_1$, $x_3 = q_2$, $x_4 = \dot{q}_2$, $\cdots$, $x_{2i-1} = q_i$, $x_{2i} = \dot{q}_i$, $\cdots$, $x_{2n-1} = q_n$, $x_{2n} = \dot{q}_n$, $x = [x_1\ x_2 \ldots x_{2n}]^T \in R^{2n}$, the original dynamic system (1) is transformed to the following state space model:

$$\dot{x} = A_x x + b_x u, \tag{2}$$

$$x_1 = c_1^T x, \tag{3}$$

where

$$\begin{aligned} b_x &= m_n^{-1}c_{2n}, \\ A_x &= [A_1^T\ A_2^T\ \ A_{2i-1}^T\ \ A_{2i}^T \ldots A_{2n-1}^T\ A_{2n}^T]^T, \\ A_1 &= c_2^T, \\ A_2 &= -m_1^{-1}(k_1c_1^T + b_1c_2^T - k_1c_3^T), \\ A_{2i-1} &= c_{2i}^T, \\ A_{2i} &= m_i^{-1}(k_{i-1}c_{2i-3}^T - (k_{i-1}+k_i)c_{2i-1}^T - b_ic_{2i}^T) \\ &\quad + m_i^{-1}k_ic_{2i+1}^T \quad (i=2,3\ldots n-1), \\ A_{2n-1} &= c_{2n}^T, \\ A_{2n} &= m_n^{-1}(k_{n-1}c_{2n-3}^T - k_{n-1}c_{2n-1}^T - b_nc_{2n}^T), \end{aligned}$$

and c_j is the j th column vector of identity matrix I_{2n} ($j = 1, 2, \cdots 2n$).

Applying Laplace transformation on equations (2) and (3), the system dynamic model in the s domain is obtained:

$$X_1(s) = H_{2n}(s)U(s), \tag{4}$$

where $X_1(s)$ and $U(s)$ are the Laplace transformation of x_1 and u respectively, and H_{2n} is the transfer function defined as

$$H_{2n}(s) = c_1^T(sI_{2n} - A_x)^{-1}b_x = \frac{d_{2n}}{s^{2n} + \Sigma_{j=0}^{2n-1}a_{2n,j}s^j}. \tag{5}$$

The coefficients in the transfer function are expressed as the explicit functions of the system parameters in Appendix 1. For clarity, their detailed and involved derivations are omitted.

Remark 1: For a CMMSD of arbitrary any degrees of freedom, we have obtained a closed-form description (5) that is explicitly expressed as the functions of the system parameters. This closed-form expression is not only essential for the control design discussed in this paper, but also very convenient and useful for simulation and system analysis.

Re-writing the state space equations in observer canonical form for system (4), we have

$$\begin{aligned} y &= Ay + B(y_1, u)^T\theta, \\ y_1 &= x_1 = c_1^T y, \end{aligned} \tag{6}$$

where

$$\dot{y}_1 = y_2 - a_{2n,2n-1}y_1$$
$$\vdots \quad \vdots$$
$$\dot{y}_i = y_{i+1} - a_{2n,2n-i}y_1$$
$$\vdots \quad \vdots$$
$$\dot{y}_{2n} = d_{2n}u$$
$$A = \begin{bmatrix} 0 & & \\ \vdots & I_{2n-1} & \\ 0 & \cdots & 0 \end{bmatrix}$$
$$B^T = [c_{2n}u \ \ -I_{2n}y_1]$$
$$\theta = [d_{2n} \ \ a_{2n,2n-1} \ \ a_{2n,2n-2} \ \cdots a_{2n,1} \ \ a_{2n,0}]^T.$$

Note that $B \in R^{(2n+1)\times 2n}$ is a matrix formed by the input u and the output y_1, and $\theta \in R^{2n+1}$ is a vector of the coefficients d_{2n} and $a_{2n,j}$ ($j = 0, 1, 2, \cdots 2n$).

Remark 2: It is essential to re-write the system model in observer canonical form (6) so that the states can be reconstructed for output feedback control design that follows. As clearly shown, this is only possible after the closed-form description (5) is obtained.

3. CONTROLLER DESIGN

In this section, adaptive output feedback is investigated based on the observer canonical form (6) of the CMMSD system. The control objective is to regulate the output y_1 to zero when the system parameters are unknown and the system states are not measurable.

In the following, both backstepping and MRAC approaches are investigated, respectively.

3.1. Adaptive output feedback control using backstepping

First, let us consider the following K-filters [9]:

$$\dot{\xi} = A_0\xi + \lambda y_1, \tag{7}$$

$$\dot{\Omega}^T = A_0\Omega^T + B(y_1, u)^T, \tag{8}$$

where $\xi \in R^{2n}$ and $\Omega^T \in R^{2n\times(2n+1)}$ are the outputs of the filters, and $\lambda = [\lambda_1 \, \lambda_2 \cdots \lambda_{2n}]^T \in R^{2n}$ are parameters that are chosen such that

$$\begin{aligned} &A_0 = A - \lambda c_1^T \in R^{2n\times 2n} \\ &PA_0 + A_0^T P = -Q < 0 \end{aligned} \tag{9}$$

for any given symmetric positive definite matrices $P \in R^{2n\times 2n}$ and $Q \in R^{2n\times 2n}$. For simplicity and clarity, we will take $Q = I_{2n}$ in the paper.

To reduce the order of the filters, Ω^T is chosen as

$$\Omega^T = [v \ \ \Omega_2],$$

where $v = [v_1 \ \ v_2 \ldots v_{2n}]^T \in R^{2n}$, $\Omega_2 = [\eta_1 \ \ \eta_2 \ \ldots \eta_{2n}] \in R^{2n\times 2n}$, and $\eta_j \in R^{2n}$ ($j = 1, 2, \ldots, 2n$).

