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상태변수 시간지연을 갖는 선형시스템의 분수 모델 축소

(A Fractional Model Reduction for Linear Systems with State Delay)

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요 약

본 논문에서는 시변 시간지연을 갖는 선형시스템의 분수 모델 간략화를 다룬다. 이를 위해 선형 시간지연 시스템의 축소된 소인수 분해를 정의하고 선형 행렬부등식의 해를 이용하여 구한다. 축소된 소인수의 일반화 가제어성, 가관측성 그래미안을 이용하여 시스템의 균형화된 상태공간 모델을 구현한다. 모델 차수축소는 균형화된 상태공간 모델의 일부 상태변수를 절삭하여 얻어지며 모델 오차의 상한치를 제시한다. 제안된 방법의 효용성을 수치 예를 통하여 입증한다.

Abstract

This paper deals with a fractional model reduction for linear systems with time varying delayed states. A contractive coprime factorization of linear time delayed systems is defined and obtained by solving linear matrix inequalities. Using generalized controllability and observability gramians of the contractive coprime factor, a balanced state space realization of the system is derived. The reduced model will be obtained by truncating states in the balanced realization and an upper bound of model approximation error is also presented. In order to demonstrate efficacy of the suggested method, a numerical example is illustrated.

Keywords: contractive coprime factorization, model reduction, linear time delayed system, linear matrix inequality

I. Introduction

For linear finite dimensional systems with high orders, optimal control techniques such as LQG and H_∞ control theory, usually produce controllers with the same state dimension as the model. Accordingly the problem of model reduction is of significant practical importance in control system design and has been a focus of a wide variety of studies for decades(see[1-6] and the references, therein).

The stability analysis and control of linear time delayed(LTD) systems are problems of practical and theoretical interest since many types of processes such as steel making process and chemical process can be modeled as dynamic systems with time delay. In the last decade, the linear matrix inequality(LMI) based controller design method for LTD systems has been developed remarkably[7-9]. A drawback of the LMI based controller synthesis is that computational requirements increase rapidly as the state dimension increases. Therefore the state dimension must be kept

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as low as possible. In the last decade, several research works related to approximation of linear systems with uncertain parameters have been performed[5,10,11] But model approximation is mainly focused to quadratically stable systems. Moreover not much works for model reduction of LTD systems have been proposed as far as we know. More recently, a balanced model reduction method for quadratically stable LTD systems is suggested.[12].

This motivates our study for a fractional model reduction of LTD systems. In this paper we introduce a contractive coprime factorization of LTD systems and study an approximation technique based on balanced truncation of the contractive coprime factor. We obtain an approximation model by truncating a part of the state variables of the system's coprime factor. This implies that we are trying to keep the coprime factor of the approximation system closed to the coprime factor of the original time delayed system.

We begin by defining a LTD system and introduce preliminary definitions for model reduction in section II. A contractive right(left) coprime factorization is introduced in section III. Model reduction in the coprime factor of the LTD system is studied in Section IV. Section V gives a numerical example to validate the results developed in the previous sections.

In this paper, the notation is fairly standard. R^n denotes n dimensional real vector spaces and $R^{n\times m}$ means the set of $n\times m$ dimensional real matrices. M^T stands for the transpose of M. 0 and I_n denote the zero matrix and the $n\times m$ dimensional identity matrix respectively. $diag(A,B)$ means the block diagonal matrix $\begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$. In a symmetric block matrix, $*$ in the (i,j) block means the transpose of the submatrix in the (j,i) block. $M<(\le)0$ means that M is negative definite (semi-definite) matrix. Γ_* and Γ_*^{-1} with a subscript $*$ denote the system and the inverse system respectively. Finally $\|\Gamma_P\|_\infty$ denotes the H_∞ norm of the system Γ_P.

II. Linear systems with time delay

We consider the following class of LTD system.

