

## 2nd-order PD-type Learning Control Algorithm

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### 요 약

In this paper are proposed 2nd-order PD-type iterative learning control algorithms for linear continuous-time system and linear discrete-time system. In contrast to conventional methods, the proposed learning algorithms are constructed based on both time-domain performance and iteration-domain performance. The convergence of the proposed learning algorithms is proved. Also, it is shown that the proposed method has robustness in the presence of external disturbances and the convergence accuracy can be improved. A numerical example is provided to show the effectiveness of the proposed algorithms.

**Key words** : Iterative Learning Control, Robustness, Convergence, Iteration-Domain Performance

### 1. 서 론

Ever since Arimoto suggested iterative learning control(ILC) methodology, there have been a number of efforts to improve and apply ILC method. In fact, ILC can be easily applied to the repetitive tasks since it requires less a priori knowledge about the controlled system in the controller design phase and it has the capability of modifying an unsatisfactory control input signal based on the knowledge of previous operations of the same task [1-12]. Also, ILC is known to guarantee an eventual uniform tracking performance as the algorithm repetitively applies.

External disturbances such as state disturbances, measurement noise are inevitable in the real control systems. These disturbances can make the system diverge by its iterative property. Therefore the robustness problem of ILC has been studied by many researchers. Heinzinger et al. have studied the robustness properties of a class of learning control algorithm for the nonlinear system [11]. Saab proved the convergence and the robustness of both P-type learning control for the nonlinear time varying system and D-type learning control for the linear discrete-time system[13, 15]. Bien and Hur proposed the higher-order ILC method that utilize more than one past error generated at prior iterations [7]. The higher-order ILC can improve the convergence performance and the robustness to the disturbances by using the multiple past-history data pairs at the expense of additional storage. However, this method can be applied to the dynamic system that has the direct linkage between the input and the output and there may arise some difficulty

in finding the suitable weighting matrices satisfying the convergence conditions, especially when the number of past-history data pairs is large [7, 14].

In this paper, we propose 2nd-order PD-type ILC algorithms based on both time-domain performance and iteration-domain performance for linear continuous-time system and linear discrete-time system. The control law based on the iteration-domain performance can improve the robustness to the disturbances by using the past-history data pair like high-order ILC algorithm [7]. The convergence of the proposed algorithms is proved. A numerical example is given to show that the proposed method has robustness in the presence of the external disturbances and the convergence performance is changed according to parameter change.

In the sequel, the following notational convention is adopted :  $k$  is the iteration number;  $x(t)$ ,  $x(i)$  are state vectors,  $u(t)$ ,  $u(i)$  are control input vectors and  $y(t)$ ,  $y(i)$  are output vectors for continuous and discrete-time systems respectively;  $I_r$  is  $r \times r$  identity matrix;  $\|x\|$  denotes the Euclidean norm of a vector  $x$ ;  $\|A\|$  denotes the induced matrix norm of a matrix  $A$ ; and the following norms are defined.

**Definition 1** We define the  $\lambda_c$  norm for a time function  $f : [0, T] \rightarrow R^n$

$$\|f(\cdot)\|_{\lambda_c} = \sup_{t \in [0, T]} e^{-\lambda t} \|f(t)\|$$

where  $\lambda > 0$ .

**Definition 2** We define the  $\lambda_d$  norm for a time function  $g : [0, M] \rightarrow R^n$

$$\|g(\cdot)\|_{\lambda_d} = \sup_{i \in [0, M]} e^{\lambda i} \|g(i)\|$$

where  $\lambda > 0$  if  $a > 1$  and  $\lambda < 0$  if  $a < 1$ .

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**Remark 1** From above definitions, it is obvious that  $\|f\|_{\lambda_c} \leq \|f\|_{\infty} \leq e^{\lambda T} \|f\|_{\lambda_c}$  and  $\|f\|_{\lambda_d} \leq \|f\|_{\infty} \leq e^{\lambda T} \|f\|_{\lambda_d}$ . These inequalities imply that the defined  $\lambda_c$ ,  $\lambda_d$  norm and  $\|\cdot\|_{\infty}$  norm are equivalent [17]. Therefore, the convergence can be proved employing the defined  $\lambda_c$  norm and  $\lambda_d$  norm.

