Journal of the Korean Society of Marine Engineers, Vol.28, No.2, pp.200~209, 2004(ISSN 1226-9549)

Implementation of Hardware Circuits for Fuzzy Controller Using α-Cut Decomposition of fuzzy set

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(Manuscript : Received NOV 25, 2003 ; Revised DEC 30, 2003)

Abstract : The fuzzy control based on α-level fuzzy set decomposition. It is known to produce quick response and calculating time of fuzzy inference.

This paper derived the embodiment computational algorithm for defuzzification by min-max fuzzy inference and the center of gravity method based on α-level fuzzy set decomposition. It is easy to realize the fuzzy controller hardware, based on the calculation formula.

In addition, this study proposed a circuit that generates PWM actual signals ranging from fuzzy inference to defuzzification. The fuzzy controller was implemented with mixed analog-digital logic circuit using the computational fuzzy inference algorithm by min-min-max and defuzzification by the center of gravity method. This study confirmed that the fuzzy controller worked satisfactorily when it was applied to the position control of a dc servo system.

Key words : α-level fuzzy set, PWM, the fuzzy controller hardware

1. Introduction

Fuzzy control reflects experts' experiential control knowledge obtained from control in the past, thus it can be applied in complicated plant control and other similar areas.

In the case when fuzzy inference operation is processed based on software in fuzzy control, the operation takes time. Therefore software-based fuzzy control is not suitable for systems that require quick responses. Therefore, servo control systems must implement its fuzzy controller based on hardware. And also there has been research on high-speed hardware-based fuzzy logic processing [1], [2].

Hardware scheme is largely divided into digital circuits and analog circuits.

A digital circuit for a fuzzy controller is good for fuzzy logics and often process operational functions such as min-max, which have been difficult to process using

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conventional calculators, on CPU. On the other hand, an analog circuit represents values of membership functions using analogue quantities such as voltage and current to process min · max operation on an electronic circuit. The method is suitable when the cardinality is small in digital data of the union set [3], [4].

When fuzzy control by an analogue circuit is used in a control loop, it is possible until the value of membership function can be obtained from continuous values entered, but after then, it is difficult to perform operation continuously, and consequently digital data of the union set must consider the speed of operation [5], [6].

In fuzzy control, inference by α-level set quantifies the membership function using two values, so it is easy to implement the operation in hardware. In addition, because a group of α-level sets can be processed in parallel, it is possible to perform high-speed operation.

Generally in a convex fuzzy set of real numbers, α-level becomes the interval and two values represent the upper and lower limits. When the upper and lower limits are dealt with as analogue values using this expression.

If the number of quantitative steps is too high when a membership function is quantitative α-level, the operation circuit becomes large. Thus it is desirable to make the number of quantitative steps subjectively determined small.

This paper presented embodiment algorithm that calculates from fuzzy inference to defuzzification in an integrative way using the α-level decomposition of fuzzy sets.

In addition, it examined α-cut number of a membership function optimal for realize of a fuzzy controller circuit by input-output characteristics of a fuzzy controller and the simulation of unit step responses. In addition, this study proposed a method of constructing a fuzzy controller operation circuit based on the derived calculation algorithm. It implemented a mixed analog-digital fuzzy controller circuit that generates PWM actual signals by fuzzy inference and defuzzification in a DC servo system. It was verified that power control is easy in the controller and the controller is useful for DC servo control.

2. Fuzzy control by α-cut decomposition

2.1 Fuzzy inference by α-level set

In our system, the fuzzy inference algorithm is a min-min-max type and the ith rule can be written as follows:

$$R_i : \textit{if } e \textit{ is } A_i \textit{ and } \Delta e \textit{ is } B_i \textit{ then } u \textit{ is } C_i \quad (i=1,\cdots,n) \tag{1}$$

Where e and Δe are the input variable, u is the ith output variable, A_i, B_i and C_i are fuzzy sets. Here, fuzzy inference algorithm by the min-min-max algorithm can be formally written as eq. (2).

$$\begin{aligned} \mu_c{}'(u) &= \bigvee_{i=1}^{n} [\mu_{Ai}(e) \wedge \mu_{Bi}(\Delta e)] \wedge \mu_{Ci}(u) \\ &= \bigvee_{i=1}^{n} \mu_{Ci}{}'(u) \end{aligned} \tag{2}$$