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Fu(t) + Bu(t) \\ z(t) &= Gx(t) \\ y(t) &= Cx(t) + Du(t) \\ w(t) &= \Theta z(t) \end{aligned} \tag{1}$$

where $x(t)\in R^n$ is the state variables, $u(t)\in R^m$ is the control input, $y(t)\in R^p$ is the measured output, $w(t)\in R^k$, $z(t)\in R^k$ are variables related to the time delayed state variables, Θ is a delay operator such that $\Theta z(t)=z(t-d(t))$, $d(t)$ is the time varying delay satisfying

$$0\le d(t)<\infty,\quad |\dot{d}(t)|\le m<1, \tag{2}$$

and A, B, C, D are constant matrices with compatible dimensions. $F\in R^{n\times k}$ and $G\in R^{k\times n}$ are not necessarily full rank matrices.

In a packed matrix notation, we express the LTD system (1) as follows :

$$\Gamma_P = \left[\begin{array}{c|c|c} A & F & B \\ \hline G & 0 & 0 \\ \hline C & 0 & D \end{array}\right] \tag{3}$$

Now we state some definitions for our development.

Definition 1(Bounded Real Lemma) : If there exist $X=X^T>0$, $S=S^T>0$ satisfying LMI (4), $\|\Gamma_P\|_\infty\le\gamma$ is achieved in LTD system (3).

$$\begin{bmatrix} L_{11} & * & * \\ F^TX & -(1-m)S & * \\ B^TX+\gamma^{-2}D^TC & 0 & \gamma^{-2}D^TD-I \end{bmatrix}\le 0 \tag{4}$$

where $L_{11}=A^TX+XA+\gamma^{-2}C^TC+G^TSG$

Definition 2(Generalized Gramian) : Suppose that Γ_P in (3) is quadratically stable.

1. When there exist $Q=Q^T>0$, $R=R^T>0$ satisfying LMI (5), we say that Q is a generalized observability gramian of the system Γ_P

$$L_{\dot{\sigma}}=\begin{bmatrix} A^TQ+QA+C^TC & * & * \\ F^TQ & -(1-m)R & * \\ RG & 0 & -R \end{bmatrix}<0 \tag{5}$$

2. If there exist $P=P^T>0$ and $S=S^T>0$ such that LMI (6) holds, P is a generalized controllability gramian of the system Γ_P

$$L_c:=\begin{bmatrix} PA^T+AP+BB^T & * & * \\ SF^T & -(1-m)S & * \\ GP & 0 & -S \end{bmatrix}<0 \quad (6)$$

Remark : Generalized controllability and obser-vability gramians defined in definition 2 are not unique contrary to the linear time invariant case.

Now we briefly state a balanced model reduction scheme using generalized controllability and observability gramians. Suppose that the system Γ_p in (3) is quadratically stable. Then $Q=Q^T>0$, $R=R^T>0$, $P=P^T>0$ and $S=S^T>0$ can be computed from LMI's (5) and (6). Let nonsingular matrices T and U be such that

$$T^{-1}PT^{-T}=T^TQT=\Sigma=diag(\sigma_1,\sigma_2,\cdots,\sigma_n)$$

$$\sigma_1\geq\sigma_2\geq\cdots\geq\sigma_n \quad (7)$$

$$U^{-1}SU^{-T}=U^TRU=\Pi=diag(\pi_1,\pi_2,\cdots,\pi_k) \quad (8)$$

Using T and U defined in (7) and (8), change of coordinates in Γ_P gives

$$\Gamma_P=\left[\begin{array}{c:c|c} T^{-1}AT & T^{-1}FU & T^{-1}B \\ \hdashline U^{-1}GT & 0 & 0 \\ \hline CT & 0 & D \end{array}\right]$$

$$=:\left[\begin{array}{cc:c|c} A_{b11} & A_{b12} & F_{b1} & B_{b1} \\ A_{b21} & A_{b22} & F_{b2} & B_{b2} \\ \hdashline G_{b1} & G_{b2} & 0 & 0 \\ \hline C_{b1} & C_{b2} & 0 & D_b \end{array}\right] \quad (9)$$

where $A_{b11}\in R^{r\times r}$, $A_{b22}\in R^{(n-r)\times(n-r)}$ and the other matrices are compatibly partitioned.

