Sliding Mode Control for Robust Stabilization of Uncertain Input-Delay Systems

Young-Hoon Roh and Jun-Ho Oh

Abstract: This paper is concerned with a delay-dependent sliding mode scheme for the robust stabilization of input-delay systems with bounded unknown uncertainties. A sliding surface based on a predictor is proposed to minimize the effect of the input delay. Then, a robust control law is derived to ensure the existence of a sliding mode on the surface. In input-delay systems, uncertainties given during the delayed time are not directly controlled by the switching control because of causality problem of them. They can influence the stability of the system in the sliding mode. Hence, a delay-dependent stability analysis for reduced order dynamics is employed to estimate maximum delay bound such that the system is globally asymptotically stable in the sliding mode. A numerical example is given to illustrate the design procedure.

Keywords: uncertain input-delayed systems, sliding mode control, delay-dependent stability, robust stabilization, predictor-based control

I. Introduction

Time delays can be found in various engineering systems such as chemical processes, pneumatic/hydraulic systems, biological systems, and economic systems. These delays cab be frequently a source of instability. Time delay also limits the achievable bandwidth and the use of high gain feedback. Another major problem in real-world systems is the robust control when there is uncertainty in the systems. Several authors deal with the control problem of the time-delay systems via predictor-based controllers [4][8][10]. Predictor-based controllers include a predictor to compensate for time delay, and so overcome the effect of the time delay. Under a predictor-based controller, therefore, a time-delay system can be transformed into a delay-free system in which the delay is eliminated from the closed loop system. This approach enables us to characterize the design procedure by the delay-free system. However, if there are uncertainties, the uncertain time-delay system is hardly transformed into an uncertain delay-free system because of causality problem of the uncertainty terms.

Recently, robust stability and robust stabilization for time-delay systems have received considerable attention. The stability criteria can be classified into two categories according to the dependence on the size of delays; delay-independent criteria[1][2][15] and delay-dependent criteria [8][16]. One of robust stabilization techniques for uncertain time-delay systems is to use the memoryless state feedback control. Many results can be found in the literature; the Reccati equation approaches [11], the linear matrix inequality (LMI) approaches [8], the H_∞ control theory [17]. These approaches do not consider compensation for input delay. A sliding mode control (SMC) has attractive features such as fast response and good transient response [3][6][14]. It is also insensitive to uncertainty in system. Other SMC schemes are proposed for uncertain linear systems with state delay only [7][13]. However, their methods cannot ensure the robust stabilization of uncertain input-delay systems because their controllers do not use any predictor to compensate for the input delay. A sliding mode control with a state predictor-based sliding surface has been proposed for the robust stabilization of uncertain linear input-delay systems [11]. The state predictor is applied to compensate for the delay of the system in the sliding mode. Input-delay systems have a delayed control loop and so are stabilized by the control input after the delayed time. Hence, if there are unknown uncertainties entering the system, it is not easy to solve the robustness problem because of lack of causality of them. In these types of systems, there usually exists a delay bound $\bar{\tau}$ such that the systems are stabilizable for any τ satisfying $0 \le \tau \le \bar{\tau}$.

This paper deals with delay-dependent condition for robust stabilization of uncertain input-delay systems with a predictor-based SMC proposed in the literature [11]. A robust control law is derived to ensure the existence of a sliding mode and to overcome the effects of the delay and uncertainty in the sliding mode dynamics. Then there is a maximum delay bound for robust stabilization of the uncertain input-delay system under the control. Delay-dependent stability analysis for reduced order dynamics in the sliding mode is employed to estimate the delay bound.

II. System description

Let us consider a linear uncertain system with input delay described by

$$\dot{x}(t) = Ax(t) + Bu(t-\tau) + f_0(x(t),t) + f_1(x(t-\tau),t) \quad (1)$$

where $x \in \Re^n, u(t) \in \Re^m$ and $\tau \in ([0,\infty), \Re)$ are the state vector, the input vector and the delay time, respectively, and A, B are constant matrices with appropriate dimensions. The unknown uncertainties $f_0(x(t),t)$ and $f_1(x(t-\tau),t)$ represent the nonlinear perturbations with respect to the current state and the delayed state, respectively. In addition to (1), the initial conditions are given by

$$x(0) = x^0,\ x_0(\theta) = \phi(\theta),\ u_0(\theta) = v(\theta),\ -\tau \le \theta \le 0 \quad (2)$$

where $x_t(\theta) = x(t+\theta)$ and $u_t(\theta) = u(t+\theta)$.

