A Study on Individual Tap-Power Estimation for Improvement of Adaptive Equalizer Performance

Nam-Yong Kim

Abstract

In this paper we analyze convergence constraints and time constant of IT-LMS algorithm and derive a method of making it's time constant independent of signal power by using input variance estimation. The method for estimating the input variance is to use a single-pole low-pass filter(LPF) with common smoothing parameter value, θ. The estimator is with narrow bandwidth for large θ but with wide bandwidth for small θ. This small θ gives long term average estimation(low frequency) of the fluctuating input variance well as short term variations (high frequency) of the input power. In our simulations of multipath communication channel equalization environments, the method with large θ has shown not as much improved convergence speed as the speed of the original IT-LMS algorithm. The proposed method with small θ=0.01 reach its minimum MSE in 100 samples whereas the IT-LMS converges in 200 samples. This shows the proposed, tap-power normalized IT-LMS algorithm can be applied more effectively to digital wireless communication systems.

Key words : Adaptive Equalizer, LMS, IT-LMS, Tap-Power Estimation.

Ⅰ. Introduction

Most digital satellite broadcasting systems and general digital communication systems use Microwave radio as a transmission medium. But radio communication systems are subject to performance degradation associated with multipath propagation. In line-of-sight (LOS) microwave links and satellite communication links the multipath propagation is associated with much of the physical phenomena and many of the meteorological correlates. Layering of the lower atmosphere creates sharp refractive index gradients and, hence, a multiplicity of signal paths with differing relative amplitude and delays[1].

In the early 1950's, propagation research showed that microwave fading could be explained in terms of multipath-transmission and the individual lay paths that occur during fading were detected using a narrow-beam vertically-scanned lens antenna[2]. Additional swept-frequency observations clearly showed the frequency selective nature of multipath propagation and deep fading exhibits a high degree of frequency selectivity. For digital transmission, frequency-selective fading causes severe amplitude and delay distortion which degrades system reliability.

Dispersion due to multipath propagation degrades digital transmission via the generation of inter-symbol interference(ISI). A remedy for these is to equalize a channel for amplitude and delay distortion[3]. Among many equalizer algorithms, IT-LMS algorithm introduced in [4] has fast convergence and simple updating mechanism. By using input variance estimation we make it possible for the IT-LMS algorithm time constant to be independent of the input signal variance. One method for estimating the input variance is to use a single-pole low-pass filter(LPF). We need long term average estimation(low frequency) of the fluctuating input variance well as short term variations(high frequency) of the input power. The IT-LMS using the proposed input variance estimation method shows more rapid convergence speed in our simulations of multipath communication channel equalization environments.

This paper is organized in the following manner. Section Ⅱ presents multipath channel communication systems and equalization methods. Section Ⅲ introduces IT-LMS algorithm briefly and in section Ⅳ convergence constraints of IT-LMS algorithm are analyzed. The tap-power normalized IT-LMS structure is explained in section Ⅴ and experimental results and discussions are presented in section Ⅵ.

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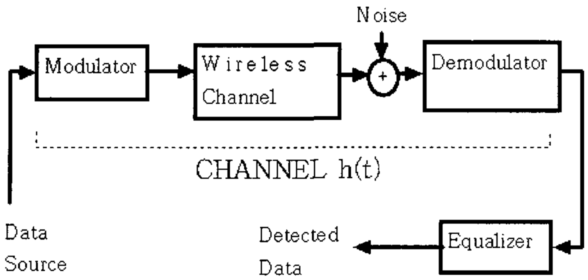

Fig. 1. Typical digital communication system.

II. Multipath Channel and Equalizers

In Fig. 1 a block diagram of a typical digital communication system is shown. We can consider the whole system between the data source and the receiver as a discrete channel with additive noise. the channel impulse response is denoted by $h(t)$.