## 2. 2nd-order PD-type ILC for linear continuous-time system

In this section, we present a 2nd-order ILC algorithm for linear continuous-time systems. Consider the linear time-invariant dynamical system described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \quad (1) \\ y(t) &= Cx(t) \end{aligned}$$

where  $x \in R^n$ ,  $u \in R^r$  and  $y \in R^r$  denote the state vector, input vector and output vector respectively.  $A$ ,  $B$  and  $C$  are constant matrices with appropriate dimensions. It is assumed that  $CB$  is nonsingular.

Let  $x_d$  be the desired state trajectory which is continuously differentiable on  $[0, T]$  and assume that

$$x_d(0) \equiv 0. \quad (2)$$

Then we consider an ILC algorithm based on both time-domain performance and iteration-domain performance. At first, we consider PD-type learning law in the time domain such as Oh *et al.*[6], Bien *et al.*[8] and Hwang *et al.*[9] as follows :

$$u_{k+1}(t) = u_k(t) + \Gamma[\delta\dot{y}_k(t) + \Lambda\delta y_k(t)] \quad (3)$$

where  $\Gamma$  and  $\Lambda$  are learning parameters, and

$$\dot{x}_k(t) = Ax_k(t) + Bu_k(t) \quad (4)$$

$$y_k(t) = Cx_k(t),$$

$$y_d(t) = Cx_d(t) \quad (5)$$

$$\delta y_k(t) = y_d(t) - y_k(t) \quad (6)$$

Also, we consider PD-type control law in the iteration domain as follows :

$$u_{k+1}(t) = u_k(t) + \Phi(\delta y_k(t) - \Theta\delta y_{k-1}(t)). \quad (7)$$

where  $\Phi$  and  $\Theta$  are learning parameters. Above iteration-domain control law uses the past-history data pairs like higher-order ILC algorithm [7] We propose a new PD-type control law of the form

$$\begin{aligned} u_{k+1}(t) &= u_k(t) + \Gamma[\delta\dot{y}_k(t) + \Lambda\delta y_k(t) \\ &+ \Phi(\delta y_k(t) - \Theta\delta y_{k-1}(t))]. \end{aligned} \quad (8)$$

If it is assumed that

$$\|I_r - \Gamma CB\| \leq \rho < 1 \quad (9)$$

and

$$y_k(0) = y_d(0) \equiv 0, \quad k = 0, 1, 2, \dots, \quad (10)$$

Arimoto's control law can make the error between  $y_k(t)$  and  $y_d(t)$  approach to zero as  $k \rightarrow \infty$  [1]. In this paper, we assume the conditions (9) and (10).

**Theorem 1** Suppose that we can choose  $\Gamma$  such that (9) and (10) holds, and that the learning law (8) is repetitively applied to (1). Then, for a given desired output  $y_d(t)$ ,  $0 \leq t \leq T$ , the learning law (8) guarantees that for each  $t \in [0, T]$ ,

$$\lim_{k \rightarrow \infty} y_k(t) = y_d(t). \quad (11)$$

**Proof**

Let  $u^*(t)$  be a control input such that

$$y_d(t) = Ce^{At}x_0 + C \int_0^t e^{A(t-\tau)} Bu^*(\tau) d\tau. \quad (12)$$

where  $x_0 = x_d(0)$ . The proof is completed if one can show  $\lim_{k \rightarrow \infty} u_k(t) = u^*(t)$ . For this, let us define