It is assumed that the system is controllable, i.e., $\text{rank}[A,\ \exp(-s\tau)B] = n$ for any s, and the states are available for feedback. We also assume that the uncertainties $f_0, f_1 : \Re^n \times \Re_+ \to \Re^n$ satisfy the matching condition, i.e.,

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Young-Hoon Roh: Digital Appliance Research Laboratory, LG Electronics Inc., Korea.
Jun-Ho Oh: Dept. of Mechanical Engineering, KAIST

$$f_0(x(t),t) = Be_0(x(t),t) \\ f_1(x(t-\tau),t) = Be_1(x(t-\tau),t) \tag{3}$$

where

$$\|e_0(x(t),t)\| \le \rho_0 \|x(t)\| \\ \|e_1(x(t-\tau),t)\| \le \rho_1 \|x(t-\tau)\| \tag{4}$$

for positive constants $\rho_0, \rho_1 > 0$. Here $\|\cdot\|$ denotes the 2 norm. Input-delay systems have a delayed control loop and so are stabilized by the control input after the delayed time. Hence, if there are unknown uncertainties that enter the system, it is not easy to ensure the robust stabilization because of lack of information and causality of them. And input-delay systems are not also controllable for the initial time, $t \in [-\tau, 0]$. During the initial time, nonzero initial states and uncertainties can affect the stability of the systems. In these types of systems it is usually needed that the condition, $v(\theta) \in L^1((-\tau, 0), \Re^m)$ exists.

III. Design of sliding mode controller with delay compensation

We consider a predictor (predictive state), $\bar{x} \in \Re^n$ as follows

$$\bar{x}(t) = x(t) + \int_{-\tau}^{0} e^{-A(\tau+\theta)} Bu(t+\theta) d\theta \cdot \tag{5}$$

The sliding surface is defined as

$$\sigma = S\bar{x} = 0 \tag{6}$$

where $\sigma = [\sigma_1, \ldots, \sigma_m]^T \in \Re^m$ and $S = [S_1^T, \ldots, S_m^T]^T \in \Re^{m \times n}$. It is noted that the proposed sliding surface includes the state predictor, and so yields sliding mode dynamics that compensates for the input delay. It is assumed that matrix S and B are of full rank and $Se^{-A\tau}B$ is non-singular. Then, the matrix S is chosen such that the dynamics on the sliding surface has the desired closed-loop behaviors.

After selecting the sliding surface, the next step is to choose the control law such that it satisfies the condition for the existence of the sliding mode; $\sigma^T \dot{\sigma} < 0$. This condition ensures that the control law will force system trajectories toward the sliding surface in finite time and maintain them on the surface afterwards. We consider the following control structure of the form shown in fig. 1

$$u(t) = u_{eq} + u_N \tag{7}$$

where u_{eq} is an equivalent control for the nominal system of (1) without the uncertainty and u_N is a switching control to overcome the uncertainties of the system.

The equivalent control law u_{eq} is derived by $\dot{\sigma} = 0$ for the nominal system of (1). The derivative of σ along the nominal system of (1) is

$$\begin{aligned}\dot{\sigma} &= S[\dot{x}(t) + A\int_{-\tau}^{0} e^{-A(\tau+\theta)} Bu(t+\theta)d\theta + e^{-A\tau}Bu(t) \\ &\quad - Bu(t-\tau)] \\ &= S[Ax(t) + A\int_{-\tau}^{0} e^{-A(\tau+\theta)} Bu(t+\theta)d\theta + e^{-A\tau}Bu(t)]\end{aligned} \tag{8}$$

Then, the equivalent control is obtained by

$$\begin{aligned}u_{eq} &= -\left[Se^{-A^{\tau}}B\right]^{-1} SA[x(t) + \int_{-\tau e}^{0} Ze^{-A(\theta+\tau)} Bu(t+\theta)d\theta] \\ &= -\left[Se^{-A\tau}B\right]^{-1} SA\,\bar{x}.\end{aligned} \tag{9}$$