With v and Ω_2 so defined, we have

$$\begin{aligned} &\dot{v} = A_0 v + c_{2n}u, \\ &\Omega_2 = A_0\Omega_2 - I^{2n}y_1. \end{aligned} \tag{10}$$

Due to the special structure of A_0 and from equations (7) and (10), we have

$$\begin{aligned} &\dot{\eta}_{2n} = A_0\eta_{2n} - c_{2n}y_1, \\ &\eta_j = A_0^{2n-j}\eta_{2n}, \\ &\xi = A_0^{2n}\eta_{2n}. \end{aligned} \tag{11}$$

With K filters determined above, the estimate of y is given by

$$\hat{y} = \xi + \Omega^T\theta. \tag{12}$$

It can be showed that the state estimation error follows

$$\dot{\varepsilon} = A_0\varepsilon. \tag{13}$$

Defining a Lyapunov function

$$V_\varepsilon = \varepsilon^T P\varepsilon \tag{14}$$

and differentiating it with respect to time t, since A_0 is symmetric we have

$$\dot{V}_\varepsilon = 2\varepsilon^T P\dot{\varepsilon}.$$

From equations (9) and (13), it can be concluded that $\dot{V}_\varepsilon = -\|\varepsilon\|^2$ and ε approach zero while $t \to \infty$.

From equations (11) and (12), and considering equations (6) and (8), we have

$$\begin{aligned} \dot{y}_1 &= c_2^T A_0^{2n}\eta_{2n} + w^T\theta + \varepsilon_2 \\ &= c_2^T A_0^{2n}\eta_{2n} + d_{2n}v_2 + \overline{w}^T\theta + \varepsilon_2, \end{aligned} \tag{15}$$

$$\dot{v}_2 = v_3 - \lambda_2 v_1, \tag{16}$$

$$\begin{aligned} &\dot{v}_i = v_{i+1} - \lambda_i v_1 \ (i = 3, 4 \ldots 2n-1), \\ &\dot{v}_{2n} = -\lambda_{2n}v_1 + u, \end{aligned} \tag{17}$$

where

$$w = [v_2 \ \ \eta_{2n}^T A_\eta^T - y_1 c_1^T]^T,$$
$$\overline{w} = [0 \ \ \eta_{2n}^T A_\eta^T - y_1 c_1^T]^T,$$
$$A_\eta = [(A_0^{2n-1})^T c_2 \ldots A_0^T c_2 \ \ c_2]^T.$$

Equations (15) to (17) represent a transformed dynamic system with the measurable v and y_1 being its states. To facilitate the controller design with backstepping method, we also need the following coordinate transformation:

$$z_1 = y_1, \tag{18}$$

$$z_i = v_i - \alpha_{i-1}, \quad i \geq 2, \qquad z = [z_1 \; z_2 \ldots z_{2n}]^T, \tag{19}$$

where z_i is the new state and α_i is the so called *stabilization function* or *virtual control* to be determined in the steps of the controller design.

The backstepping design involves $2n$ steps. A Lyapunov function is constructed, and from which, a stabilizing function α_i is determined in every step. In addition, a function called *tuning function* τ_i is also generated in each step to estimate the uncertain system parameters. In the last step $2n$, the control input u is derived.

Step 1. In this step, we begin with the study of the tracking error z_1. Its equation can be derived from equations (15), (18) and (19), such that

$$\dot{z}_1 = d_{2n}\alpha_1 + c_2^T A_0^{2n}\eta_{2n} + \bar{w}^T\theta + d_{2n}z_2 + \varepsilon_2. \tag{20}$$

Considering the following virtual control:

$$\alpha_1 = \hat{d}\bar{\alpha}_1, \tag{21}$$

where $\hat{d}$ is the estimate of $1/d_{2n}$, and substituting it into equation (20) leads to

$$\dot{z}_1 = \bar{\alpha}_1 + c_2^T A_0^{2n}\eta_{2n} + \bar{w}^T\theta - d_{2n}(\tilde{d}\bar{\alpha}_1 - z_2) + \varepsilon_2. \tag{22}$$

Consider the Lyapunov function candidate:

$$V_1 = \frac{1}{2}z_1^2 + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} + \frac{d_{2n}}{2\gamma}\tilde{d}^2 + V_\varepsilon,$$

where $\tilde{\theta} = \theta - \hat{\theta}$, $\hat{\theta}$ is the estimate of θ, $\Gamma = \Gamma^T > 0$, $\gamma > 0$ are the gain matrix and gain respectively, and V_ε is defined in equation (14). Note that it has been proved that $\dot{V}_\varepsilon = -\|\varepsilon\|^2$.

Differentiating V_1 with respect to time t, we have

$$\dot{V}_1 = z_1\dot{z}_1 - \tilde{\theta}\Gamma^{-1}\dot{\hat{\theta}} - \frac{d_{2n}}{\gamma}\tilde{d}\dot{\hat{d}} - \|\varepsilon\|^2.$$

It can be easily verified that along the solution of (22),

$$\begin{aligned}\dot{V}_1 &\leq -\zeta_1 z_1^2 + c_1^T\hat{\theta}z_1z_2 + \tilde{\theta}^T(\tau_1 - \Gamma^{-1}\dot{\hat{\theta}}) - \frac{1}{2}(z_1 - \varepsilon_2)^2 + \frac{1}{2}\varepsilon_2^2 - \|\varepsilon\|^2 \\ &\leq -\zeta_1 z_1^2 + c_1^T\hat{\theta}z_1z_2 + \tilde{\theta}^T(\tau_1 - \Gamma^{-1}\dot{\hat{\theta}})\end{aligned}$$

if we choose

$$\bar{\alpha}_1 = -(\zeta_1 + \frac{1}{2})z_1 - c_2^T A_0^{2n}\eta_{2n} - \bar{w}^T\hat{\theta}, \tag{23}$$

$$\dot{\hat{d}} = -\gamma\bar{\alpha}_1 z_1, \qquad \tau_1 = w - \hat{d}\bar{\alpha}_1 z_1, \tag{24}$$

where $\zeta_1 > 0$ is a control parameter and $\hat{\theta}$ is the estimate of parameters θ.