$$\delta u_k(t) \triangleq u^*(t) - u_k(t). \quad (13)$$

Then it follows from (8) and (12) that

$$\begin{aligned} \delta u_{k+1}(t) &= u^*(t) - u_k(t) \Gamma [\delta\dot{y}_k(t) + \Lambda\delta y_k(t) \\ &+ \Phi(\delta y_k(t) - \Theta\delta y_{k-1}(t))] \\ &= u^*(t) - u_k(t) \Gamma [\delta\dot{y}_k(t) \\ &+ (\Lambda + \Phi)\delta y_k(t) - \Phi\Theta\delta y_{k-1}(t)] \\ &= (I - \Gamma CB)\delta u_k(t) \\ &- \Gamma CA \int_0^t e^{A(t-\tau)} B\delta u_k(\tau) d\tau \\ &- \Gamma(\Lambda + \Phi)C \int_0^t e^{A(t-\tau)} B\delta u_k(\tau) d\tau \\ &+ \Gamma\Phi\Theta C \int_0^t e^{A(t-\tau)} B\delta u_{k-1}(\tau) d\tau \\ &= (I - \Gamma CB)\delta u_k(t) - \Gamma(CA + \Lambda C \\ &+ \Phi C) \int_0^t e^{A(t-\tau)} B\delta u_k(\tau) d\tau \\ &+ \Gamma\Phi\Theta C \int_0^t e^{A(t-\tau)} B\delta u_{k-1}(\tau) d\tau \end{aligned} \quad (14)$$

Taking the norm  $\|\cdot\|$  on both side of (14), we have

$$\begin{aligned} \|\delta u_{k+1}(t)\| &\leq \|I - \Gamma CB\| \cdot \|\delta u_k(t)\| \\ &+ \|\Gamma(CA + \Lambda C + \Phi C)\| \\ &\cdot \int_0^t \|e^{A(t-\tau)}\| \|B\| \|\delta u_k(\tau)\| d\tau \\ &+ \|\Gamma\Phi\Theta C\| \\ &\cdot \int_0^t \|e^{A(t-\tau)}\| \|B\| \|\delta u_{k-1}(\tau)\| d\tau \\ &+ h_0 \int_0^t e^{a(t-\tau)} \|\delta u_k(\tau)\| d\tau \\ &+ h_1 \int_0^t e^{a(t-\tau)} \|\delta u_{k-1}(\tau)\| d\tau \end{aligned} \quad (15)$$

where  $\rho \triangleq \|I - \Gamma CB\|$ ,  $h_0 \triangleq \|\Gamma(CA + \Lambda C + \Phi C)\| \cdot$

$\|B\|$ ,  $h_1 \triangleq \|\Gamma\Phi\Theta C\| \cdot \|B\|$ ,  $a \triangleq \|A\|$ .

By multiplying both side of (15) by  $e^{-\lambda t}$  and taking the norm  $\|\cdot\|_{\lambda}$ ,

$$\begin{aligned} \|\delta u_{k+1}(t)\|_{\lambda} &= \sup_{t \in [0, t]} e^{-\lambda t} \|\delta u_{k+1}(t)\| \\ &\leq \rho \|\delta u_k(t)\|_{\lambda} \\ &\quad + h_0 \sup_{t \in [0, T]} \int_0^t e^{-(a-\lambda)(t-\tau)} \\ &\quad \sup_{\tau \in [0, T]} e^{-\lambda \tau} \|\delta u_k(\tau)\| d\tau \\ &\quad + h_1 \sup_{t \in [0, T]} \int_0^t e^{-(a-\lambda)(t-\tau)} \\ &\quad \sup_{\tau \in [0, T]} e^{-\lambda \tau} \|\delta u_{k-1}(\tau)\| d\tau \\ &= (\rho + h_0 \frac{1-e^{-(a-\lambda)T}}{\lambda-a}) \|\delta u_k(t)\|_{\lambda} \\ &\quad + (h_1 \frac{1-e^{-(a-\lambda)T}}{\lambda-a}) \|\delta u_{k-1}(t)\|_{\lambda}, \end{aligned} \quad \text{for } \lambda \neq a. \quad (16)$$

Now, we can show that  $\lim_{k \rightarrow \infty} \|\delta u_k(t)\|_{\lambda} = 0$ , if

$$[\rho + h_0 \frac{1-e^{-(a-\lambda)T}}{\lambda-a}] + [h_1 \frac{1-e^{-(a-\lambda)T}}{\lambda-a}] < 1. \quad (17)$$

Noting that the inequality (16) can be represented by a non-negative sequence  $x_k$  with the property

$$x_{k+2} \leq r x_{k+1} + s x_k, \quad k=1, 2, 3, \dots, \quad (18)$$

where  $r, s > 0$  and the convergence condition is equivalent to the condition that eigenvalues of  $x_{k+2} = r x_{k+1} + s x_k$  are all in the unit circle in the complex plane, we can easily show that the above sequence converges to zero if  $r+s < 1$  holds.