Here, μ_F: Membership function of

fuzzy set F

Formula (3) is the F_α level set of fuzzy set F. It is also called α-cut.

$$F_\alpha = \{x \mid \mu_F(x) \geq \alpha\} \ , \alpha \in [0, 1] \qquad (3)$$

Here, $0 \langle \alpha \langle 1$. eq. (2) is transformed into eq. (4) using the α-level set of eq. (3).

$$\begin{aligned} C'_\alpha &= \{u \mid \mu_{C'}(u) \geq \alpha\} \\ &= \bigcup_{i=1}^{n} \{u \mid \mu_{C'_i}(u) \geq \alpha\} \\ &= \bigcup_{i=1}^{n} C'_{i\alpha} \end{aligned} \qquad (4)$$

Where, C'_i represents inference results of each control rule

C'_α represents a final inference results

$C'_{i\alpha}$ represents results of α-cut inference of each rule.

If the number of control rules is m, and the number of α-cut is n (i = 1, 2,....... , m, = 1, 2,..... , n). Inference results of each control rule can be formally written as eq. (5).

$$\begin{aligned} C'_{i\alpha} &= \{u \mid (\mu_{Ai}(e) \wedge \mu_{Bi}(\Delta e)) \wedge \mu_{Ci}(u) \geq \alpha\} \\ &= \begin{cases} C'_{i\alpha} & ,\mu_{Ai}(e) \wedge \mu_{Bi}(\Delta e) \geq \alpha \\ \phi & otherwise \end{cases} \\ &= \begin{cases} C'_{i\alpha} & ,e \in A_{i\alpha} \ and \ \Delta e \in B_{i\alpha} \\ \phi & ,otherwise \end{cases} \end{aligned} \qquad (5)$$

Here, $A_{i\alpha}$, $B_{i\alpha}$, $C'_{i\alpha}$ and C'_α are α-level fuzzy sets, assuming a convex condition, the α-level set of a convex fuzzy set F is a closed interval $[l_{F\alpha}, r_{F\alpha}]$, so the operation above can be substituted with interval end point operation.

2.2 Deffuzification by the center of gravity

The center of gravity can be represented as eq. (6) by using a α-cut set.

$$u = \frac{\int_M u \, d\alpha du}{\int_M 1 d\alpha du} = \frac{\int_0^1 \left[\int_{C_\alpha'} u \, du \right] d\alpha}{\int_0^1 \left[\int_{C_\alpha'} 1 du \right] d\alpha}$$

$$[\ M = \{(\alpha, u) \mid u \in C_\alpha' \}] \qquad (6)$$

Here, u represents a definite integral, the integral interval of which is the whole set of U. Also, if C′ is taken as a convex fuzzy set, calculation can be written as follows:

$$C'_\alpha = [l_\alpha, r_\alpha] \qquad (7)$$

l_α and r_α represents the leftmost value and rightmost value of the a-level fuzzy set.

The final crisp output value u is obtained eq. (8)

$$u = \frac{\int_0^1 1/2(r_\alpha^2 - l_\alpha^2) d\alpha}{\int_0^1 (r_\alpha - l_\alpha) d\alpha} = \frac{N}{D} \qquad (8)$$

Here, D and N stand for denominator and nominator Formula (8). We can see that this form is realized to easy with electrical circuits.

3. Evaluation by simulation

3.1 Characteristics of input and output

Fuzzy inference is performed on the basis of control rules in Table 1 and membership functions in Fig. 1.

Table 1 Control rules.