At the receiver the filtered signal, which is distorted by multipath fading and corrupted by additive noise, w(t). The transfer function of the multipath channel with M elements can be written as [5]

$$H(z)=\sum_{i=0}^{M-1} h_i\, z^{-i} \qquad (1)$$

The equalizer input is sampled every T seconds and this sampled signal, $x(k)$, is presented to the TDL (Tapped Delay Line) equalizer. The output, $y(k)$, which is to be a good approximation to the transmitted symbol $d(k)$. The equalizer input is given by

$$x(k)=\sum_i h_i d(k-i)+w(k) \qquad (2)$$

In the equation (2) $w(k)$ is additive white gaussian noise. The output sample $y(k)$ at time k of the TDL equalizer is

$$y(k)=\sum_{i=0}^{L} c_i(k)x(k-i) = C(k)^T X(k) \qquad (3)$$

where

$$X(k)=[x(k),x(k-1),\ldots,x(k-L)]^T$$

$$C(k)=[c_0(k),\ldots,c_i(k),\ldots,c_L(k)]^T$$

$c_i(k)$ is the i-th tap coefficient

For simplicity, we assume all the values are real. The error related to this symbol becomes

$$e(k)=d(k)-y(k) \qquad (4)$$

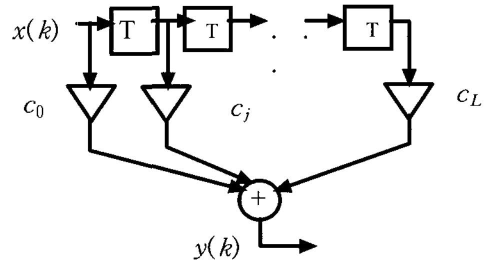

Fig. 2. TDL equalizer structure.

III. Individual-TAP-Controlled-LMS Algorithm (IT-LMS) in TDL Equalizer

Letting i-th coefficient distance $v_i(k)$ be $v_i(k)=c_i(k)-c_i^o(k)$ where $c_i^o(k)$ is the i-th optimum coefficient, MSE, E[$e(k)^2$] can be expressed in terms of the coefficient distance vector $V(k)$.

$$MSE=MSE_{\min}+V(k)^T R\ V(k) \qquad (5)$$

where $V(k)=[v_0(k), v_1(k),\ldots,v_L(k)]^T$,

$R=E[X(k)X(k)^T]$ is input correlation matrix and E[ ] denotes the ensemble averaging operation.

One of its element $r(i-l)$ is defined as $r(i-l)=E[x(k-i)x(k-l)]$. The input correlation matrix R is real, symmetric and positive definite. Arrangement of this equation in terms of $v_i(k)$ gives the following equation.

$$MSE(\ v_i(k)\ )=A\ v_i^2(k)\ +2B\ v_i(k)\ +F \qquad (6)$$

where $A=r(0)$, $B=\sum_{l=0,l\neq i}^{L} v_l(k)\,r(l-i)$ and

$$F=\sum_{l=0,l\neq i}^{L}\ \sum_{j=0,j\neq i}^{L} v_l(k)\,v_j(k)\,r(l-j)\ +\ MSE_{\min}$$

Equation (6) shows that MSE is a parabolic function of each tap coefficient in one dimensional space; i.e., when one tap changes while all others are kept constant, the locus of MSE becomes a parabolic function in terms of the i-th coefficient. This locus, which is unique for each tap coefficient at each state of the channel, is called the characteristic function of that tap coefficient[6].

Based on these characteristics, an iterative algorithm for adjusting the equalizer coefficients one by one can be developed. The algorithm uses the steepest descent method to update the i-th tap coefficient, holding all other coefficients constant. The process continues for the other tap coefficients at time k and all the L+1 tap coefficients are updated. As the time k increases, the coefficient vector will be approach the Wiener optimum $C^o(k)$.