Since  $0 \leq \rho < 1$  by assumption, it is possible to choose  $\lambda$  sufficiently large so that

$$\rho + h_0 \frac{1-e^{-(a-\lambda)T}}{\lambda-a} + h_1 \frac{1-e^{-(a-\lambda)T}}{\lambda-a} < 1. \quad (19)$$

Thus,

$$\lim_{k \rightarrow \infty} \|\delta u_k(t)\|_{\lambda} = 0.$$

By definition of  $\|\cdot\|_{\lambda}$ , this implies

$$\lim_{k \rightarrow \infty} u_k(t) = u^*(t).$$

This completes the proof.

Theorem 1 shows that the proposed learning algorithm (8) guarantees convergence of the output in tracking as  $k$  increases.

**Remark 2** The proposed algorithm (8) looks more complex than 1st-order methods [1, 2, 8, 9, 6, 16]. However, it gives more freedoms for adjustment of both convergence speed and tracking accuracy. With  $\Phi=0$ , the proposed method is much the same as the learning law proposed by Oh *et al.*[6], Bien *et al.*[8] and Lee

*et al.*[16]. Also, when  $\Phi=0$  and  $\Lambda=0$ , the proposed method is essentially the same as the learning law proposed by Arimoto *et al.*[1]. Thus, the proposed method can be considered as a generalization of the previous works [1, 6, 8, 16]. The convergence conditions in theorem 1 are similar to the previous works [1, 6, 8, 16] and more simple than the higher-order ILC method [7].

### 3. 2nd-order PD-type ILC for linear discrete-time system

Consider the linear discrete time dynamical system described by

$$\begin{aligned} x(i+1) &= Ax(i) + Bu(i) \\ y(i) &= Cx(i) \end{aligned} \quad (20)$$

where  $x \in R^n, u \in R^r$  and  $y \in R^r$  denote the state vector, input vector and output vector respectively.  $A, B$  and  $C$  are constant matrices with appropriate dimensions. It is assumed that  $CB$  is nonsingular.

Then the 2nd-order PD type ILC control law for the system(20) can be described as follows :

$$u_{k+1}(i) = u_k(i) + \Gamma [\delta y_k(i+1) + \Lambda \delta y_k(i) + \Phi(\delta y_k(i) - \Theta \delta y_{k-1}(i))] \quad (21)$$

where  $\Gamma, \Lambda, \Phi$  and  $\Theta$  are learning parameters and

$$\delta y_k(i) = y_d(i) - y_k(i) \quad i=0, 1, \dots, N. \quad (22)$$

We assume that

$$\|I_r - \Gamma CB\| \leq \rho < 1, \quad (23)$$

and

$$y_k(0) = y_d(0) \equiv 0, \quad k=0, 1, 2, \dots. \quad (24)$$

**Theorem 2** Suppose that we can choose  $\Gamma$  such that (23) and (24) holds, and that the learning law (21) is repetitively applied to (20). Then, for a given desired output  $y_d(i), i=0, 1, \dots, N$ , the learning law (21) guarantees that for each  $i \in [0, N]$ ,

$$\lim_{k \rightarrow \infty} y_k(i) = y_d(i). \quad (25)$$

**Proof**

Let  $u^*(i)$  be a control input such that

$$y_d(i) = CA^i x_0 + C \sum_{j=0}^{i-1} A^{i-j-1} B u^*(j). \quad (26)$$

where  $x_0 = x_d(0)$ . The proof is completed if  $\lim_{k \rightarrow \infty} u_k(i) = u^*(i)$ . For this, we define