Note that if $v_2 = \alpha_1$ is the actual control and

$$z_2 = 0, \; \dot{\hat{\theta}} = \Gamma\tau_1$$

it leads to

$$\dot{V}_1 \leq -\zeta_1 z_1^2 \leq 0.$$

Step 2. From equations (16), (19) and (21), we have

$$\dot{z}_2 = \alpha_2 + z_3 - \gamma_2(w^T\tilde{\theta} + \varepsilon_2) - \hat{d}\frac{\partial\bar{\alpha}_1}{\partial\hat{\theta}}\dot{\hat{\theta}} - \beta_2, \tag{25}$$

$$\gamma_2 = \hat{d}\frac{\partial\bar{\alpha}_1}{\partial y_1},$$

$$\beta_2 = \lambda_2 v_1 + \hat{d}\frac{\partial\bar{\alpha}_1}{\partial\eta_{2n}}(A_0\eta_{2n} - c_{2n}y_1) - \gamma\bar{\alpha}_1^2 z_1 + \gamma_2(c_2^T A_0^{2n}\eta_{2n} + w^T\hat{\theta}). \tag{26}$$

Equation (25) describes the behavior of the tracking error z_2. To find out the virtual control α_2 to stabilize z_2, consider the following Lyapunov function candidate:

$$V_2 = V_1 + \frac{1}{2}z_2^2 + V_\varepsilon.$$

Differentiating V_2 with respect to time t along the solutions of (22) and (25), we have

$$\begin{aligned}\dot{V}_2 \leq &-\zeta_1 z_1^2 + z_2z_3 + \tilde{\theta}^T(\tau_2 - \Gamma^{-1}\dot{\hat{\theta}}) \\ &+ z_2(\alpha_2 + c_1^T\hat{\theta}z_1 - \beta_2 - \hat{d}\frac{\partial\bar{\alpha}_1}{\partial\hat{\theta}}\dot{\hat{\theta}}) - \gamma_2z_2\varepsilon_2 - \|\varepsilon\|^2,\end{aligned}$$

where $\tau_2 = \tau_1 - \gamma_2 w z_2$.

If we select

$$\alpha_2 = -(\zeta_2 + \frac{\gamma_2^2}{4})z_2 - c_1^T\hat{\theta}z_1 + \beta_2 + \hat{d}\frac{\partial\bar{\alpha}_1}{\partial\hat{\theta}}\Gamma\tau_2, \; \zeta_2 > 0,$$

it follow that

$$\begin{aligned}\dot{V}_2 \leq &-\zeta_1 z_1^2 - \zeta_2 z_2^2 + z_2z_3 + \tilde{\theta}^T(\tau_2 - \Gamma^{-1}\dot{\hat{\theta}}) \\ &+ z_2\hat{d}\frac{\partial\bar{\alpha}_1}{\partial\hat{\theta}}(\Gamma\tau_2 - \dot{\hat{\theta}}).\end{aligned}$$

If $v_3 = \alpha_2$ is the actual control and

$$z_3 = 0, \; \dot{\hat{\theta}} = \Gamma\tau_2,$$

we have

$$\dot{V}_2 \leq -\zeta_1 z_1^2 - \zeta_2 z_2^2 \leq 0.$$

Step i ($3 \le i \le 2n-1$) In this step, we generalize the design procedure for any step $i \ge 3$ with the assumption that stabilizing functions $\alpha_1, \alpha_2, \ldots \alpha_{i-1}$ and tuning functions $\tau_1, \tau_2, \ldots \tau_{i-1}$ are derived in previous steps.

It can be showed that the tracking error z_i satisfies

$$\dot{z}_i = \alpha_i + z_{i+1} - \gamma_i (w^T \tilde{\theta} + \varepsilon_2) - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} - \beta_i , \quad (27)$$

where

$$\gamma_i = \frac{\partial \alpha_{i-1}}{\partial y_1},$$

$$\beta_i = \Sigma_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial v_j}(v_{j+1} - \lambda_j y_1) + \gamma_i (w^T \hat{\theta} + c_2^T A_0^{2n} \eta_{2n})$$
$$+ \lambda_i y_1 + \frac{\partial \alpha_{i-1}}{\partial \eta_{2n}}(A_0 \eta_{2n} - c_{2n} y_1) - \frac{\partial \alpha_{i-1}}{\partial \hat{d}} \gamma \bar{\alpha}_1 z_1 .$$

Consider the Lyapunov function candidate:

$$V_i = V_{i-1} + \frac{1}{2} z_i^2 + V_\varepsilon .$$

Following the same procedure as in previous steps, the derivate of V_i with respective to time t along the solution of equation (27) is rendered as

$$\dot{V}_i \le -\Sigma_{j=1}^{i} \zeta_j z_j^2 + z_i z_{i+1} + \tilde{\theta}^T (\tau_i - \Gamma^{-1} \dot{\hat{\theta}})$$
$$+ \Sigma_{j=2}^{i} z_j \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} (\Gamma \tau_i - \dot{\hat{\theta}})$$

by selecting

$$\alpha_i = -(\zeta_i + \frac{\gamma_i^2}{4}) z_i - z_{i-1} + \beta_i + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma \tau_i$$
$$- \Sigma_{j=2}^{i-1} z_j \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \Gamma \gamma_i w,$$

where $\zeta_i > 0$ is a control parameter and

$$\tau_i = \tau_{i-1} - \gamma_i w z_i .$$

Obviously, if $v_{i+1} = \alpha_i$ is the actual control and

$$z_{i+1} = 0, \ \dot{\hat{\theta}} = \Gamma \tau_i ,$$

we have

$$\dot{V}_i \le -\Sigma_{j=1}^{i} \zeta_j z_j^2 \le 0.$$

Step 2n. This is the final step of backstepping control design where the control input is determined. Following the same way as done for α_i in the previous steps, the control input can be set as

$$u = \alpha_{2n} = -\zeta_{2n} z_{2n} - z_{2n-1} + \beta_{2n} + \frac{\partial \alpha_{2n-1}}{\partial \hat{\theta}} \Gamma \tau_{2n}$$
$$- \Sigma_{j=2}^{2n-1} z_j \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \Gamma \gamma_{2n} w, \ \zeta_{2n} > 0. \quad (28)$$