The gradient of $v_i(k)$ with all others fixed is presented in equation (7),

$$\begin{aligned}\frac{\partial MSE}{\partial v_i} &= 2E[x(k-i)x(k-i)](c_i(k)-c_i^o) \\ &\quad + 2\sum_{n=0,\, n\neq i}^{L} E[x(k-n)x(k-i)](c_n(k)-c_n^o) \\ &= 2E[x(k-i)(\sum_{n=0}^{L} x(k-n)c_n(k) - \sum_{n=0}^{L} x(k-n)c_n^o)] \\ &= -2E[x(k-i)e_i(k)] \qquad (7)\end{aligned}$$

In updating the i-th tap coefficient while holding all others fixed, the steepest descent using the measured or estimated gradient, $-2x(k-i)e_i(k)$, can be used. This is known as an implementation of the steepest descent using the measured or estimated gradient[7]:

$$new\ c_i(k) = c_i(k) + 2\mu\ e_i(k)x(k-i) \qquad (8)$$

where $e_i(k)$ is calculated when all other coefficients are fixed. The parameter μ is a step size (convergence factor) that controls stability and rate of adaptation.

Using the samples taken from the TDL and some past samples, the i-th tap coefficient can be updated N+1 times at sample time k. Introducing index j, (8) can become (9).

$$\begin{aligned} c_i(k, j+1) &= c_i(k,j) + 2\mu\ e_i(k-N+j)x(k-N+j-i) \\ 0 \le j < N,&\quad c_i(k,N) = c_i(k+1) \qquad (9)\end{aligned}$$

Continuing this process from tap $i=0$ to tap $i=L$, all the tap coefficients are updated one by one and the output sample $y(k)$ is made from the TDL equalizer. For vector representations, a new input vector $X(k-N+j)$ and coefficient vector $C_i(k,j)$ are defined as

$$\begin{aligned} X(k-N+j) = [&x(k-N+j), x(k-N+j-1), \\ &\ldots, x(k-N+j-L)]^T \qquad (10)\end{aligned}$$

$$C_i(k,j) = [c_0(k,j), \ldots, c_i(k,j), \ldots, c_L(k,j)]^T \qquad (11)$$

where $c_l(k,j)$ for $l<i$ are already in updated state, only $c_i(k,j)$ changes and $c_p(k,j)$ for $p>i$ are to be updated.

Then the temporary output $y_i(k,j)$ and error $e_i(k-N+j)$ during $c_i(k,j)$ adaptation are expressed as

$$y_i(k,j) = X^T(k-N+j)C_i(k,j) \qquad (12)$$

$$e_i(k-N+j) = d(k-N+j) - y_i(k,j) \qquad (13)$$

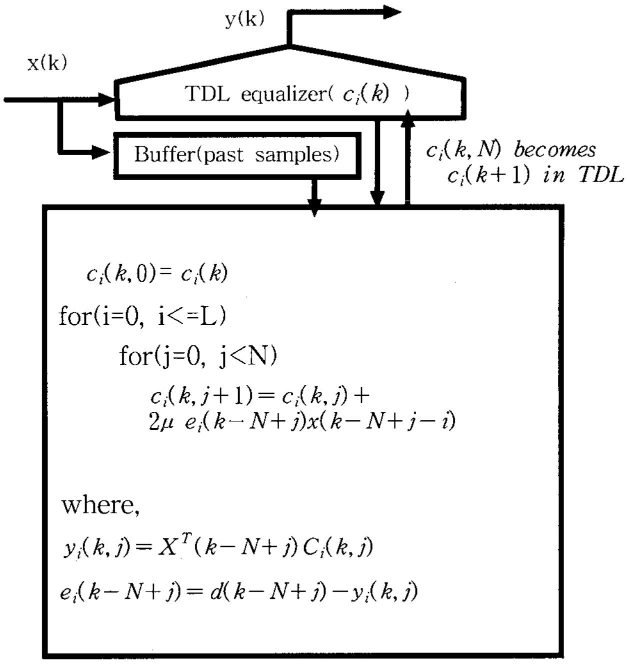

Fig. 3. IT-LMS algorithm.

Also IT-LMS algorithm (9) can be expressed as the following vector form.

$$C_i(k,j+1) = C_i(k,j) + 2\mu(\ d(k-N+j) - X^T(k-N+j)C_i(k,j)\ )\begin{bmatrix}0\\ \vdots \\ 0\\ x(k-N+j-i)\\ 0\\ \vdots\\ 0\end{bmatrix} \qquad (14)$$

The operation of the algorithm at time k can be summarized as equation (9), (12) and (13).