$$\delta u_k(i) \triangleq u^*(i) - u_k(i). \quad (27)$$

Then it follows from (21) and (26) that

$$\begin{aligned}
 \delta u_{k+1}(i) &= u^*(i) - u_k(i) \Gamma [\delta y_k(i+1) + \Lambda \delta y_k(i) \\
 &\quad + \Phi (\delta y_k(i) - \Theta \delta y_{k-1}(i))] \\
 &= u^*(i) - u_k(i) \Gamma [\delta y_k(i+1) \\
 &\quad + (\Lambda + \Phi) \delta y_k(i) - \Theta \delta y_{k-1}(i)] \\
 &= (I - \Gamma C B) \delta u_k(i) \\
 &\quad - \Gamma C A \sum_{j=0}^{i-1} A^{i-j-1} B \delta u_k(j) \\
 &\quad - \Gamma (\Lambda + \Phi) C \sum_{j=0}^{i-1} A^{i-j-1} B \delta u_k(j) \\
 &\quad + \Gamma \Phi \Theta C \sum_{j=0}^{i-1} A^{i-j-1} B \delta u_{k-1}(j) \quad (28) \\
 &= (I - \Gamma C B) \delta u_k(i) - \Gamma (C A + \Lambda C \\
 &\quad + \Phi C) \sum_{j=0}^{i-1} A^{i-j-1} B \delta u_k(j) \\
 &\quad + \Gamma \Phi \Theta C \sum_{j=0}^{i-1} A^{i-j-1} B \delta u_{k-1}(j) \quad (29)
 \end{aligned}$$

Taking the norm  $\|\cdot\|$  on both side of (29), we have

$$\begin{aligned}
 \|\delta u_{k+1}(i)\| &\leq \|I - \Gamma C B\| \cdot \|\delta u_k(i)\| \\
 &\quad + \|\Gamma (C A + \Lambda C + \Phi C)\| \\
 &\quad \cdot \sum_{j=0}^{i-1} \|A^{i-j-1}\| \|B\| \|\delta u_k(j)\| \\
 &\quad + \|\Gamma \Phi \Theta C\| \\
 &\quad \cdot \sum_{j=0}^{i-1} \|A^{i-j-1}\| \|B\| \|\delta u_{k-1}(j)\| \\
 &= \rho \|\delta u_k(i)\| + h_0 \sum_{j=0}^{i-1} a^{i-j-1} \|\delta u_k(j)\| \\
 &\quad + h_1 \sum_{j=0}^{i-1} a^{i-j-1} \|\delta u_{k-1}(j)\| \quad (30)
 \end{aligned}$$

where  $\rho \triangleq \|I - \Gamma C B\|$ ,  $h_0 \triangleq \|\Gamma (C A + \Lambda C + \Phi C)\| \cdot \|B\|$ ,  
 $h_1 \triangleq \|\Gamma \Phi \Theta C\| \cdot \|B\|$ ,  $a \triangleq \|A\|$ .

By multiplying both side of (30) by  $a^{-\lambda i}$  and taking the norm  $\|\cdot\|_{\lambda_d}$ ,

$$\begin{aligned}
 \|\delta u_{k+1}(i)\|_{\lambda_d} &= \sup_{i \in [0, M]} a^{-\lambda i} \|\delta u_{k+1}(i)\| \\
 &\leq \rho \|\delta u_k(i)\|_{\lambda_d} + h_0 \sup_{i \in [0, M]} a^{-(\lambda-1)i} \\
 &\quad \cdot \sum_{j=0}^{i-1} a^{(\lambda-1)j} \sup_{j \in [0, M]} a^{-\lambda j} \|\delta u_k(j)\| \\
 &\quad + h_1 \sup_{i \in [0, M]} a^{-(\lambda-1)i} \\
 &\quad \cdot \sum_{j=0}^{i-1} a^{(\lambda-1)j} \sup_{j \in [0, M]} a^{-\lambda j} \|\delta u_{k-1}(j)\| \\
 &\leq (\rho + h_0 \frac{1 - a^{-(\lambda-1)N}}{a^{\lambda-1} - 1}) \|\delta u_k(t)\|_{\lambda_d} \\
 &\quad + (h_1 \frac{1 - a^{-(\lambda-1)N}}{a^{\lambda-1} - 1}) \|\delta u_{k-1}(t)\|_{\lambda_d}. \quad (31)
 \end{aligned}$$