We obtain an r dimensional reduced order system of Γ_p as follows:

$$\hat{\Gamma}_P=\left[\begin{array}{c:c|c} A_{b11} & F_{b1} & B_{b1} \\ \hdashline G_{b1} & 0 & 0 \\ \hline C_{b1} & 0 & D_b \end{array}\right] \quad (10)$$

It was proved that the approximation error is bounded by $\|\Gamma_p-\hat{\Gamma}_p\|_\infty\leq 2\sum_{i=r+1}^{n}\sigma_i$ [12]

III. Contractive Coprime Factorization

In this section, we extend some of the results on coprime factorization of linear time invariant system to the LTD system. For model reduction, we focus on the contractive coprime factorization which is analo-gous to the normalized coprime factorization of linear time invariant systems.

Definition 3 : Let Γ_P be a LTD system given in (3). Γ_P admits a quadratically stable proper right coprime(left coprime) factorization if there exist quadratically stable LTD systems Γ_X, Γ_Y, Γ_N, Γ_M ($\tilde{\Gamma}_X$, $\tilde{\Gamma}_Y$, $\tilde{\Gamma}_N$, $\tilde{\Gamma}_M$) such that $\Gamma_P=\Gamma_N\Gamma_M^{-1}$, $\Gamma_X\Gamma_N+\Gamma_Y\Gamma_M=I$ ($\Gamma_P=\tilde{\Gamma}_M^{-1}\tilde{\Gamma}_N$, $\tilde{\Gamma}_N\tilde{\Gamma}_X+\tilde{\Gamma}_M\tilde{\Gamma}_Y=I$). Moreover we say that the pair (Γ_N,Γ_M) (($\tilde{\Gamma}_N$, $\tilde{\Gamma}_M$)) is a contractive right coprime(left coprime) factorization if

$$\left\|\begin{matrix}\Gamma_N\\ \Gamma_M\end{matrix}\right\|_\infty\leq 1 \quad (\ \|\tilde{\Gamma}_N\ \ \tilde{\Gamma}_M\|_\infty\leq 1).$$

First, we obtain a coprime factorization of the LTD system Γ_P.

Lemma 1 : Suppose that K and L are quadratically stabilizing state feedback and output injection matrix of the system Γ_P.

Define

$$\begin{bmatrix}\Gamma_N & \tilde{\Gamma}_Y\\ \Gamma_M & \tilde{\Gamma}_X\end{bmatrix}:=\left[\begin{array}{c:c|cc} A+BK & F & B & -L \\ \hdashline G & 0 & 0 & 0 \\ \hline C+DK & 0 & D & I \\ K & 0 & I & 0 \end{array}\right],$$

$$\begin{bmatrix}\Gamma_X & \Gamma_Y\\ \tilde{\Gamma}_M & -\tilde{\Gamma}_N\end{bmatrix}:=\left[\begin{array}{c:c|cc} A+LC & F & L & -(B+LD) \\ \hdashline G & 0 & 0 & 0 \\ \hline K & 0 & 0 & I \\ C & 0 & I & -D \end{array}\right] \quad (11)$$

Then

$$\begin{bmatrix} \Gamma_X & \Gamma_Y \\ \Gamma_{\tilde{M}} & -\Gamma_{\tilde{N}} \end{bmatrix} \begin{bmatrix} \Gamma_N & \Gamma_{\tilde{Y}} \\ \Gamma_M & \Gamma_{\tilde{X}} \end{bmatrix} = I \quad . \tag{12}$$