A drawback of IT-LMS algorithm is some burden of computation. The computational complexity of the IT-LMS algorithm is presented in [8].

IV. Convergence Constraints of IT-LMS

The adaptation equation of i-th tap coefficient, (14), can be rewritten as (15).

$$C_i(k,j+1) = C_i(k,j) + 2\mu X^T(k-N+j)[C^o - C_i(k,j)]\begin{bmatrix}0\\ \vdots \\ 0\\ x(k-N+j-i)\\ 0\\ \vdots\\ 0\end{bmatrix} \qquad (15)$$

We initially consider the updating process of the first coefficient, i=0, then we can obtain the i-th coefficient updating equation by inductively expanding the update process.

$$C_0(k,j+1)=C_0(k,j)+2\mu X^T(k-N+j)[C^o-C_0(k,j)]\cdot\begin{bmatrix} x(k-N+j)\\ 0\\ \cdot\\ \cdot\\ \cdot\\ 0\end{bmatrix} \tag{16}$$

We can rearrange the equation (16) as following;

$$C_0(k,j+1)=[1-2\mu\ x(k-N+j)\ X^T(k-N+j)]C_0(k,j) + 2\mu\ x(k-N+j)\ X^T(k-N+j)C^o \tag{17}$$

For the i-th tap coefficient the updating equation can be expressed as

$$C_i(k,j+1)=[1-2\mu\ x(k-N+j-i)\ X^T(k-N+j)]C_i(k,j) + 2\mu\ x(k-N+j-i)\ X^T(k-N+j)C^o \tag{18}$$

Taking E[.] of the equation (18), we obtain

$$E[C_i(k,j+1)]=[1-2\mu\ r^T(i)]\cdot E[C_i(k,j)] + 2\mu\ r^T(i)C^o \tag{19}$$

where

$$r^T(i)=[r(i),r(i-1),\dots,r(1),r(0),r(1),\dots,r(L-i)]$$

and (20)

$$r(i-l)=E[x(k-N+j-i)x(k-N+j-l)]$$

Equation (19) can become (21) by rewriting it with the elements $r(i)$.

$$\begin{aligned} E[C_i(k,j+1)] = E[C_i(k,j)] &-2\mu\sum_{n=0}^{i-1} r(i-l)(c_l(k,j)-c_l^o)\\ &-2\mu\ r(0)(c_i(k,j)-c_i^o)\\ &-2\mu\sum_{p=i+1}^{L} r(p-i)(c_p(k,j)-c_p^o)\end{aligned} \tag{21}$$

The tap coefficients are updated one by one, so, in (21), $c_l(k,j)$ for $l<i$ are already in updated state, whereas $c_p(k,j)$ for $p>i$ are to be updated. When i=0, $c_l(k,j)$ for $1\le l\le L$ are not updated yet and when i=1, $c_0(k,j)$ is in updated state but $c_l(k,j)$ for $2\le l\le L$ are not updated yet. For the $c_0(k,j)$, equation (21) becomes

$$\begin{aligned} E[C_0(k,j+1)] = E[C_0(k,j)] &-2\mu\ r(0)(c_0(k,j)-c_0^o)\\ &-2\mu\sum_{p=1}^{L} r(p-i)(c_p(k,j)-c_p^o)\end{aligned} \tag{22}$$

The elements of $C_0(k,j+1)$ on the left side and $C_0(k,j)$ on the right side are the same except for $c_0(k,j)$ which is in the updating process. From (21), defining $w_i(k,j)$ as $E[c_i(k,j)-c_i^o]$ and update-finished $w_i(k)$ as $E[c_i(k,N+1)-c_i^o]$, we can acquire for i=0, 1.

$$\begin{aligned} w_0(k,j+1) = w_0(k,j) &-2\mu\ r(0)w_0(k,j)\\ &-2\mu\sum_{p=1}^{L} r(p-i)\ w_p(k,j)\end{aligned} \tag{23}$$

$$\begin{aligned} w_1(k,j+1) = [1-2\mu r(0)]w_1(k,j) &-2\mu\ r(1)w_0(k)\\ &-2\mu\sum_{p=2}^{L} r(p-i)\ w_p(k,j)\end{aligned} \tag{24}$$