		Δe				
		NB	NS	ZE	PS	PB
E	PB	PB				
	PS	NM	ZE	PS	PM	PB
	ZE	NB	NS	ZE	PS	PB
	NS	NB	NM	NS	ZE	PM
	NB					

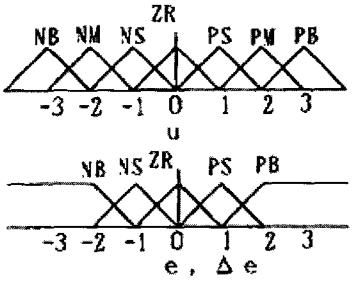
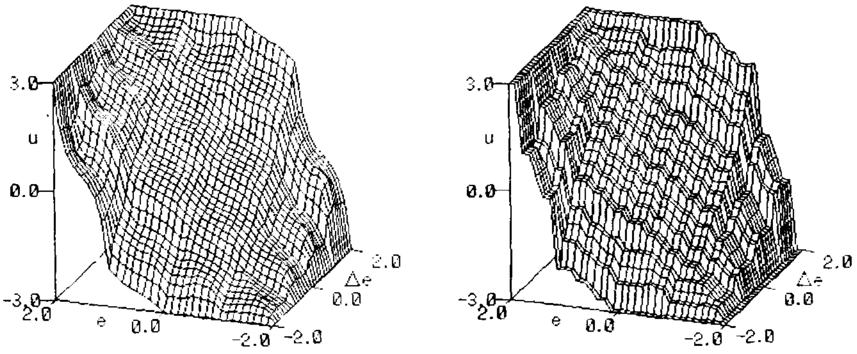


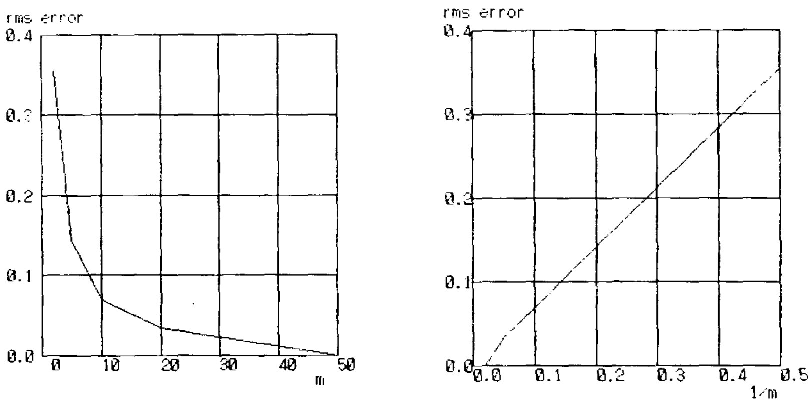
Fig. 1 Membership functions for variables (normalized to fit in [-3, 3]).

Fig. 2 shows the result of the simulation of the fuzzy inference relationship between input e and Δe and



(a) (b)

Fig. 2 Input-output characteristics. ((a) α =50step, (b) α=4 step).



(a) (b)

Fig. 3 Calculation of errors.

output u. Fig. (a) and Fig. (b) show a case when the number of quantitative steps is 50 and 4 respectively. By the effects of quantitative, the control surface of output is flatness and almost linear in the 50-step quantitative, but it looks like a stair with many shifts in the 4-step quantitative.

Fig. 3 is the result of calculating [rms] errors by comparing outputs from different numbers of α-cut with α-cut 50 steps. Fig. (a) is calculation errors of m-step quantitative compared with 50-step quantitative, which shows that the error is large when α-cut is less than 4 steps. It is possibly because calculation errors are determined by the width and shape of quantitative of the level of membership functions.

Fig. (b) is calculation errors of 1/m-step quantitative compared with 50-step quantitative. The quantitative step for 1/m is close to a straight line. The result shows what consideration should be made in 1/m-step quantitative.

3.2 Position response of servo system

A servo system is position control by PWM of DC servomotor using a fuzzy controller. The transfer function of a servo system is eq. (9).

$$G(s) = \frac{K_m}{s(1+T_m s)} \tag{9}$$

Here, motor gain constant $K_m = 5$, and mechanical time constant $T_m = 0.5[\mathrm{sec}]$. Fuzzy controller input $e = (\theta^* - \theta_r) \times \pi[rad]$, its change is Δe, and sampling cycle $\Delta T = 0.05[\mathrm{sec}]$. In addition, actual variable u is voltages.

Fig. 4 shows the result of position response simulation of a dc servomotor with 4-step and 50-step quantitative of the membership function. According to the result, the difference is largely insignificant. As presented above, the input-output characteristics of a fuzzy controller and the position response of a servo system indicate that the number of α-cut steps of quantizer must be over a certain level.

In case hardware is used in this study, the optimal number of α-cut steps is 4 considering the simplicity and stability of the circuit.