With control input u in equation (28), parameter updating laws for $\hat{d}$ and $\hat{\theta}$ in equations (24) and (33) respectively, combined with α_i and $\dot{\hat{\theta}} - \Gamma \tau_i$ in each step, the resulting close loop system with state vector $[z_1 \ z_2 \cdots z_{2n}]^T$ can be described by

$$\dot{z}_1 = -(\zeta_1 + \frac{1}{2}) z_1 + c_1^T \hat{\theta} z_2 + \varepsilon_2 + \bar{w}^T \tilde{\theta} - d_{2n} \bar{\alpha}_1 \tilde{d} \ , \quad (29)$$

$$\dot{z}_2 = -c_1^T \hat{\theta} z_1 - (\zeta_2 + \frac{\gamma_2^2}{4}) z_2 + z_3$$
$$+ \hat{d} \frac{\partial \bar{\alpha}_1}{\partial \hat{\theta}} \Sigma_{j=3}^{2n} \Gamma \gamma_j w z_j - \gamma_2 (w^T \tilde{\theta} + \varepsilon_2), \quad (30)$$

$$\dot{z}_i = -\Sigma_{j=2}^{i-2} \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \Gamma \gamma_i w z_j - (1 + \frac{\partial \alpha_{i-2}}{\partial \hat{\theta}} \Gamma \gamma_i w) z_{i-1}$$
$$- (\zeta_i + \frac{\gamma_i^2}{4}) z_i + z_{i+1} + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Sigma_{j=i+1}^{2n} \Gamma \gamma_j w z_j \quad (31)$$
$$- \gamma_i (w^T \tilde{\theta} + \varepsilon_2) \ \ 3 \le i \le 2n-1,$$

$$\dot{z}_{2n} = -\Sigma_{j=2}^{2n-2} \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \Gamma \gamma_{2n} w z_j$$
$$- (1 + \frac{\partial \alpha_{2n-2}}{\partial \hat{\theta}} \Gamma \gamma_{2n} w) z_{2n-1} \quad (32)$$
$$- (\zeta_{2n} + \frac{\gamma_{2n}^2}{4}) z_{2n} - \gamma_{2n} (w^T \tilde{\theta} + \varepsilon_2).$$

To prove the asymptotic stability of the closed-loop system represented from equations (29) to (32), consider the following Lyapunov function candidate:

$$V_{2n} = V_{2n-1} + \frac{1}{2} z_{2n}^2 + V_\varepsilon$$
$$= \frac{1}{2} z^T z + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} + \frac{d_{2n}}{2\gamma} \tilde{d}^2 + 2n V_\varepsilon .$$

Differentiating V_{2n} with respective to time t along the solutions of (29) to (32), we have

$$\dot{V}_{2n} \le -\Sigma_{j=1}^{2n} \zeta_j z_j^2 + \tilde{\theta}^T (\tau_{2n} - \Gamma^{-1} \dot{\hat{\theta}})$$
$$+ \Sigma_{j=2}^{2n} z_j \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} (\Gamma \tau_{2n} - \dot{\hat{\theta}}),$$

where $\tau_{2n} = \tau_1 - \Sigma_{j=2}^{2n} r_j w z_j$ and $\gamma_1 = -1$ is a new constant introduced to keep the consistency in expression.

Letting

$$\dot{\hat{\theta}} = \Gamma \tau_{2n} \quad (33)$$

it follows that

$$\dot{V}_{2n} \le -\Sigma_{j=1}^{2n} \zeta_j z_j^2 \le 0 .$$

From equation (33), it can be concluded that V_{2n} is non-increasing. Hence, z, $\tilde{\theta}$, $\tilde{d}$ and ε are all bounded. From their boundedness, it can be proved that other signals in the control loop are also bounded. Based on LaSalle-Yoshizawa theorem [9], $z \to 0$ when $t \to \infty$. Obviously $y_1 \to 0$ when $t \to \infty$.

The above results can be summarized in the following theorem.

Theorem 1: For the chained multiple mass-spring-damper system (6) and the re-constructed dynamic model represented by equations (15) to (17), the regulation of the position y_1 is achieved ($y_1 \to 0$ when $t \to \infty$) under the control law (28) and the parameter adaptation laws (24) and (33).

Remark 3: The CMMSD system considered is assumed to be free of external disturbances. To keep the robustness of the controlled system under the external disturbances, various robustification approaches can be used, such as dead-zone modification or δ-modification [10,14], though the resulting controllers tend to be more complicated. As pointed out in [10] and [11], the adaptive controller developed with backstepping methods shows much higher degree of robustness than that of conventional adaptive controller even in the absence of robustification tools.

3.2. Model Reference Adaptive Output Feedback Control

In this section, adaptive output feedback control is to be designed based on MRAC approach. To begin with, the system transfer function (5) is re-written using the differential operator $p = d/dt$ such that

$$y_1 = \frac{d_{2n}}{A_p} u, \tag{34}$$

where

$$A_p = p^{2n} + \Sigma_{j=0}^{2n-1} a_{2n,j} p^j.$$

Assume that the desired behavior of the controlled system is specified by the following reference model:

$$y_{1m} = \frac{B_m}{A_m} u_c, \tag{35}$$

where $B_m = d_m$, $A_m = p^{2n} + \Sigma_{j=0}^{2n-1} a_{m,j} p^j$, u_c is the command input and y_{1m} is the desired output. Note that A_m is a stable monic polynomial with the same order as that of A_p and it should be selected such that when $u_c = 0$, $y_{1m} \to 0$.

The task now is to find a control input u for system (34) such that the controlled system follows the reference model (35). Following the pole placements procedure in [15], this objective can be achieved by making the control u to meet the equations:

$$\begin{aligned} P_r u &= P_t u_c - P_s y_1, \\ P_s &= s_{2n-1} p^{2n-1} + s_{2n-2} p^{2n-2} + \ldots + s_1 p + s_0, \\ P_t &= t_{2n-1} p^{2n-1} + t_{2n-2} p^{2n-2} + \ldots + t_1 p + t_0, \\ P_r &= p^{2n-1} + r_{2n-2} p^{2n-2} + \ldots + r_1 p + r_0, \end{aligned} \tag{36}$$

where P_s and P_t are polynomials and their coefficients s_i, t_i and r_i ($i = 0.1, \ldots 2n-1$) can be obtained from the following equations:

$$A_p P_r + d_{2n} P_s = P_o A_m, \tag{37}$$

$$P_t = P_o B_m / d_{2n} \tag{38}$$

with P_o being a pre-defined observer polynomial $P_o = p^{2n-1} + o_{2n-2} s^{2n-2} + \ldots + o_1 s + o_0$ with o_i being its coefficients. Equation (37) is normally called *Diophantine* equation.