From these results we can generalize it to the i-th tap coefficient distance equation.

$$\begin{aligned} w_i(k,j+1) &= w_i(k,j)-2\mu\sum_{n=0}^{i-1} r(i-l)w_l(k)-2\mu\ r(0)w_i(k,j) -2\mu\sum_{p=i+1}^{L} r(p-i)\ w_p(k-1)\\ &= -2\mu\sum_{n=0}^{i-1} r(i-l)w_l(k) +(1-2\mu\ r(0))w_i(k,j) -2\mu\sum_{p=i+1}^{L} r(p-i)\ w_p(k-1)\end{aligned} \tag{25}$$

Noticing equation (9) where the i-th tap coefficient can be updated N ($0\le j<N$) times at sample time k using the current and some past samples. the equation (25) can be expressed a power series.

$$\begin{aligned} w_i(k,j+1) = (1-2\mu\ r(0))^{j+1}\ w_i(k,0) &-[\sum_{n=0}^{i-1}\frac{r(i-l)}{r(0)}w_l(k)+ \sum_{p=i+1}^{L}\frac{r(p-i)}{r(0)}w_p(k-1)]\\ &\cdot[1-(1-2\mu r(0))^{j+1}]\end{aligned} \tag{26}$$

Defining $\beta=1-2\mu\ r(0)$ and $a_{p-i}=\frac{r(p-i)}{r(0)}$, the equation (26) becomes

$$w_i(k,j+1)-\beta^{j+1}\ w_i(k,0) = -[\sum_{n=0}^{i-1} a_{i-l}w_l(k)+\sum_{p=i+1}^{L} a_{p-i}w_p(k-1)]\cdot[1-\beta^{j+1}] \tag{27}$$

where $w_i(k,0)$ can be replaced with $w_i(k-1,N)$. When $j=N-1$, we finish the adaptation of the i-th tap coefficient as expressed in (28).

$$w_i(k,N)-\beta^N w_i(k-1,N)= -[1-\beta^N]\cdot[\sum_{n=0}^{i-1} a_{i-n}w_n(k)+\sum_{p=i+1}^{L} a_{p-i}w_p(k-1)] \quad (28)$$

Also,

$$w_i(k+1)-\beta^N w_i(k)= -[1-\beta^N]\cdot[\sum_{n=0}^{i-1} a_{i-n}w_n(k)+\sum_{p=i+1}^{L} a_{p-i}w_p(k-1)] \quad (29)$$

Equation (29) is stable if and only if $|\beta|=|1-2\mu\, r(0)|<1$. This condition can be also expressed as

$$0<\mu<\frac{1}{r(0)} \quad (30)$$

As mentioned earlier this algorithm updates the i-th tap coefficient, holding all other coefficients constant. So, we can define the right side of (30) $[1-\beta^N][\sum_{n=0}^{i-1} a_{i-n}w_n(k)+\sum_{p=i+1}^{L} a_{p-i}w_p(k-1)]$ as a constant K during updating the i-th tap coefficient. Taking the Z-transform of the equation (29) with respect to k yields the transfer function $W(i, z)$ and time constant τ_i given by

$$W(i,z)=\frac{z^{-1}}{1-\beta^N z^{-1}}$$

$$\tau_i=\frac{-1}{\ln(\beta^N)}=\frac{-1}{\ln([1-2\mu r(0))^N]} \simeq \frac{1}{2\mu r(0)} \quad (31)$$

This means the convergence speed of i-th tap coefficient is dependent of signal power of i-th tap.