SMC

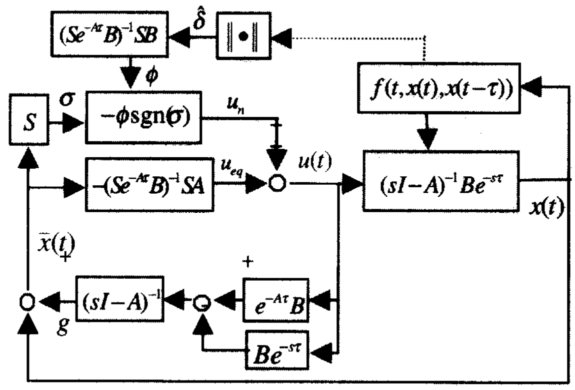

Fig. 1. Block diagram of the proposed SMC.

The equation (9) represents a state predictor-based feedback control that compensates for the delay of the nominal system of (1). Now, we need to eliminate the effect of the uncertainties in spite of the delays, and also to force the system trajectories toward the designed sliding surface. Then, the switching control u_N is chosen by

$$u_N = \begin{cases} -\dfrac{(se^{-A\tau}B)^{-1}SBB^TS^T\sigma}{\|B^TS^T\sigma\|}\hat{\delta}(x,t) & \text{if } \|B^TS^T\sigma\| \neq 0 \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

where $\hat{\delta}(x,t) = \rho\|x\| + \beta$, for $\rho = \rho_0 + \rho_1 q$, $q > 1$, $\beta > 0$, is the upper bound on the norm of the lumped uncertainty of the system. The switching control (10) is determined to remove the effect of input delay and uncertainty by the sliding surface. The uncertainty given at time t can not be cancelled by the switching control until the time, $t + \tau$, because of the delayed input. It implies that the uncertainties given in the system are sequentially cancelled by the corresponding switching controls after each delay interval.

Remark 1 : Since the control input enters the system with the interval of delay, τ, the reaching motion of the sliding mode is generated after the delay, and so the sliding mode exists at $t_s > t_0 + \tau$ for initial time t_0.

In order to ensure the existence of the sliding mode, we consider the time derivative of σ along the uncertain input-delay system (1) as

$$\begin{aligned}\dot{\sigma} = S[&Ax(t) + A\int_{-\tau}^{0} e^{-A(\theta+\tau)} Bu(t+\theta)d\theta + e^{-A\tau}Bu(t) \\ &+ f_0(x(t),t) + f_1(x(t-\tau),t)]\end{aligned} \tag{11}$$

The dynamics (11) is constrained to the sliding surface. In the dynamics (11), the uncertainty given at time t is considered but the uncertainties (dotted arrows) given during the delayed period are not considered because of lack of causality of them as shown in fig. 2. It is said that the dynamics (11) is in ideal sliding mode. Now we are ready for the following.

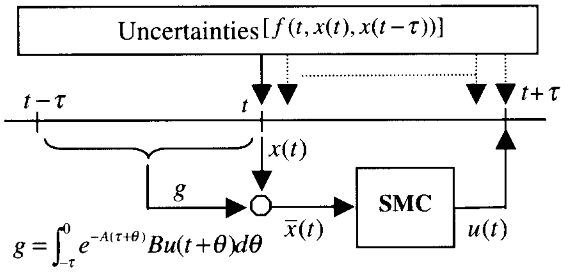

Fig. 2. Control property of the proposed SMC scheme.

Theorem 1: If the control law (7) is used for the system (11), then a sliding mode always exists, i.e., the dynamics (11), is asymptotically stable.

Proof: We choose the Lyapunov function

$$V(\sigma,t)=\frac{1}{2}\sigma^{T}\sigma. \tag{12}$$

The time derivative of V along the trajectories of the system (1) is

$$\dot{V}=\sigma^{T}S[Ax(t)+A\int_{-\tau}^{0}e^{-A(\theta+\tau)}Bu(t+\theta)d\theta+e^{-A\tau}Bu(t) \\ +f_0(x(t),t)+f_1(x(t-\tau),t)]. \tag{13}$$

Substituting (3) and (7) into the above equation yields

$$\dot{V}=\sigma^{T}[Se^{-A\tau}Bu_N+SB\{e_0(x(t),t)+e_1(x(t-\tau),t)\}]. \tag{14}$$