From equations (37) and (38), the coefficients of P_r, P_s and P_t are obtained:

$$r_4 = o_4 + a_{m,2n-1} - a_{2n-1},$$

$$r_i = o_i + \sum_{j=i+1}^{2n-1} (o_j a_{m,2n+i-j} - r_j a_{2n+i-j}) \quad (i = 0, \ldots 2n-2),$$

$$s_i = \sum_{j=0}^{i} (o_j a_{m,i-j} - r_j a_{i-j}) / d_{2n} \quad (i = 0, \ldots 2n-1),$$

$$t_i = d_m o_i / d_{2n} \quad (i = 0, \ldots 2n-2).$$

Obviously r_i, s_i and t_i are the functions of $a_{2n,i}$, $a_{m,i}$, o_i and d_{2n}. In the following, we will develop the parameter adaptation laws to estimate these parameters.

From equations (34), (35) and (36), the error between the output of the controlled loop and the reference model is obtained:

$$e = y_1 - y_{1m} = \frac{d_{2n}}{P_o A_m} (P_r u + P_s y_1 - P_t u_c). \tag{39}$$

To express this error in a linear-in-parameter (LIP) form , re-arrange equation (39) such that

$$e = d_{2n} \left(\frac{1}{P_1} u + \frac{P_r - P_2}{P_f} u + \frac{P_s}{P_f} y_1 - \frac{P_t}{P_f} u_c \right),$$

where $P_f = P_1 P_2$, $P_1 = A_m$ and $P_2 = P_o$. Obviously $P_r - P_2$ is a polynomial of p with coefficients being $r_i = r_i - o_i$ ($i = 0, 1, \cdots, 2n-1$).

Define a vector consisting of coefficients of the polynomials of $P_r - P_2$, P_s and P_t such that

$$\theta_f = [r_{2n-2} \cdots r_0 \; s_{2n-1} \cdots s_0 \; t_{2n-1} \cdots t_0]^T$$

and another vector consisting of filtered input, output and the command inputs such that

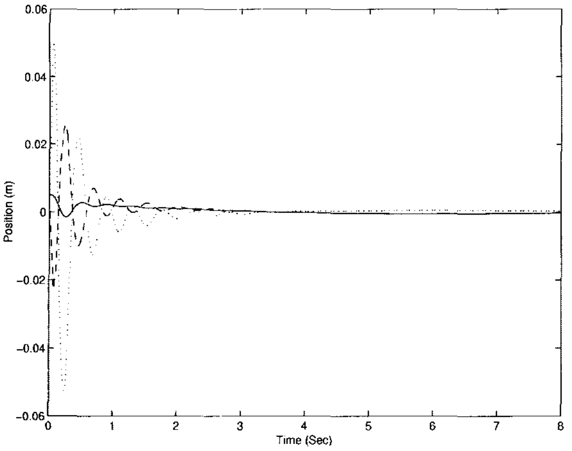

Fig. 2. Position responses (Solid: q_1, dashed: q_2, dashdot: q_3).

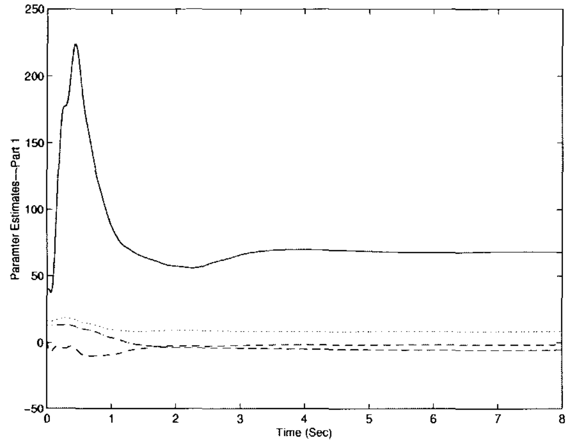

Fig. 3. Parameter estimations-Part 1 (Solid: $\hat{\theta}_1$, dashed: $\hat{\theta}_2$, dotted: $\hat{\theta}_3$, dashdot: $\hat{\theta}_4$).

$$\varphi = [\frac{p^{2n-2}}{P_f}u \ldots \frac{1}{P_f}u \;\; \frac{p^{2n-1}}{P_f}y_1 \ldots \frac{1}{P_f}y_1 \;\; \frac{-p^{2n-1}}{P_f}u_c \ldots \frac{-1}{P_f}u_c]^T.$$

With φ and θ_f defined above, the error e is expressed as

$$e = d_{2n}(\frac{1}{P_1}u + \varphi^T \theta_f). \tag{40}$$

Letting the output feedback control law be

$$u = -\hat{\theta}_f^T(P_1\varphi) \tag{41}$$

and substituting it in the equation (40), we have

$$\begin{aligned} e &= \varepsilon + d_{2n}\eta, \\ \varepsilon &= d_{2n}\varphi^T \tilde{\theta}_f, \\ \eta &= -\frac{1}{P_1}u - \varphi^T \hat{\theta}_f, \end{aligned} \tag{42}$$

where $\tilde{\theta}_f = \theta_f - \hat{\theta}_f$ and $\hat{\theta}_f$ is the estimate of θ_f. It is obvious that ε is linear in the parameters. The adaptation law for θ is then obtained through *gradient approach* [15]:

$$\dot{\hat{\theta}}_f = \gamma_f \varphi \varepsilon,$$

where $\gamma_f > 0$ is the adaptation gain. Note that the unknown parameter d_{2n} is absorbed in γ_f.

The calculation of ζ requires unknown parameter d_{2n}. To estimate it as well as θ_f, the augmented error ε is replaced by *prediction error*

$$\varepsilon_p = e - \hat{d}_{2n}(\varphi^T \hat{\theta}_f + \frac{1}{P_1}u)$$

and the estimates of θ_f and d_{2n} are thus given by

$$\dot{\hat{\theta}}_f = \gamma_f \varphi \varepsilon_p, \tag{43}$$

$$\dot{\hat{d}}_{2n} = \gamma_f(\varphi^T \hat{\theta}_f + \frac{1}{P_1}u)\varepsilon_p. \tag{44}$$

Given $u_c = 0$ and following the same lengthy procedure in proving the stability of general MRAC controllers [15,16], it can be showed that under the controller (41) and parameter adaptation laws (43) and (44), $y_1 \to y_{1m}$ asymptotically and the signals in the controlled system are all bounded. As $y_{1m} \to 0$ when $u_c = 0$, thus we can conclude $y_1 \to 0$ asymptotically. The regulation of the output y_1 with u is thus achieved.