V. Tap-Power-Normalized IT-LMS

Convergence condition of the IT-LMS equalizer in the last section can be summarized as (30), (31). For some applications we might need the IT-LMS equalizer algorithm whose time constant τ is independent of the input signal variance $\sigma^2_{x,i}$, i,e, $r(0)$. Now if we define the convergence parameter as

$$\mu=\lambda/\sigma^2_{x,i} \quad (32)$$

the time constant τ_i becomes independent of $\sigma^2_{x,i}$, i.e.,

$$\tau_i=1/2\lambda \quad (33)$$

Substituting (32) into (29), the sufficient condition for the algorithm to converge is given by

$$0<\lambda<1 \quad (34)$$

Assuming that the variance of the reference input signal changes slowly, one common method for estimating the variance for the i-th tap is to use a single-pole low-pass filter, i.e.,

$$\sigma^2_{x,i}(k+1)=\theta\ \sigma^2_{x,i}(k)+(1-\theta)\ x^2(k-i) \quad (35)$$

where $0<\theta<1$ is a smoothing parameter which controls the bandwidth and time constant of the variance-estimation system $S(z)$ with its input $x^2(k-i)$.

$$S(z)=(1-\theta)\frac{z}{z-\theta} \quad (36)$$

Now we can summarize the proposed, tap-power normalized IT-LMS algorithm, as

$$c_i(k,j+1)=c_i(k,j)+\frac{2\lambda}{\sigma^2_{x,i}(k)}\ e_i(k-N+j)x(k-N+j-i) \quad (37)$$

where,

$$\sigma^2_{x,i}(k)=\theta\sigma^2_{x,i}(k)+(1-\theta)\ x^2(k-i) \quad (38)$$

$$y_i(k,j)=X^T(k-N+j)\,C_i(k,j) \quad (39)$$

$$e_i(k-N+j)=d(k-N+j)-y_i(k,j) \quad (40)$$

VI. Simulation Results

Its performance has been investigated in multipath channel equalization applications through computer simulations and compared. For multipath channel environments we used discrete time-dispersive channels which are shown in [5]. The channel impulse responses are

$$H_1(z)=0.26\ +\ 0.93z^{-1}\ +\ 0.26z^{-2} \quad (41)$$

$$H_2(z)=0.304\ +\ 0.903z^{-1}\ +\ 0.304z^{-2} \quad (42)$$

The spectral characteristics for the channels in Fig. 4 possess high frequency nulls as typical spectral characteristics of multipath fading channels[5]. The number of coefficient taps, L, for the TDL equalizer is 11 on the channel model. AWGN variance is 0.001 and SNR=30 dB. The convergence parameter 2μ = 0.02 as in [5] and the smoothing parameter θ is considered with values (0.01~0.99). The number of past samples for multiple updating, N, is 6.

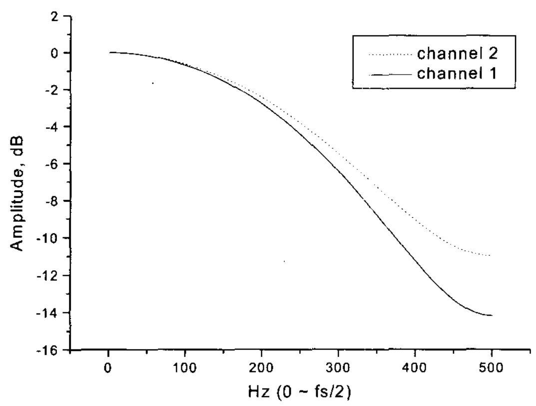

Fig. 4. Convergence performances.

In our simulations of multipath communication channel equalization environments, IT-LMS algorithm has shown fast convergence but we can speed up its convergence by using the proposed, tap-power normalized IT-LMS algorithm whose time constant τ is independent of the input signal variance.

For various θ we have shown their convergence results. From the results, for large θ, the variance-estimation system $S(z)$ has small high-frequency components and large low-frequency components. For small θ, the variance-estimation system $S(z)$ has more of high-frequency components, the instant i-th tap signal power is more reflected in the i-th tap coefficient updating process. This gives long term average estimation (low frequency) of the fluctuating input variance well as short term variations (high frequency) of the input power. In Fig. 5, the proposed, small θ, has shown more rapid convergence speed. The original IT-LMS converges after 200 samples and the proposed, large θ=0.70, reaches minimum MSE in 150. The proposed, small θ=0.01, converges in 100 samples. Also we have investigated it's performance in SNR=33 dB for channel model 1 and 2 in Fig. 6 and 7, respectively. In the cases of channel 1, 2 of SNR=33 dB, we have similar results except that we can acquire larger performance increases in severer channel environments. In Fig. 7, IT-LMS algorithm converges in about 400 samples whereas the proposed method with theta=0.1 converges in about 150 samples.