Proof : A state space realization of the composite system $\Gamma_X\Gamma_N+\Gamma_Y\Gamma_M$ is given by

$$\Gamma_X\Gamma_N+\Gamma_Y\Gamma_M = \left[\begin{array}{c|c|c} A_c & F_c & B_c \\ \hline G_c & 0 & 0 \\ \hline C_c & 0 & D_c \end{array}\right] \tag{13}$$

where

$$A_c=\begin{bmatrix} A+LC & L(C+DK) & 0 & 0 \\ 0 & A+BK & 0 & 0 \\ 0 & 0 & A+LC & -(B+LD)K \\ 0 & 0 & 0 & A+BK \end{bmatrix},$$

$$B_c=\begin{bmatrix} LD \\ B \\ -(B+LD) \\ B \end{bmatrix}, \quad C_c^T=\begin{bmatrix} K^T \\ 0 \\ K^T \\ K^T \end{bmatrix}, \quad D_c=I,$$

$$F_c=diag(F,F,F,F), \quad G_c=diag(G,G,G,G).$$

Define the coordinate transformation matrix T as follows:

$$T=\begin{bmatrix} I & I & I & 0 \\ 0 & I & 0 & -I \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}. \tag{14}$$

Change of coordinate in the composite system $\Gamma_X\Gamma_N+\Gamma_Y\Gamma_M$ gives

$$TA_cT^{-1}= diag\left(\begin{bmatrix} A+LC & (B+LD)K \\ 0 & A+BK \end{bmatrix}, \begin{bmatrix} A+LC & -(B+LD)K \\ 0 & A+BK \end{bmatrix}\right),$$

$$TB_c=[0 \ 0 \ -(B+LD)^T \ B^T]^T,$$

$$C_cT^{-1}=[K \ -K \ 0 \ 0],$$

$$TF_cG_cT^{-1}=diag(FG, FG, FG, FG).$$

Therefore we can conclude that $\Gamma_X\Gamma_N+\Gamma_Y\Gamma_M=I$. Similarly we can obtain $\Gamma_X\Gamma_{\tilde{Y}}+\Gamma_Y\Gamma_{\tilde{X}}=0$, $\Gamma_{\tilde{M}}\Gamma_N-\Gamma_{\tilde{N}}\Gamma_M=0$ and $\Gamma_{\tilde{M}}\Gamma_{\tilde{Y}}-\Gamma_{\tilde{N}}\Gamma_{\tilde{X}}=I$. This completes the proof.

The next theorem states that a contractive coprime factorization of the LTD system Γ_P can be obtained using solutions of LMI's.

Theorem 2 : Suppose that there exist $Z_1=Z_1^T>0$, $R_1=R_1^T>0$, $Z_2=Z_2^T>0$, $R_2=R_2^T>0$ satisfying matrix inequalities (15) and (16).

$$\begin{aligned} &Z_1(A-BD_1^{-1}D^TC)^T+(A-BD_1^{-1}D^TC)Z_1 \\ &\quad -BD_1^{-1}B^T+\frac{1}{1-m}FR_1F^T \\ &\quad +Z_1G^TR_1^{-1}GZ_1+Z_1C^TD_2^{-1}CZ_1<0 \end{aligned}$$

$$\begin{aligned} &(A-BD^TD_2^{-1}C)^TZ_2+Z_2(A-BD^TD_2^{-1}C) \\ &\quad -C^TD_2^{-1}C+G^TR_2G \\ &\quad +\frac{1}{1-m}Z_2FR_2^{-1}F^TZ_2+Z_2BD_1^{-1}B^TZ_2<0 \end{aligned}$$

where $D_1=I+D^TD$, $D_2=I+DD^T$.

Then the pair (Γ_N, Γ_M) $((\Gamma_{\tilde{N}}, \Gamma_{\tilde{M}}))$ is a contractive right coprime(left coprime) factorization of Γ_P where (15)

$$\begin{bmatrix} \Gamma_N \\ \Gamma_M \end{bmatrix} = \left[\begin{array}{c|c|c} A+BK & F & BD_1^{-1/2} \\ \hline G & 0 & 0 \\ \hline C+DK & 0 & DD_1^{-1/2} \\ K & 0 & D_1^{-1/2} \end{array}\right], \tag{16}$$

$$K=-D_1^{-1}(B^TZ_1^{-1}+D^TC) \tag{17}$$

$$\begin{bmatrix} \Gamma_{\tilde{N}} & \Gamma_{\tilde{M}} \end{bmatrix} = \left[\begin{array}{c|c|cc} A+LC & F & B+LD & L \\ \hline G & 0 & 0 & 0 \\ \hline D_2^{-1/2}C & 0 & D_2^{-1/2}D & D_2^{-1/2} \end{array}\right],$$

$$L=-(Z_2^{-1}C^T+BD^T)D_2^{-1/2} \tag{18}$$

Proof : We restrict ourselves to the contractive right coprime case. The proof for the left coprime case can be performed similarily.