The above results are summarized in the following theorem.

Theorem 2: For the chained mass-spring-damper system with transfer function (34), the position y_1 is controlled such that $y_1 \to 0$ when $t \to \infty$ under the control law (41) and the parameter adaptation laws (43) and (44).

Remark 4: For adaptive feedback control with backstepping, MRAC based approach needs more control parameters such as those of the reference model, observer polynomials, filters and the adaptation gains. Several filters are also needed to filter the outputs, command inputs and the control inputs respectively.

Remark 5: Observer polynomial P_o should be selected such that it is stable with faster dynamic response than that of A_m.

4. SIMULATION STUDY

Consider a system with three mass-spring-damper units. The system parameters are selected as $m_1 = m_2 = m_3 = 2kg$, $b_1 = b_2 = b_3 = 0.8 N/ms^{-1}$ and $k_1 = k_2 = 40N/m$. Based on these system parameters, it can be calculated that $\theta = [200\ 1.0\ 80\ 64\ 1212\ 480\ 0]^T$ and $d = 0.005$. As θ and d are unknown, their initial estimates are assumed to be 20% of their true values, that is, $\hat{\theta} = [40\ 0.2\ 16\ 13\ 240\ 96\ 0]^T$ and $\hat{d} = 0.0015$ respectively. Assume initially $y_1 = q_1 = 0.02$ ($0.02m$ displacement from its equilibrium position) and $q_2 = q_3 = \dot{q}_1 = \dot{q}_2 = \dot{q}_3 = 0$.

First, the simulation is done for the adaptive backstepping output feedback controller. The input to the system is obtained by setting $n = 3$ and setting control parameters $c_1 = 2$, $c_2 = 3$, $c_3 = 5$, $c_4 = 4$, $c_5 = 6$, $c_6 = 7$, $\lambda_1 = 10$, $\lambda_2 = 50$, $\lambda_3 = 20$, $\lambda_4 = 30$, $\lambda_5 = 20$, $\lambda_6 = 20$, $\Gamma = 25I^6$ and $\gamma = 20$. The system responses are shown from Figs. 2 to 4. The control u is shown in Fig. 5. The estimates for $\hat{\theta}$ are shown in Figs. 3 and 4. $\hat{d}$ is plotted together with its value under disturbances in Fig. 10.

It can be seen that positions q_i ($i = 1,2,3$) approach to zero in approximately 4 seconds. It is interesting to note that, during the transient period, q_3 demonstrates larger oscillations than those of q_2 and q_1 while that of q_1 is the minimum. It is understandable as q_3 is directly affected by the input u. The control input is in a reasonable range, though it shows a sharp change to overcome the initial position error at the beginning. Under the adaptation laws, all the parameter estimates become stable around 2.5 seconds, though they do not approximate their true values.

To test the robustness of the controller under the bounded external disturbances, a small bounded disturbance $\Delta(t) = [\delta_1(t)\ \delta_2(t) \ldots \delta_6(t)]^T$ is added to the system dynamics such that

$$\dot{x} = A_x x + b_x u + \Delta(t),$$
$$x_1 = c_1^T x,$$

where $\delta_i (i = 1\ 2 \ldots 6)$ is defined as

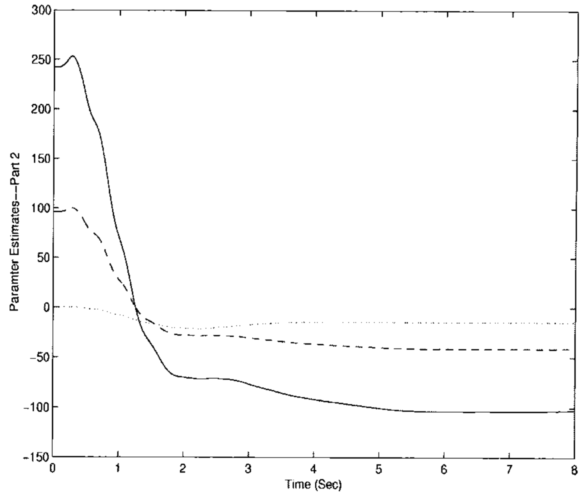

Fig. 4. Parameter estimates-Part 2 (Solid: $\hat{\theta}_5$, dashed: $\hat{\theta}_6$, dotted: $\hat{\theta}_7$).

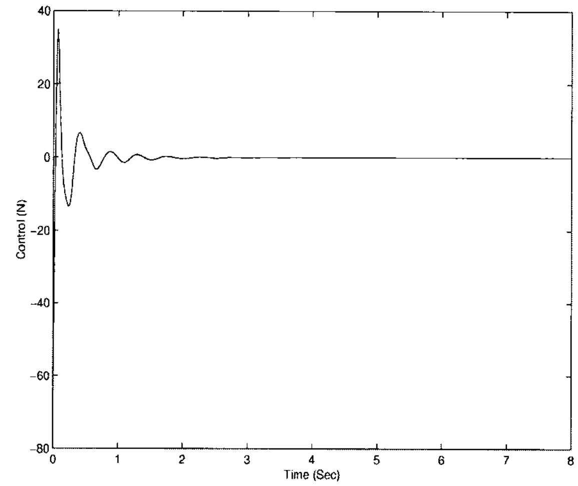

Fig. 5. Control input u.

$$\delta_i = \begin{cases} 0.15 & t \le 2.5\ Sec \\ 0.1 & t > 2.5\ Sec. \end{cases}$$

The simulation results are plotted from Figs. 6 to 8 and Fig. 10. The control effort is plotted in Fig. 9. It can be observed that frequent oscillations are superimposed with the position signals for a longer time compared with those when the system is external disturbance free. Though the asymptotic regulation of the positions are not achieved, the position errors tend to be bounded within a narrow bound. The parameter estimates are stabilized. Though it is observed that the output of the system becomes divergent if the disturbances is large, the above simulation results still show that the controller demonstrates has some degrees of robustness to some bounded disturbances. For larger disturbances, robust control approach should be used for compensations.