Substituting (10) into the above equation yields

$$\dot{V}\le-\|B^{T}S^{T}\sigma\|\{\rho\|x\|+\beta-\|e_0(x(t),t)+e_1(x(t-\tau),t)\|\}. \tag{15}$$

From the Razumikhin theorem 0, $\|x_t(\theta)\|<q\|x(t)\|$, $q>1$, $-\tau\le\theta\le0$, then

$$\|e_1(x(t-\tau),t)\|\le\rho_1\|x(t-\tau)\|\le\rho_1 q\|x(t)\|. \tag{16}$$

Thus

$$\|e_0(x(t),t)+e_1(x(t-\tau),t)\|\le\|e_0\|+\|e_1\|\le\rho\|x(t)\| \tag{17}$$

where $\rho=\rho_0+\rho_1 q>0$. We can finally obtain the following inequality:

$$\dot{V}\le-\beta\|B^{T}S^{T}\sigma\|<0 \tag{18}$$

for $\sigma\ne0$. Since β is positive, $\sigma\to0$ as $t\to\infty$. ■
From the theorem 1 the sliding mode of the dynamics (11) along the sliding surface $\sigma=0$ always exists in the finite time.

IV. Global stability of system

We see that the ideal sliding mode dynamics includes uncertainty given at time, t. However, the uncertainties given during the delayed period, which are not considered in the ideal sliding mode dynamics, can influence the stability of the actual system in the sliding mode though they will be cancelled out after each interval of the delay. It is needed to investigate the effect of them upon the ideal sliding mode dynamics. We first transform the uncertain input-delay system (1) to an uncertain delay-free system by differentiating the state predictor (5) along the trajectories of the system (1) as follows

$$\dot{\bar{x}}(t)=A\bar{x}(t)+e^{-A\tau}Bu(t)+f_0(x(t),t)+f_1(x(t-\tau),t). \tag{19}$$

Since the system (19) considers only the uncertainty at time t, the relation

$$f_0(\bar{x}(t),t+\tau)+f_1(\bar{x}(t-\tau),t+\tau) \\ =e^{A\tau}\{f_0(x(t),t)+f_1(x(t-\tau),t)\} \tag{20}$$

is derived. Then, the delay-free system (19) is rewritten as

$$\dot{\bar{x}}=A\bar{x}+e^{-A\tau}Bu(t)+e^{-A\tau}\{f_0(\bar{x}(t),t+\tau) \\ +f_1(\bar{x}(t-\tau),t+\tau)\}. \tag{21}$$

Substituting (3) into the above equation yields

$$\dot{\bar{x}}=A\bar{x}+\tilde{B}u(t)+\tilde{B}\{e_0(\bar{x}(t),t+\tau)+e_1(\bar{x}(t-\tau),t+\tau)\} \tag{22}$$

where $\tilde{B}=e^{-A\tau}B$. Let us consider transformation matrix $\bar{T}$ as

$$\bar{T}\tilde{B}=\begin{bmatrix}0\\ \tilde{B}_2\end{bmatrix} \tag{23}$$

where $\tilde{B}_2\in\Re^{m\times m}$ is nonsingular. And define $\bar{z}=\bar{T}\bar{x}=\begin{bmatrix}\bar{z}_1\\ \bar{z}_2\end{bmatrix}$

where $\bar{z}_1\in\Re^{n-m}$, $\bar{z}_2\in\Re^{m}$. Then, the system is represented by

$$\dot{\bar{z}}_1(t)=\bar{A}_{11}\bar{z}_1(t)+\bar{A}_{12}\bar{z}_2(t) \tag{24}$$

$$\dot{\bar{z}}_2(t)=\bar{A}_{21}\bar{z}_1(t)+\bar{A}_{22}\bar{z}_2(t)+\tilde{B}_2\{u(t) \\ +e_0(\bar{T}^{-1}\bar{z}(t),t+\tau)+e_1(\bar{T}^{-1}\bar{z}(t-\tau),t+\tau)\} \tag{25}$$

where $\bar{T}A\bar{T}^{-1}=\begin{bmatrix}\bar{A}_{11} & \bar{A}_{12}\\ \bar{A}_{13} & \bar{A}_{14}\end{bmatrix}$. Let us define a matrix $K\in\Re^{m\times(n-m)}$ and consider the following constraint