Now, we can show that  $\lim_{k \rightarrow \infty} \|\delta u_k(i)\|_{\lambda_d} = 0$ , if

$$\rho + h_0 \frac{1 - a^{-(\lambda-1)N}}{a^{\lambda-1} - 1} + [h_1 \frac{1 - a^{-(\lambda-1)N}}{a^{\lambda-1} - 1}] < 1. \quad (32)$$

Since  $0 \leq \rho < 1$  by assumption, it is possible to choose  $\lambda$  sufficiently large so that

$$\rho + h_0 \frac{1 - a^{-(\lambda-1)N}}{a^{\lambda-1} - 1} + h_1 \frac{1 - a^{-(\lambda-1)N}}{a^{\lambda-1} - 1} < 1. \quad (33)$$

Thus,

$$\lim_{k \rightarrow \infty} \|\delta u_k(i)\|_{\lambda_d} = 0.$$

By definition of  $\|\cdot\|_{\lambda_d}$ , this implies

$$\lim_{k \rightarrow \infty} u_k(i) = u^*(i).$$

This completes the proof.

**Remark 3** With  $\Phi=0$ , the proposed method is the same as the learning law proposed by Hwang *et al.* [9] and with  $\Phi=0$  and  $\Lambda=0$ , it is essentially the same as the learning law proposed by Saab [15]. Therefore the proposed method can be considered as a generalization of the previous works [9, 15]. Also, the convergence conditions in theorem 2 are similar to the previous works [9, 15].

### 4. Simulation Example

In the following, we shall consider linear continuous time-invariant dynamic system [16]

$$\begin{aligned}
 \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w(t) \\
 y(t) &= [0 \ 1] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + 0.3 v(t). \quad (34)
 \end{aligned}$$

Also, suppose the desired output trajectory is given by

$$y_d(t) = 12t(1-t) \quad 0 \leq t \leq 1$$

and let

$$y_k(0) = y_0 = 0 \quad k=0, 1, 2, \dots$$

Let us assume that  $CB=1.3$  and  $\Gamma$  is chosen as 0.7 based on the condition (9).  $A$  is chosen as 0.7 based on the condition (19). As shown in Figure 1, the output  $y(t)$  approaches the desired output  $y_d(t)$  as the learning law (8) is repetitively applied. The result in Figure 2 shows  $\sum_{k=1}^{10} \int_0^T |e_k(t)| dt$  according to the parameters,  $\Phi$  and  $\Theta$ .

Figure 3 shows  $\int_0^T |e_{10}(t)| dt$  according to the parameters,  $\Phi$  and  $\Theta$  at 10th iteration. We can show that the tracking performance tends to depend on the choice of learning parameters and can improve them by choosing the suitable parameters,  $\Lambda$ ,  $\Phi$  and  $\Theta$ . The proposed method is more free to adjust both convergence speed and tracking accuracy. Figure 4 and Figure 5 shows that the proposed method is robust to the state disturbance and measurement noise that are random numbers whose elements are normally distributed with mean 0 and variance 1.

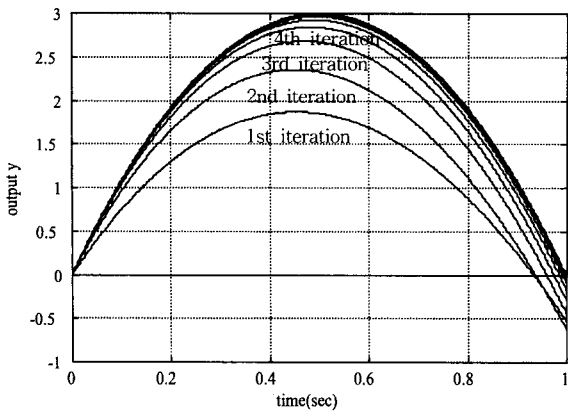


Figure 1. Output trajectories when  $\Gamma=0.7$   
 $\Lambda=0.7$ ,  $\Phi=1.0$ ,  $\Theta=0.7$