The simulation is also done for MRAC adaptive output feedback controller. The initial states of the sys-

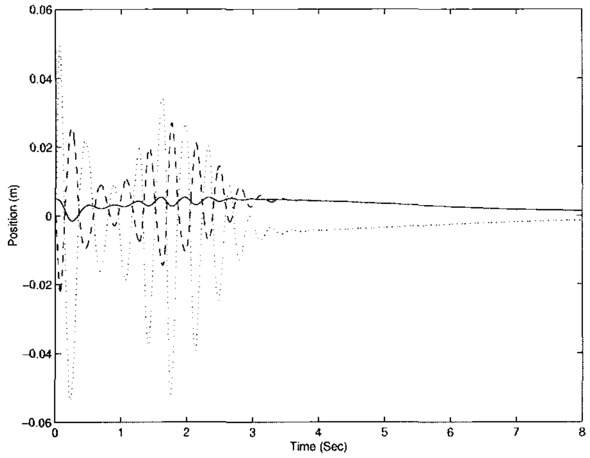

Fig. 6. Position responses under disturbances (So-lid: q_1, dashed: q_2, dashdot: q_3).

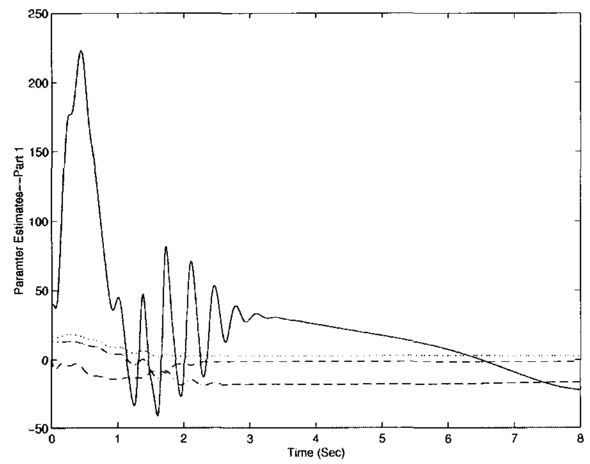

Fig. 7. Parameter estimates under disturbances-Part 1 (Solid: $\hat{\theta}_1$, dashed: $\hat{\theta}_2$, dotted: $\hat{\theta}_3$, dashdot: $\hat{\theta}_4$).

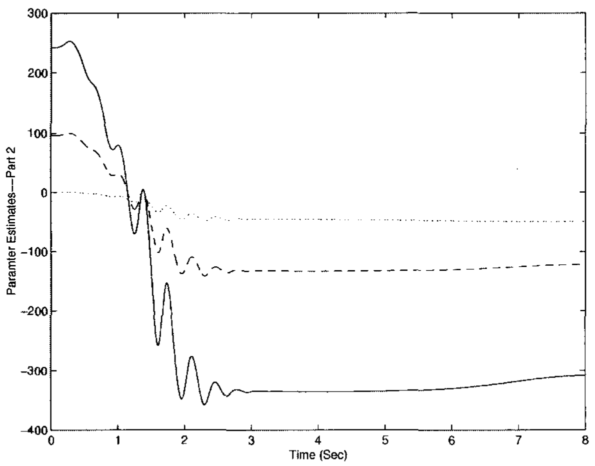

Fig. 8. Parameter estimates under disturbances-Part 2 (Solid: $\hat{\theta}_5$, dashed: $\hat{\theta}_6$, dotted: $\hat{\theta}_7$).

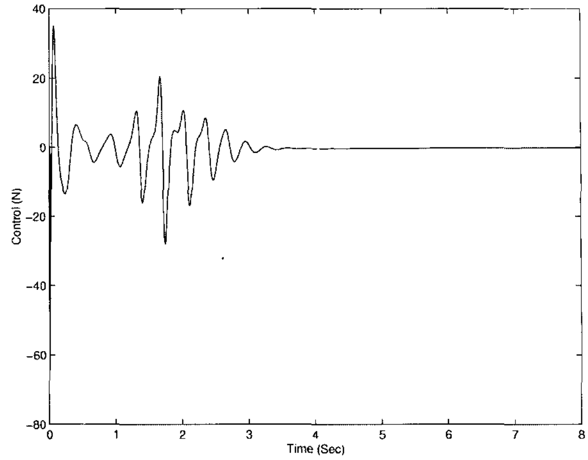

Fig. 9. Control input under disturbances u.

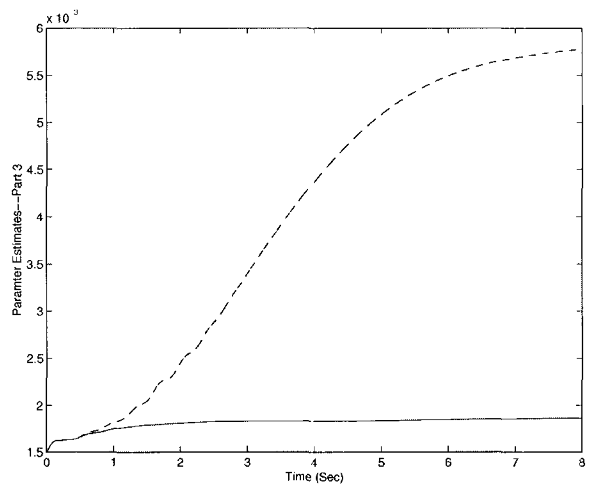

Fig. 10. Parameter estimates - $\hat{d}$ (Solid: $\hat{d}$ when free of disturbances, dashed: $\hat{d}$ under disturbances).

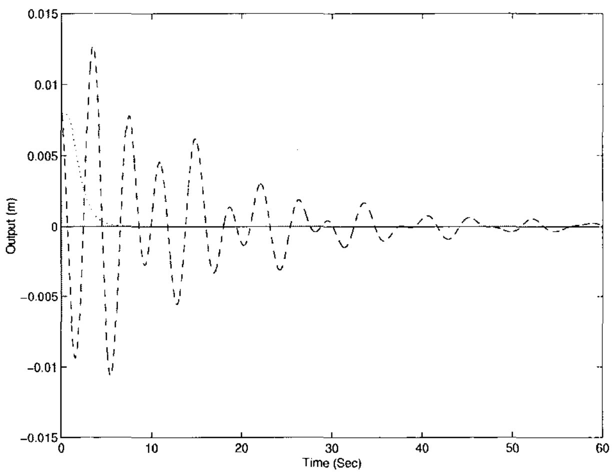

Fig. 11. Position response under the MRAC adaptive output feedback control(Solid: q_1, dashed: q_2, dashdot: q_3).