With the definition K in (17) and by defining $P_1=Z_1^{-1}$ and $S_1=R_1^{-1}$, matrix inequality (15) is equivalent to the existence of $P_1=P_1^T>0$ and $S_1=S_1^T>0$ satisfying

$$\begin{aligned} &(A+BK)^TP_1+P_1(A+BK)+(C+DK)^T(C+DK) \\ &\quad +K^TK+G^TS_1G+\frac{1}{1-m}P_1FS_1^{-1}F^TP_1<0 \end{aligned} \tag{19}$$

Thus K is a quadratically stabilizing state feedback gain matrix. It is easy to show $\Gamma_P=\Gamma_N\Gamma_M^{-1}$, so will be omitted.

For a quadratically stabilizing output injection matrix L, define quadratically stabilizing LTD systems Γ_X and Γ_Y as follows:

$$[\Gamma_X \quad \Gamma_Y]=\left[\begin{array}{c|c|cc} A+LC & F & L & -(B+LD) \\ \hline G & 0 & 0 & 0 \\ \hline D_1^{1/2}K & 0 & 0 & D_1^{1/2} \end{array}\right]. \quad (20)$$

Then it is obvious that $\Gamma_X\Gamma_N+\Gamma_Y\Gamma_M=I$ from Lemma 1. Now it suffices to prove $\|\Gamma_R\|_\infty\le 1$, where

$$\Gamma_R=\begin{bmatrix}\Gamma_N\\ \Gamma_M\end{bmatrix}=\left[\begin{array}{c|c|c} A+BK & F & BD_1^{-1/2} \\ \hline G & 0 & 0 \\ \hline C+DK & 0 & DD_1^{-1/2} \\ K & 0 & D_1^{-1/2} \end{array}\right]. \quad (21)$$

From definition 1 and using Schur complement, $\|\Gamma_R\|_\infty\le 1$ is equivalent to the existence of $X=X^T>0$ and $S=S^T>0$ satisfying

$$\begin{bmatrix} L_{11} & * \\ L_{21} & D_1^{-1/2}(I+D^TD)D_1^{1/2}-I \end{bmatrix}\le 0 \quad (22)$$

where

$$\begin{aligned} L_{11}=&(A+BK)^TX+X(A+BK)\\ &+(C+DK)^T(C+DK)+K^TK\\ &+G^TSG+\frac{1}{1-m}XFS^{-1}F^TX \end{aligned}$$

$$L_{21}=D_1^{-1/2}(B^TX+D^T(C+DK)+K)$$

We know that $X=P_1$, $S=S_1$ and $K=-D_1^{-1}(B^TP_1+D^TC)$ solve matrix inequality (22). This completes the proof

Ⅳ. Fractional Balanced Model Reduction

Up to now we defined a contractive coprime factorization. In this section, we present a balanced realization and a model reduction scheme in the contractive coprime factor of LTD system Γ_P.

Generalized controllability gramian P and observability gramian Q of the right coprime factor Γ_R can be obtained as solutions satisfying LMI's (23) and (24).

$$L_c:=\begin{bmatrix} L_{c11} & * & * \\ SF^T & -(1-m)S & * \\ GP & 0 & -S \end{bmatrix}<0 \quad (23)$$

$$L_o:=\begin{bmatrix} L_{o11} & * & * \\ F^TQ & -(1-m)R & * \\ RG & 0 & -R \end{bmatrix}<0 \quad (24)$$

where

$$L_{c11}=P(A+BK)^T+(A+BK)P+BD_1^{-1}B^T$$

$$\begin{aligned} L_{o11}=&(A+BK)^TQ+Q(A+BK)\\ &+(C+DK)^T(C+DK)+K^TK \end{aligned}$$

The following lemma states that P, Q, R, S satisfying LMI's (23) and (24) can also be obtained from solutions of matrix inequalities (15) and (16).