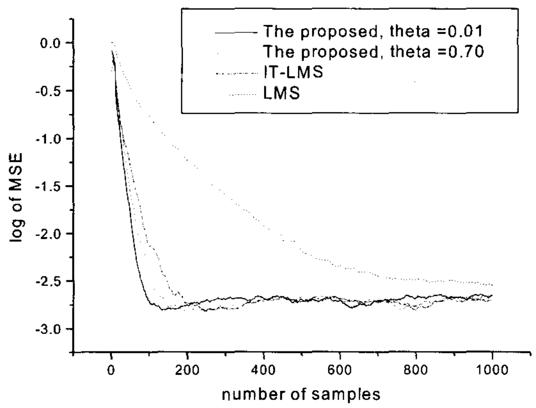

Fig. 5. Convergence performances for channel 1 and SNR = 30 dB.

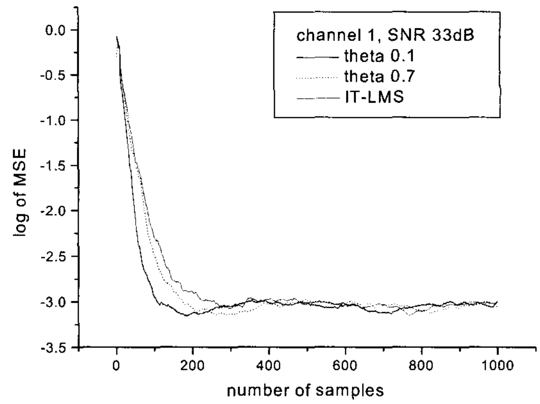

Fig. 6. Convergence performances for channel 1, SNR = 33 dB.

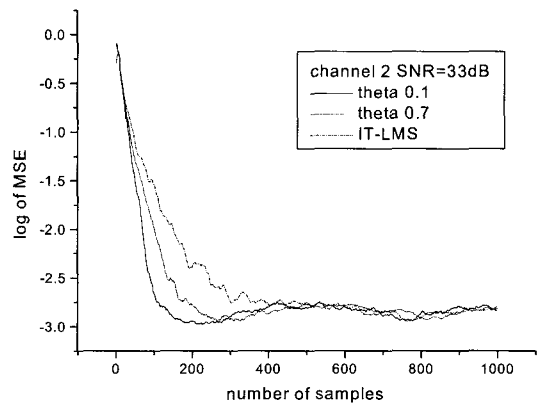

Fig. 7. Convergence performances for channel 2, SNR = 33 dB.

VII. Conclusions

A method of improving MSE convergence speed of the IT-LMS algorithm is investigated. By using input variance estimation we can make the IT-LMS algorithm's i-th tap coefficient time constant independent of the input signal variance. We analyze convergence constraints and time constant of IT-LMS algorithm and derive a method of making it's time constant independent of signal power by using input variance estimation. The method for estimating the input variance is to use a single-pole low-pass filter (LPF) with common smoothing parameter value, θ. The estimator is with narrow bandwidth for large θ but with wide bandwidth for small θ. This gives long term average estimation (low frequency) of the fluctuating input variance well as short term variations (high frequency) of the input power. In our simulations of multipath communication channel equalization environments, the method with large θ has shown not as much improved convergence speed as the speed of the original IT-LMS algorithm. The original IT-LMS and the method with θ=0.70 converge around 200 samples but the proposed method with small θ=0.01 reach its minimum MSE in 100 samples. This shows the proposed, tap-power normalized IT-LMS algorithm can be applied more effectively to digital wireless communication systems.

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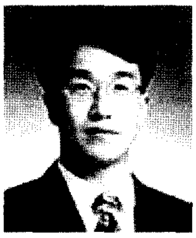

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