$$\bar{z}_2=-K\bar{z}_1. \tag{26}$$

Since (A,B) is a controllable pair, the matrix pair $(\bar{A}_{11},\bar{A}_{12})$ is also controllable. Combining (24) and (26) give the reduced order dynamics of the system (19) in the sliding mode as

$$\dot{\bar{z}}_1(t)=(\bar{A}_{11}-\bar{A}_{12}K)\bar{z}_1(t). \tag{27}$$

It is said that the dynamics (27) is in ideal sliding mode. In general, the matrix K is chosen so that assigns eigenvalues of the reduced order system (27) to the left-half plane.

We now will investigate the stability of the reduced order dynamics of the actual system with uncertainties that are not considered in the ideal sliding mode dynamics. Let us define $Z=Tx=\begin{bmatrix}Z_1\\ Z_2\end{bmatrix}$ where $T=\bar{T}e^{-A\tau}$. Then, the reduced order dynamics of the actual system in the sliding mode, which includes uncertainties given for a period of the delay, is rewritten as

$$\dot{z}_1(t) = (A_{11} - A_{12}K)z_1(t) + \hat{f}_0(z_1(t+\theta), t) + \hat{f}_1(z_1(t+\theta-\tau), t) \quad (28)$$

where $\bar{T}A\bar{T}^{-1} = TAT^{-1} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix}$ and the uncertainties given for $-\tau \le \theta \le 0$ are obtained by

$$\hat{f}_0(z_1(t+\theta), t) = \int_{-\tau}^{0} Tf_0\left(T^{-1}\begin{bmatrix} I \\ -K \end{bmatrix} z_1(t+s), t+s\right) ds \quad (29)$$

$$\hat{f}_1(z_1(t+\theta-\tau), t) = \int_{-\tau}^{0} Tf_1\left(T^{-1}\begin{bmatrix} I \\ -K \end{bmatrix} z_1(t+s-\tau), t+s\right) ds . \quad (30)$$

It is shown that the uncertainty terms of the above reduced order dynamics can not satisfy matching condition due to the causality problem. The equation (28) is also closed-loop dynamics of the actual system (1) under the control law (7).

Theorem 2: If the Lyapunov equation

$$P(A_{11} - A_{12}K) + (A_{11} - A_{12}K)^T P = -Q \quad (31)$$

and the following condition

$$\lambda_{\min}(Q) - 2\tau\rho\|PB_2\|\left\|T^{-1}\begin{bmatrix} I \\ -K \end{bmatrix}\right\| > 0 \quad (32)$$

are satisfied for the positive constant $\rho = \rho_0 + \rho_1 q$, $q > 1$, and positive definite matrix P and positive definite symmetric matrix Q, then system (1) under control law (7) is asymptotically stable.

Proof: Consider the Lyapunov function

$$V_1 = z_1^T P z_1 \quad (33)$$

The derivative of the above equation along the trajectories of the system (28) is given by

$$\dot{V}_1 = z_1^T P \dot{z}_1 + \dot{z}_1^T P z_1 \le -\lambda_{\min}(Q)\|z_1\|^2 + 2\|P(\hat{f}_0 + \hat{f}_1)\|\|z_1\| \quad (34)$$

where $\lambda_{\min}(\cdot)$ denotes the minimum eigenvalue of matrix $(\cdot)$. It is easily shown that

$$\|P(\hat{f}_0 + \hat{f}_1)\| \le \int_{-\tau}^{0} \left\| PTf_0(T^{-1}\begin{bmatrix} I \\ -K \end{bmatrix} z_1(t+s), t+s) \right\| + \left\| PTf_0(T^{-1}\begin{bmatrix} I \\ -K \end{bmatrix} z_1(t+s-\tau), t+s) \right\| ds \quad (35)$$

$$\le \int_{-\tau}^{0} \|PB_2\| \left\| T^{-1}\begin{bmatrix} I \\ -K \end{bmatrix} \right\| \{\rho_0\|Z_1(t+s)\| + \rho_1\|Z_1(t+s-\tau)\|\} ds. \quad (36)$$