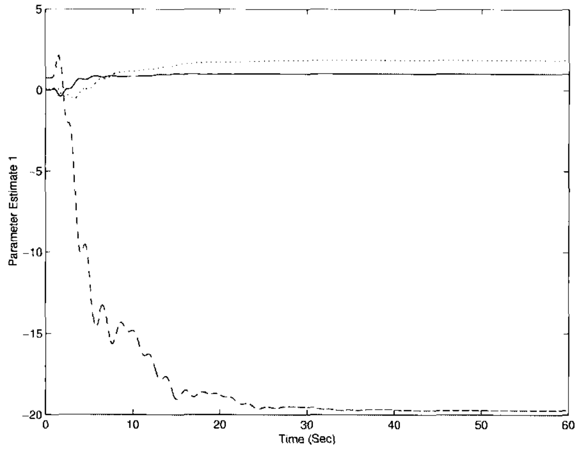

Fig. 12. Some parameter estimates under the MRAC adaptive feedback controller (Solid: $\hat{\theta}_{f1}$, dotted: $\hat{\theta}_{f2}$, dashd: $\hat{\theta}_{f3}$).

system are the same as those for backstepping controller. The controller parameters of the controller are chosen as $\gamma = 12$. The initial values of the parameter estimates $\hat{\theta}_f$ and $\hat{d}_{2n}$ are set to zeros. The simulation results are shown from Fig. 11 to Fig. 13.

It can be seen that it takes longer time (almost 50 seconds) for the output to be regulated to zero compared with that of backstepping control scheme. The position responses also show bigger oscillations (especially q_3) in a longer period. The parameter estimates are convergent, though they are not close to their true values. The control input is in a reasonable range with a magnitude much smaller than that of backstepping controller, but its oscillation lasts longer. In the simulation, it is found that the controller is sensitive to the controller parameters and the initial values of the estimated parameters. The outputs becomes divergent once the disturbance $\Delta(t)$ is added to the system.

5. CONCLUSION

In this paper, dynamic models and adaptive output feedback controller for position regulation are developed for general chained multiple mass spring damper systems. Based on backstepping and MRAC adaptive methods, two adaptive output feedback controllers–recursive and non-recursive respectively, are developed. They rely on the input and the output of the system and do not require the exact knowledge of the parameters and the internal states of the system. Under the proposed controllers, the output (position) of the system is regulated to zero. The simulations are used to verify and compare the effectiveness of the proposed control approaches.

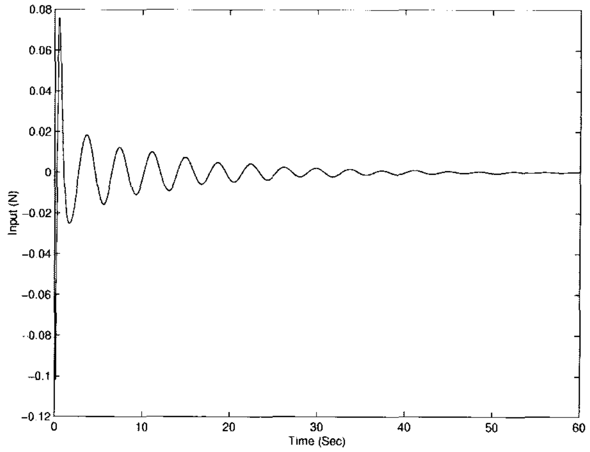

Fig. 13. Control input under the MRAC adaptive feedback controller.

APPENDIX 1:TRANSFER FUNCTION $H_{2n}(s)$

The transfer function $H_{2n}(s)$ can be derived step wise from

$$H_{2i}(s) = \frac{d_{2i}}{s^{2i} + \Sigma_{j=0}^{2i-1} a_{2i,j} s^j} \quad (i = 1\,2 \ldots n), \qquad (45)$$

where

$$\begin{aligned} d_{2i} &= m_i^{-1} d_{2i-2}, \\ a_{2i,j} &= a_{2i-2,j-2} + m_i^{-1} b_i (a_{2i-2,j-1} + \sigma(j-2i+1)) \\ &\quad + m_i^{-1} k_{i-1} (a_{2i-2,j} + \sigma(j-2i+2)) \\ &\quad - d_{2i-4}^{-1} d_{2i-2} m_i^{-1} k_{i-1} (a_{2i-4,j} + \sigma(j-2i+4)), \\ d_{2l} &= m_l^{-1} k_l d_{2l-2}, \\ a_{2l,j} &= a_{2l-2,j-2} + m_l^{-1} b_l (a_{2l-2,j-1} + \sigma(j-2l+1)) \\ &\quad + m_l^{-1} (k_{l-1} + k_l)(a_{2l-2,j} + \sigma(j-2l+2)) \\ &\quad - d_{2l-4}^{-1} d_{2l-2} m_l^{-1} k_{l-1} (a_{2l-4,j} + \sigma(j-2l+4)) \\ &\quad (l = 2, 3, \ldots i-1,\ j = 0, 1\,2, \ldots l-1), \\ d_2 &= \frac{k_1}{m_1}, \quad a_{2,0} = \frac{k_1}{m_1}, \quad a_{2,1} = \frac{b_1}{m_1} \end{aligned}$$

with function $\sigma(\bullet)$ defined as

$$\sigma(x) = \begin{cases} 1 & if \quad x = 0 \\ 0 & if \quad x \neq 0. \end{cases} \qquad (46)$$

This closed-form solution for the coefficients in the system transfer function (45) is very convenient for the design of the controllers, adaptive or nonadaptive, model free or model based. For this reason, we would like to share it with control communities.

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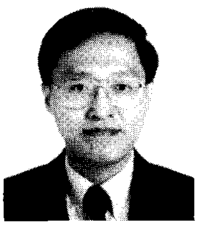

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