Lemma 3 : Let Z_1, R_1, Z_2, R_2 be solutions of (15) and (16). Then $Q=Z_1^{-1}$, $R=R_1^{-1}$, $P=(Z_1^{-1}+Z_2)^{-1}$ and $S=(R_1^{-1}+R_2)^{-1}$ solve LMI's (23) and (24).

Proof : Using $K=-D_1^{-1}(B^TZ_1^{-1}+D^TC)$, $D_2^{-1}=I-DD_1^{-1}D^T$ and Schur complement, LMI (24) is equivalent to the following matrix inequality (25).

$$\begin{aligned} &(A-BD_1^{-1}D^TC)^TQ+Q(A-BD_1^{-1}D^TC)\\ &-Z_1^{-1}BD_1^{-1}B^TQ-QBD_1^{-1}B^TZ_1^{-1}\\ &+Z_1^{-1}BD_1^{-1}B^TZ_1^{-1}+\frac{1}{1-m}QFR^{-1}F^TQ\\ &+G^TRG+C^TD_2^{-1}C<0 \end{aligned} \quad (25)$$

Pre and post multiply Z_1^{-1} to matrix inequality (15) gives

$$\begin{aligned} &(A-BD_1^{-1}D^TC)^TZ_1^{-1}+Z_1^{-1}(A-BD_1^{-1}D^TC)\\ &-Z_1^{-1}BD_1^{-1}B^TZ_1^{-1}+\frac{1}{1-m}Z_1^{-1}FR_1F^TZ_1^{-1}\\ &+G^TR_1^{-1}G+C^TD_2^{-1}C<0 \end{aligned} \quad (26)$$

Comparing (25) with (26), we know that $Q=Z_1^{-1}$ and $R=R_1^{-1}$ solve matrix inequality (25).

Matrix inequality (26) can be expressed as

$$\begin{bmatrix} M_{11} & * \\ F^T Z_1^{-1} & -(1-m)R_1^{-1} \end{bmatrix} < 0 \qquad (27)$$

where

$$M_{11} = (A - BD_1^{-1}D^TC)^T Z_1^{-1} + Z_1^{-1}(A - BD_1^{-1}D^TC) - Z_1^{-1}BD_1^{-1}B^TZ_1^{-1} + G^TR_1^{-1}G + C^TD_2^{-1}C.$$

We rewrite matrix inequality (16) as follows

$$\begin{bmatrix} \overline{M_{11}} & * \\ F^TZ_2 & -(1-m)R_2 \end{bmatrix} < 0 \qquad (28)$$

where

$$\overline{M_{11}} = (A - BD^TD_2^{-1}C)^TZ_2 + Z_2(A - BD^TD_2^{-1}C) - C^TD_2^{-1}C + G^TR_2G + Z_2BD_1^{-1}B^TZ_2$$

Note that $D_1^{-1}D^T = D^TD_2^{-1}$ by the matrix inver-sion lemma. Hence addition of (27) and (28) gives

$$\begin{bmatrix} L_{11} & * \\ F^T(Z_1^{-1}+Z_2) & -(1-m)(R_1^{-1}+R_2) \end{bmatrix} < 0 \qquad (29)$$

where

$$L_{11} = (A+BK)^T(Z_1^{-1}+Z_2) + (Z_1^{-1}+Z_2)(A+BK) + (Z_1^{-1}+Z_2)BD_1^{-1}B^T(Z_1^{-1}+Z_2) + G^T(R_1^{-1}+R_2)G$$

Hence we can conclude that $P = (Z_1^{-1}+Z_2)^{-1}$ and $S = (R_1^{-1}+R_2)^{-1}$ solve LMI (23). This completes the proof. ■