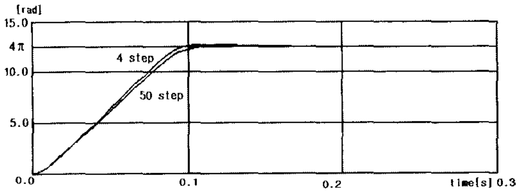


Fig. 4 Position response by simulation.

4. Fuzzy controller hardware implementation

4.1 Hardware schemes

Fig. 5 is a hardware scheme of fuzzy controller for motor drive. The fuzzy controller is composed of a fuzzy inference block and defuzzification block, and the inference block includes an input circuit, control rule circuit and membership function circuit. The defuzzification block includes PWM circuit to obtain PWM actual signals. The α-level division of the fuzzy controller is of four steps, $\alpha =$ 0.8, 0.6, 0.4, 0.2 (voltage division). The inference block is composed of four identical circuits for the parallel processing of each step.

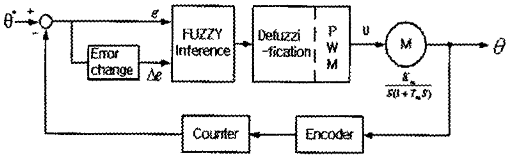


Fig. 5 Hardware scheme of fuzzy controller for motor drive.

(1) Fuzzy inference block

(a) The input circuit obtains analogue signals of 0~5V with counter output of DAC0800, and takes $e=\theta^*-\theta_r$ and its change Δe as the input signals of the controller.

(b) A membership function circuit in Fig. 1 marks the end points of an interval as with voltage in each a-level set, and set the voltage of each point with a voltage divider circuit. Under the condition of Formula (5), output is decided depending on whether the input falls within the interval comparing it with the ending points of α-level set. This is realized using a wide-range comparator (C339). Fig. 6 shows the composition circuit antecedent membership function.

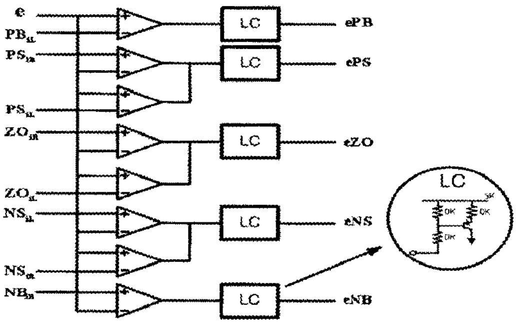


Fig. 6 Composition circuit of antecedent membership function.

(c) Fuzzy rule circuit: Fuzzy inference selects what to adopt among 17 rules through combining input 2-digit signals (labels) with logical product by corresponding to control rules in Table 1 according to Formula (5). Then the voltage of the label of membership functions in the antecedent part of each rule adopted is obtained.

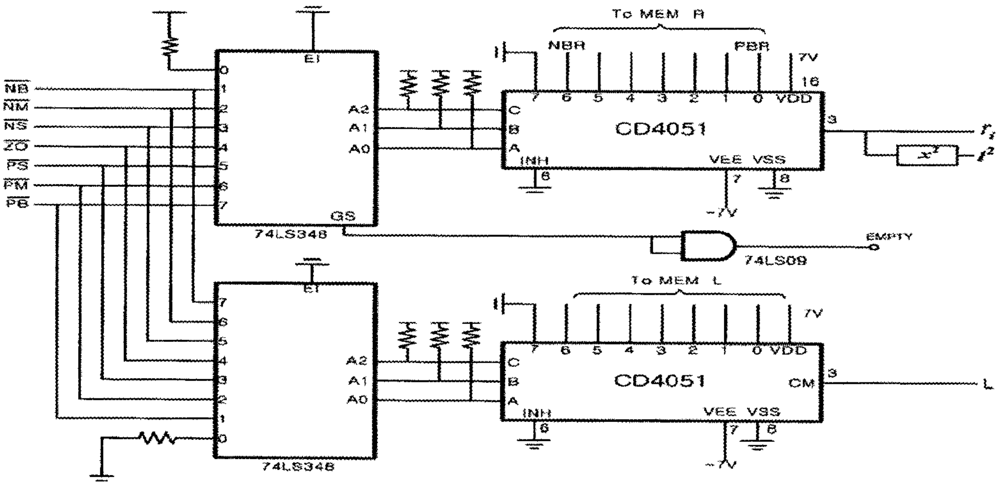


Fig. 7 A circuits of final result for l_α and l_α^2 in eq. (7).