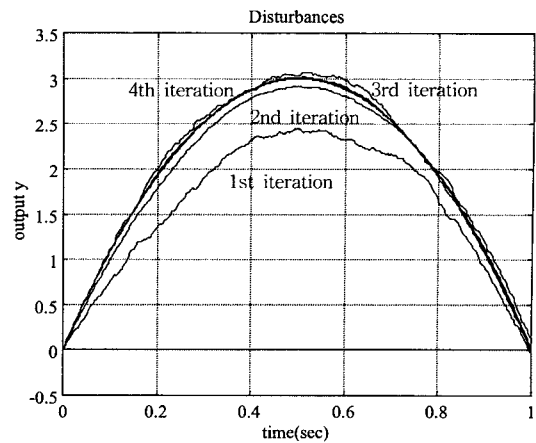


Figure 4. Output trajectories under state disturbances

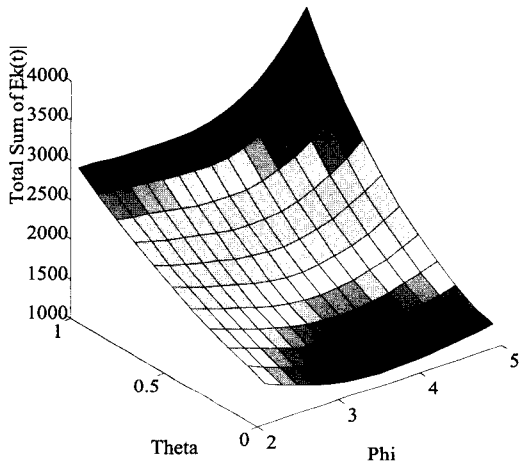


Figure 2.  $\sum_{k=1}^{10} \int_0^T e_k(t) dt$  according to the parameters

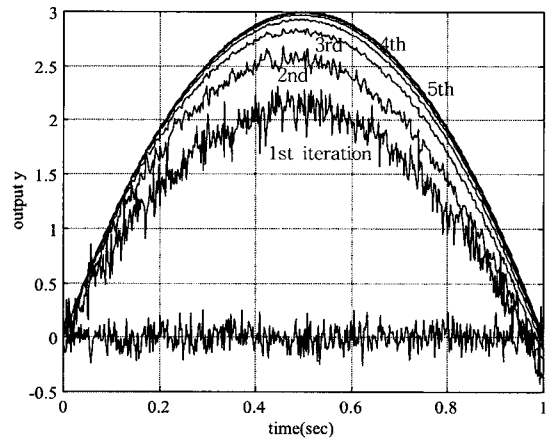


Figure 5. Output trajectories under measurement noise

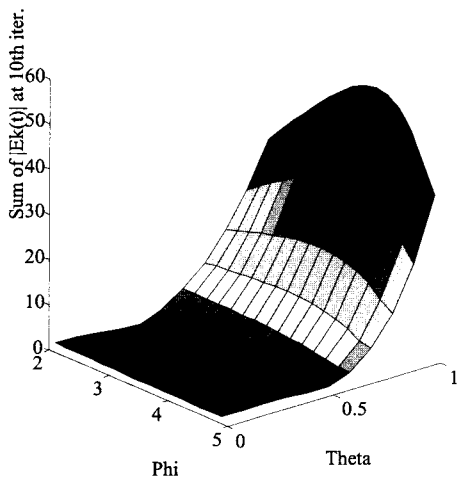


Figure 3.  $\int_0^T |e_{10}(t)| dt$  according to the parameters

### 5. Conclusion

2nd-order PD-type ILC algorithms based on both time-domain performance and iteration-domain performance are proposed. The convergence of the proposed algorithms is proved. Also, it is shown by a numerical example that the proposed method has robustness in the presence of the state disturbance and measurement noise. The proposed algorithms give more freedom for adjustment of convergence speed and tracking accuracy and can be considered as a generalization of the previous works. The proposed ILC method will be useful when ILC is applied to real control systems in the presence of external disturbance. We will study ILC methods for the dynamic systems of more general form and analyze the learning performance according to the learning parameters.

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