As described in (7) and (8), we compute T and U using P, Q, R, S obtained from lemma 3. With transformation matrices T and U define a balanced realization of the system Γ_R as

$$\Gamma_R = \left[\begin{array}{c|c|c} T^{-1}(A+BK)T & T^{-1}FU & T^{-1}BD_1^{-1/2} \\ \hline U^{-1}GT & 0 & 0 \\ \hline (C+DK)T & 0 & DD_1^{-1/2} \\ KT & 0 & D_1^{-1/2} \end{array}\right]$$

$$=: \left[\begin{array}{cc|c|c} A_{b11}+B_{b1}K_{b1} & A_{b12}+B_{b1}K_{b2} & F_{b1} & B_{b1}D_1^{-1/2} \\ A_{b21}+B_{b2}K_{b1} & A_{b22}+B_{b2}K_{b2} & F_{b2} & B_{b2}D_1^{-1/2} \\ \hline G_{b1} & G_{b2} & 0 & 0 \\ \hline C_{b1}+DK_{b1} & C_{b2}+DK_{b2} & 0 & DD_1^{-1/2} \\ K_{b1} & K_{b2} & 0 & D_1^{-1/2} \end{array}\right] \qquad (30)$$

where $A_{b11} \in R^{r \times r}$, $A_{b22} \in R^{(n-r)\times(n-r)}$ and the other matrices are compatibly partitioned.

Let $\hat{\Gamma}_R$ be the reduced order system with r states obtained by truncating Γ_R. $\hat{\Gamma}_R$ can be described as follows :

$$\hat{\Gamma}_R = \begin{bmatrix} \hat{\Gamma}_N \\ \hat{\Gamma}_M \end{bmatrix} = \left[\begin{array}{c|c|c} A_{b11}+B_{b1}K_{b1} & F_{b1} & B_{b1}D_1^{-1/2} \\ \hline G_{b1} & 0 & 0 \\ \hline C_{b1}+DK_{b1} & 0 & DD_1^{-1/2} \\ K_{b1} & 0 & D_1^{-1/2} \end{array}\right] \qquad (31)$$

Finally we associate $\hat{\Gamma}_R$ the following LTDS with r states.

$$\hat{\Gamma}_P = \left[\begin{array}{c|c|c} A_{b11} & F_{b1} & B_{b1} \\ \hline G_{b1} & 0 & 0 \\ \hline C_{b1} & 0 & D \end{array}\right] \qquad (32)$$

From [12], the following lemma is immediate.

Lemma 4 : The reduced order $\mathcal{T}_R$ is quadratically stable and balanced. Moreover, the reduction error is bounded by

$$\| \Gamma_R - \mathcal{T}_R \|_\infty \le 2 \sum_{i=r+1}^{n} \sigma_i.$$

Remark : Suppose that $\mathcal{T}_K$ is a controller stabilizing reduced order system $\mathcal{T}_P$. Thus $(I + \mathcal{T}_P \mathcal{T}_K)^{-1}\mathcal{T}_P \mathcal{T}_K$ is stable. By the small gain theorem, $(I + \Gamma_P \mathcal{T}_K)^{-1}\Gamma_P \mathcal{T}_K$ is stable if and only if $\| \mathcal{T}_{M^{-1}}(I + \mathcal{T}_K \mathcal{T}_P)^{-1}[\ \mathcal{T}_K\ I] \|_\infty < 1/\| \Gamma_R - \mathcal{T}_R \|_\infty$

.V. Numerical Example

We now describe a numerical algorithm for fractio-nal model reduction. Matrix inequalities (15) and (16) are easily converted to LMI's to solve Z_1, Z_2, R_1 and R_2. But in order to get a less reduction error bound, it is necessary for $\sigma_{r+1}, \cdots, \sigma_n$ to be small. Hence we choose a cost function as $J = trace(PQ) = \sum_{i=1}^{n} \sigma_i^2$. Thus we will minimize the nonconvex cost function subject to convex constraints. This optimization problem is very difficult to solve it. So, we suggest a suboptimal procedure using an iterative method. From the results of lemma 3, the cost function can be written as

$$J = trace(PQ) = trace(I + Z_1^{1/2} Z_2 Z_1^{1/2})^{-1} \\ = trace(I + Z_2^{1/2} Z_1 Z_2^{1/2})^{-1} \qquad (33)$$

step 1 : Set $i=0$. Initialize $Z_{2,i}$ as a positive definite matrix.

step 2 : Set $i=i+1$.