Membership functions in the precedent part obtain each label with voltage of 0~5V using an analogue switch (priority encoder 74LS348) as in Fig. 7.

Fuzzy inference is min · max, and the final result is a trapezoid-shaped set that takes as its two ends the minimum and maximum values among end values (voltage) of membership functions. Composition resulting from inference in Formula (5) is to find a union set, so the min operation is implemented in the lower limit, and the mix operation is implemented in the upper limit. The dimensional relationship between the upper limit and lower limit among precedent α-level sets is determined in the antecedent membership functions.

Therefore, the end point voltage of corresponding labels in the precedent part in an analogue multiplexed (4501) is selected by encoding labels obtained from a priority encoder 74LS348.

Fig. 7 is a circuit for final composition result to obtain l_α and l_α^2 in eq. (7), the left end values of α- level fuzzy sets resulting from the composition of min · max inference. A circuit to obtain the right end values of α-level fuzzy sets is identical.

Fig. 8 is processing circuits so that composition resulting from inference in eq. (5) is empty set.

(2) Defuzzification block

The defuzzification block is a circuit to obtain center values, which are a defuzzified value and a actual variable of motor input voltage. Center value u needs a division in Formula (8) but it is recommended to generate PWM actual signals directly without a division.

Fig. 9 shows the principle of obtaining PWM actual signals by the input of numerator and denominator in Formula (8). The principle is to generate triangle wave with an amplitude proportional to denominator D, compare it with numerator N using a comparator, and produce the average output of voltage in proportion to N /D. An integrator and hysteresis comparator are used to

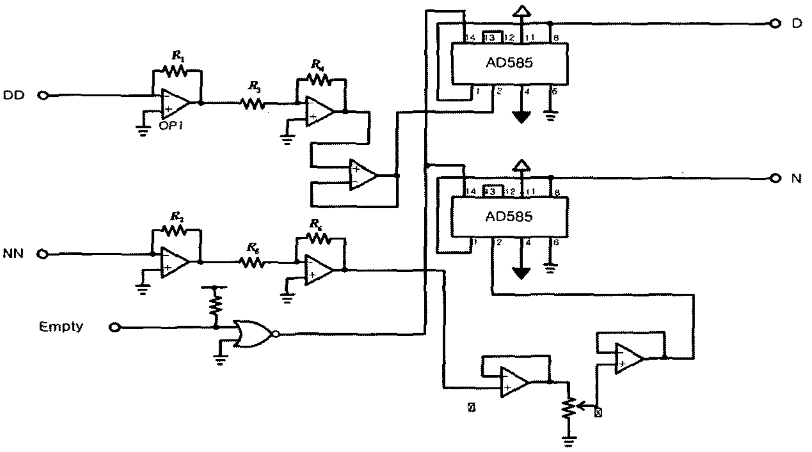


Fig. 8 Processing circuit of empty set in eq. (5).

generate triangle wave. Because the gradient of pyramidal wave is proportional to denominator D, the switching frequency is maintained constant.

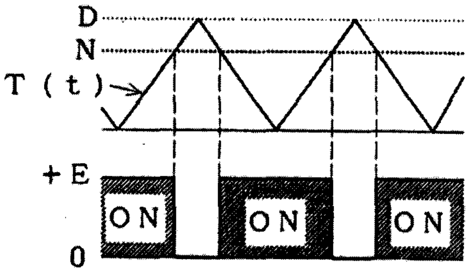


Fig. 9 Principle of PWM generation by $N \div D$.

Fig. 10 is a circuit for final composition result to obtain numerator N and denominator D in eq. (8)

Fig. 11 is a circuit to obtain PWM actual signals directly without a division based on the principle in Fig. 9.