From the Razumikhin theorem [5] we obtain

$$\|P(\hat{f}_0 + \hat{f}_1)\| \le \tau\rho\|PB_2\|\left\|T^{-1}\begin{bmatrix} I \\ -K \end{bmatrix}\right\|\|z_1(t)\| \quad (37)$$

where $\rho = \rho_0 + \rho_1 q$, $q > 1$. Then we obtain

$$\dot{V}_1 \le -\left[\lambda_{\min}(Q) - 2\tau\rho\|PB_2\|\left\|T^{-1}\begin{bmatrix} I \\ -K \end{bmatrix}\right\|\right]\|z_1\|^2 . \quad (38)$$

The system (28) is asymptotically stable if the condition (32) is satisfied; then there is a sufficiently small $q > 1$ such that condition (32) is satisfied. Consequently, the system (1) under the control law (7) is asymptotically stable. The proof is completed. ■

From theorem 2 it is possible to estimate a maximum delay bound, $\bar{\tau}$ that guarantees the asymptotic stability of the system (1) in the sliding mode as follows:

$$0 < \tau \le \bar{\tau} = \frac{\lambda_{\min}(Q)}{2\rho\|PB_2\|\left\|T^{-1}\begin{bmatrix} I \\ -K \end{bmatrix}\right\|} . \quad (39)$$

It is pointed out that the state predictor-based SMC proposed here is a powerful tool for robust stabilization of uncertain input-delay systems, while the state-based method shown in the previous work [3][6][7][14] is incapable of dealing with the input-delay systems. In addition, employing $\tau = 0$, the state predictor-based SMC is reduced to the state-based one.

V. Illustrative example

In order to illustrate the procedure of the proposed SMC scheme, we consider an unstable plant as follows:

$$\dot{x}(t) = \begin{pmatrix} 0 & 1 \\ 3 & -2 \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u(t-\tau) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} (e_0(x(t), t) + e_1(x(t-\tau), t)) \quad (40)$$

where an initial condition, $x(0) = [-1.6 \quad 1]^T$, and the nonlinear parameter perturbations are given by

$$e_0(x(t), t) = 0.5x_2(t)\sin(x_2(t))$$
$$e_1(x(t-\tau), t) = 0.3x_2(t-\tau)\sin(x_2(t-\tau)) .$$

It can be easily seen that $\rho_0 = 0.5, \rho_1 = 0.3$. The design objective is to determine the control law and maximum delay bound $\bar{\tau}$ that robustly stabilizes system (40). By the design procedure, assigning -5 as the eigenvalue of the reduced order dynamics (28) results in $K = 5$. Choosing $q = 1.1$ and $T = I_n$ yields $\|PB_2\| = 0.1$ and $\left\|T^{-1}\begin{bmatrix} I \\ -K \end{bmatrix}\right\| = 5.099$ for

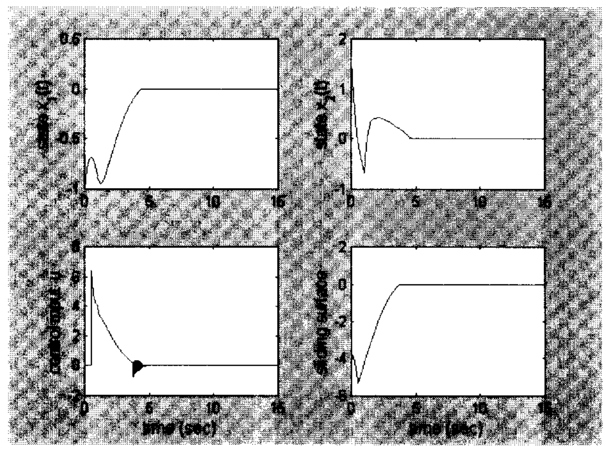

Fig. 3. Simulation results of the proposed SMC for delay time $\tau = 0.5$ sec.

$P=0.1, Q=1$. From (39) the maximum delay bound $\bar{\tau}=1.1814$ is obtained. We select $\tau=1.18$ and $S=[13.54 \quad 4.49]$ for numerical simulation. The simulation results are shown in Fig. 3-6. They show that the proposed SMC ensures the robust stabilization of input-delay system (40) with the nonlinear parameter perturbations.