1) Minimize $J_i = trace(I + Z_{2,i-1}^{1/2} Z_{1,i} Z_{2,i-1}^{1/2})^{-1}$ subject to (15).

2) Minimize $J_i = trace(I + Z_{1,i}^{1/2} Z_{2,i} Z_{1,i}^{1/2})^{-1}$ subject to (16).

step 3 : If $|J_i - J_{i-1}|$ is less than a small tolerance level stop iteration, otherwise go to step 2.

We consider a LTD system given by

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & -2 & -1 & -3 \end{bmatrix}, \quad FG = \begin{bmatrix} 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.1 \\ 0 & 0.1 & 0 & 0.1 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad C^T = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad D = 0, \quad |d(t)| \le 0.1$$

Using an initial estimate $Z_{2,0} = I$, we obtain the balanced gramian $\Sigma = diag(0.9853, 0.5217, 0.3749, 0.1085)$. By truncating the last state variable, we obtain the truncated right coprime factor T_R in (31) and the associated LTDS Γ_P in (32), where

$$A_{b11} = \begin{bmatrix} 0.3640 & 0.3833 & 0.0982 \\ -0.0784 & -0.2575 & 0.9385 \\ 0.0060 & -0.9655 & -0.2621 \end{bmatrix},$$

$$F_{b1} = \begin{bmatrix} 0.5574 & 0.0985 & 0.0299 & 0.0002 \\ 0.0105 & 0.3177 & 0.0627 & 0.0000 \\ -0.0079 & -0.0706 & 0.2627 & 0.0003 \end{bmatrix}, \quad B_{b1} = \begin{bmatrix} 1.0846 \\ 0.5881 \\ 0.3938 \end{bmatrix},$$

$$G_{b1} = \begin{bmatrix} 0.0923 & 0.2862 & 0.0885 \\ -0.0067 & 0.1629 & 0.4100 \\ 0.0241 & -0.1308 & 0.1288 \\ -0.0002 & 0.0008 & 0.0000 \end{bmatrix}, \quad C_{b1}^T = \begin{bmatrix} 0.1663 \\ -0.4506 \\ 0.2973 \end{bmatrix},$$

$$K_{b1} = [-1.0687 \quad -0.3068 \quad -0.1477]$$

When $d(t)=1$, the singular value plot of $\Gamma_R - \Gamma_{\overline{R}}$ is given in Fig. 1. Lemma 4 says that an upper bound of reduction error is 0.217. From Fig.1 we know that

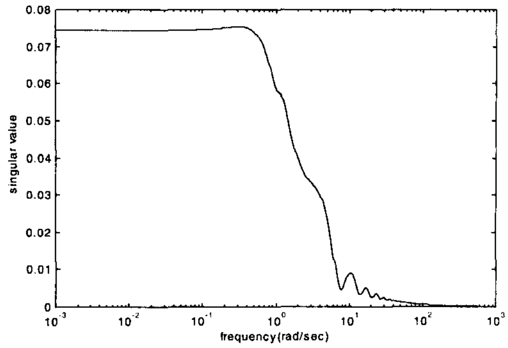

그림 1. 모델축소오차의 특이치.
Fig. 1. Singular value plot of model reduction error.

reduction error is 0.076 when the time delay is 1 second.

Ⅵ. Conclusion

In this paper, we have studied a fractional model reduction for LTD systems. A contractive coprime factorization analogous to the normalized coprime factorization of the linear time invariant systems is derived by solving LMI's. Based on the contractive coprime factor, a balanced realization is obtained from generalized controllability and observability gramians which can be computed from solutions of LMI's wh-ich was also used for a contractive coprime factoriza-tion of LTD systems. However, from a numerical example we observe that the upper bound of reduction error is conservative. The conservativeness may foll-ow from the optimization problem which has the non-covex cost function. But we think that the proposed method may be suitable for model reduction of LTD systems which are not quadratically stable.

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