4.2 Experiment of hardware-based implementation

Fig. 12 is the input/output characteristics of a hardware-based fuzzy controller and Fig. 13 is the result of position control of a dc servomotor using the controller. The input / output

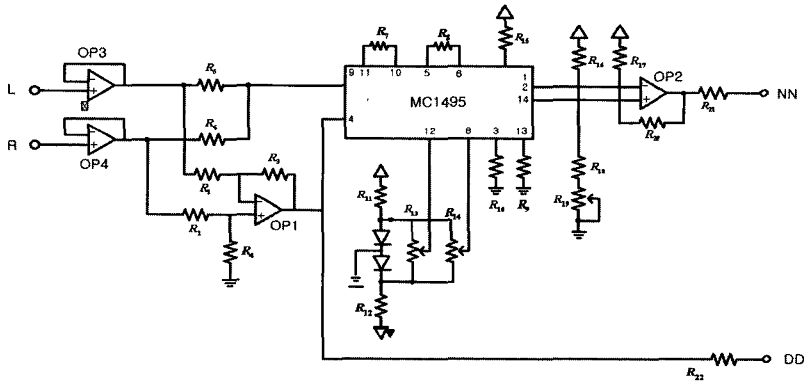


Fig. 10 Circuit of numerator N and denominator D.

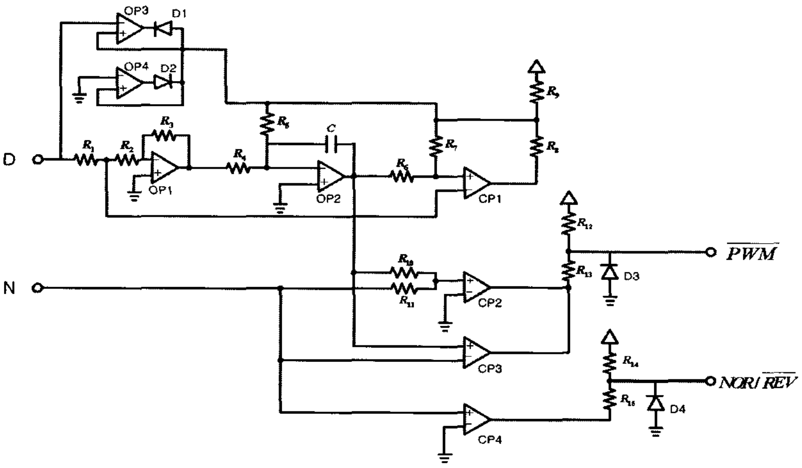


Fig. 11 Circuit for actual PWM signal generation by $N \div D$.

characteristics are identical with those from simulation, and the position control response does not show any stationary deviation and is stable without overshoot.

Because the fuzzy controller used the center method in defuzzification, the output of the controller is the average of each α-level, so the controller works even if the circuits of some levels are removed. However, in the experiment using one circuit of the highest α-level, a relatively large vibration occurred and the operation was not normal. The phenomenon is possibly caused by the fact that, because the α-level set is narrow and the input space is not sufficiently large, output from any control rules is negative.

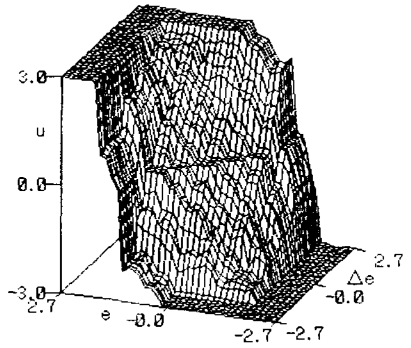


Fig. 12 Input-output characteristics of fuzzy controller.

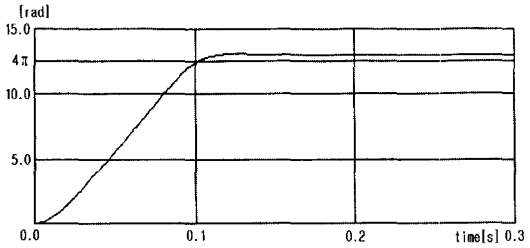


Fig. 13 Experimental result of the position control.

5. Conclusions

The present study derived the computational algorithms by min・max fuzzy inference and the center method based on α-level fuzzy sets to make it easy to create the circuit of a fuzzy controller, and based on the calculation formula, implemented fuzzy controller hardware.

In addition, this study proposed a method of generating PWM actual signals ranging from fuzzy inference to defuzzification in the fuzzy control of a dc servo system. According to the result of simulation, the quantitative α-cut of membership function necessary for hardware of fuzzy controller is four steps. The operation of the fuzzy controller was verified through an experiment of a dc servo system.

If a fuzzy controller is implemented as an analogue circuit using this method, it is applicable directly to almost every control rule.

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