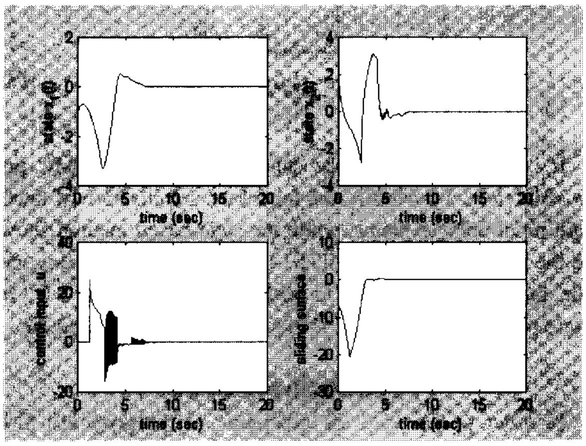

Fig. 4. Simulation results of the proposed SMC for delay time $\tau=1.1$ sec.

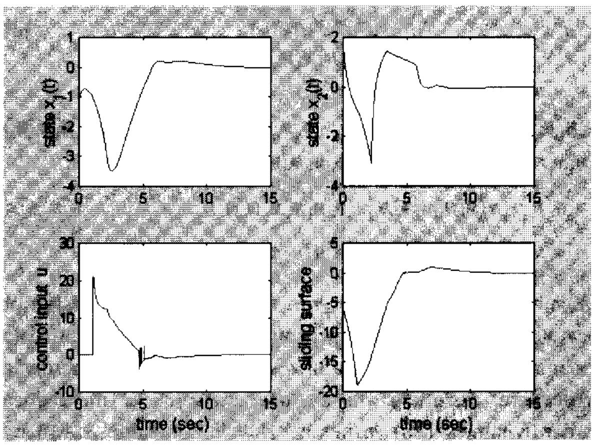

Fig. 5. Simulation results of the proposed SMC for delay time $\tau=1.1$ sec and $f_0(t,x(t))=0$.

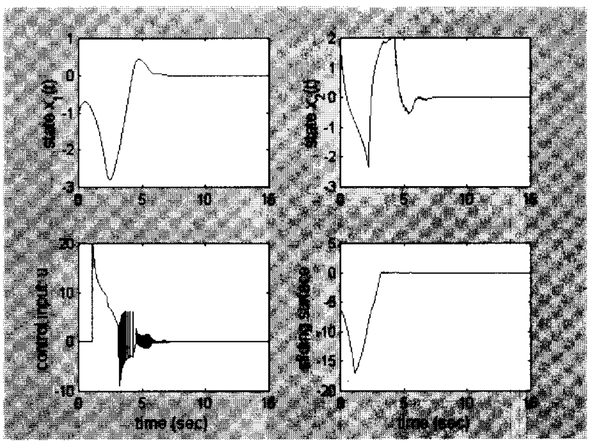

Fig. 6. Simulation results of the proposed SMC for delay time $\tau=1.1$ sec and $f_1(t,x(t-\tau))=0$.

It is also shown that the reaching motion of the sliding mode is generated after a period of delay, τ. The sliding surface diverges rapidly during the initial time, $t+\tau\leq 0$, but it is bounded in the given delay time. Once it reaches zero and then maintains it. As the delay time is getting larger, the system response is getting faster but overshoot is getting larger. It is shown in Fig. 5-6 that the uncertainty term of current state affects the system response more than the uncertainty term of delayed state.

VI. Conclusions

In this paper, we have proposed a delay-dependent sliding mode control for robust stabilization of uncertain input-delay systems. Our method uses a predictor to compensate for the input delay. A robust control law is derived to ensure the existence of the sliding mode. The proposed scheme has a reaching motion of sliding mode at $t_s > t_0+\tau$ and is dependent on the size of the delay. A maximum delay bound for robust stabilization is estimated by the delay-dependent stability analysis of the reduced order dynamics in the sliding mode. The simulation results have shown that the proposed method effectively controlled the input-delayed system with nonlinear parameter perturbations. The proposed scheme, where $\tau=0$, can be applied to systems without input delay.

References

[1] E. Cheres, S. Gutman, and Z. J. Palmor, "Stabilization of uncertain dynamic systems including state delay," *IEEE Trans. Automat. Control*, vol. AC-34, pp. 1199-1203, 1989.

[2] H. H. Choi and M. J. Chung, "Memoryless stabilization of uncertain dynamics systems with time varying delayed states and controls," *Automatica*, vol. 31, pp. 1349-1351, 1994.

[3] R. DeCarlo, S. H. Zak, and G. P. Matthews, "Variable structure control of nonlinear multivariable systems: A tutorial," *Proceedings of IEEE*, pp. 212-232, 1988.

[4] Y. A. Fiagedz and A. E. Pearson, "Feedback stabilization of linear Autonomous time lag systems," *IEEE Trans. Automat.* Control, vol. AC-31, pp. 847-855, 1986.

[5] J. K. Hale and S. M. Verduyn Lunel, *Introduction to functional differential equations*, Springer-Verlag, New York, 1993.

[6] J. Y. Hung, W. Gao, and J. C. Hung, "Variable structure control: A survey," *IEEE Trans. on Industrial Electronics*, vol. 40, pp. 1-22, 1993.

[7] A. J. Koshkouei and A. S. I. Zinober, "Sliding mode time-delay systems," *Proc. of Int. Workshop on VSS.*, Tokyo, pp. 97-101, 1996.

[8] W. H. Kwon and A. E. Pearson, "Feedback stabilization of linear systems with delayed control," *IEEE Trans. Automat. Control,* vol. AC-25, pp. 266-269, 1980.

[9] X. Li and C. E. Souza, "Delay-dependent robust stability and stabilization of uncertain linear delay systems: a linear matrix inequality approach," *IEEE Trans. Automat.* Control, vol. AC-42, pp. 1144-1148, 1997.

[10] A. Z. Manitius and A. W. Olbrot, "Finite spectrum as-

signment problem for system with delays," *IEEE Trans, Automat. Control*, vol. AC-25, pp. 266-269, 1979.

[11] Y. H. Roh and J. H. Oh, "Robust stabilization of uncertain input-delay systems by sliding mode control with delay compensation," *Automatica*, vol. 35, pp. 1861-1865, 1999.

[12] J. C. Shen, B. S. Chen, and F. C. Kung, "Memoryless stabilization of uncertain dynamic delay systems: Riccati equation approach," *IEEE Trans. Automat. Control*, vol. AC-36, pp. 638-640, 1991.

[13] K. K. Shyu and J. J Yan, "Robust stability of uncertain time-delay systems and its stabilization by variable structure control," *Internat. J. Control*, vol. 57, pp. 237-246, 1993.

[14] S. K. Spurgeon and R. Davies, "A nonlinear control strategy for robust sliding mode performance in the presence of unmatched uncertainty," *Internat. J. Control*, vol. 57, pp. 1107-1123, 1993.

[15] J. H. Su, "Further results on the robust stability of linear systems with a single time delay," *Systems Control Lett.*, vol. 23, pp. 375-379, 1994.

[16] T. J. Su and C. G. Huang, "Robust stability of delay dependence for linear uncertain systems," *IEEE Trans. Automat. Control*, vol. AC-37, pp. 1656-1659, 1992.

[17] L. Yu, J. Chu and H. Su, "Robust memoryless H_∞ controller design for linear time delay systems with norm-bounded time varying uncertainty," *Automatica*, vol. 32, pp. 1759-1762, 1996.

Young-Hoon Roh

Young-Hoon Roh was born in Korea on April 11, 1961. He received the B.S. and M.S. degrees in mechanical engineering from Inha University in 1984 and 1986, and the Ph.D. degree from KAIST in 1999, respectively. He is now a senior researcher in LG Electronics Inc. since 1988. His research Interests include robust control, nonlinear control, robust stabilization of time-delay systems, network control, and system identification and conrol in idustrial appliances.

Jun-Ho Oh

Jun-Ho Oh was born in korea on Oct. 3, 1954. He received the B.S. and M.S. degrees in mechanical engineering from Yonsei University in 1977 and 1979, and the Ph.D. degree from Univ. of California, Berkeley in 1985, respectively. He was a researcher in Korea Atomic Energy Research Institute from 1979 to 1981. From 1996-1997, he was a visiting professor in University of Texas, Austin. Since 1985, he has been a Professor of Mechanical Engineering in KAIST. His Main Interests include optimal control, neural network, fuzzy control and sliding mode control, and robot control such as quadruped walking robot, interface of sensors and systems.