기둥과 보-기둥 구조물의 비탄성 좌굴거동

Inelastic Buckling Behavior of Column and Beam-Column

이 동 식[1] · 오 순 택[2]

Lee, Dong Sik *Oh, Soon Taek*

요 약 : 에너지법을 이용하여 보-기둥 및 기둥의 비탄성 좌굴거동을 해석하였다. 미국에서 생산되는 I 형강에 적용되는 단순형 잔류응력 모델을 우리나라에서 생산되는 I 형강에 적용하였다. 먼저, 집중 압축 축하중과 균등 휨을 동시에 받는 I 형강에 대하여 비탄성 횡-비틀림 좌굴거동을 알아보고 보-기둥에서의 잔류응력의 영향을 해석하였다. 또한 기둥의 경우에 대하여 해석하였으며 얻어진 결과를 강구조편람에 의한 설계 시의 값과 비교하였다. 결론적으로 강구조편람에 의한 설계는 과설계가 됨을 알 수 있었다.

ABSTRACT : The inelastic lateral-torsional buckling behavior of the beam-columns and the columns was investigated in this paper.The energy method was deployed to study the inelastic buckling behavior of the beam-columns and columns, which requires the iterative solution of a fourth-order eigenproblem. Hitherto, the patterns of residual stress that satisfies the I-section manufacturing in Korea is not available, therefore the pattern of residual stress used in this study is a 'well-known' simplified pattern. The simplified pattern of the residual stresses is incorporated with the flow theory of plasticity to model the inelastic response.

Firstly, this study investigates the inelastic lateral-torsional buckling behavior of the I-section beam-columns under a concentric axial compressive force and uniform bending, and the effect of residual stress on the inelastic buckling behavior of beam-columns is studied. The study is then extended to the inelastic buckling of the columns by eliminating a bending moment. These results are compared it with the design method in the Korean Steel Designers' Manual (KSDM 1995). This study has found that design method in KSDM (1995) is excessively conservative.

핵 심 용 어 : 기둥, 보-기둥, 비탄성, 잔류응력, 항복

KEYWORDS : columns, beam-column, inelasticity, residual stress, yielding

1. Introduction

The buckling mode of column subjected to an axial compressive force has been classified into flexural, torsional or combination of these two mode (flexural-torsional). The lowest buckling mode of column is usually flexural about minor axis without twist. However, it is not the case for beam-column subjected to a central axial compressive force and uniform bending, the buckling mode of such I-sections have been known as lateral-torsional. When the beam-column is subjected to a concentric axial force and uniform bending, which produced the single curvature bending and geometric nonlinearity produced the $N-\delta$ effects. Therefore, the inelastic in-plane bending analysis of the beam-column is more complicated than those of the elastic analysis of beam-column because the major axis flexure rigidity is not constant due to the combination of yielding of the cross-section caused by the residual stress and applied load. Newmark (1943) presented integration technique to determine the end moment, and this study is adopted Newmark's this method to determine the end moment.

(1) 전 서울대학교 지진공학센터 연구원
(Tel. 02-970-6576, Fax. 02-975-7642, E-mail : leehansol@hotmail.com)
(2) 정회원, 서울산업대학교 구조공학과 교수(alicia@duck.snut.ac.kr)

본 논문에 대한 토의를 2004년 10월 31일까지 학회로 보내주시면 토의 회답을 게재하겠습니다.

Aim of this study is to investigate the inelastic buckling behavior of the beam-columns and the columns. The buckling behavior of the I-sections is strongly influenced by the residual stress that is produced during cooling process. Over the years, number of researcher has been proposed the theoretical model of residual stress based on the experimental study. Ketter et al. (1955) presented the simplified pattern of residual stress, the distribution of residual stress in the flange is bilinear and flange and constant tensile in the web, and this pattern of residual stresses is suited the North American I-sections. Number of researchers has been used this pattern of residual stress to analyze the inelastic lateral-torsional buckling behavior of beam-columns. Miranda and Ojalvo (1965), Fukomoto and Galambos (1966), and Abdel-Sayed and Agaln (1973) investigated the inelastic buckling behavior of the beam-columns with the simplified residual stress. The tangent modulus E_t that has been used in their study is equal to the elastic modulus for elastic regions and zero for yielded regions. Furthermore Abdel-Sayed and Agaln (1973) assumed that the yielded regions of the cross-section do not affect the torsional rigidity, and therefore Saint Venant torsional rigidity was used for the elastic and the yielded regions. Trahair and Kitipornchai (1972) argued the use of tangent modulus in the yielded regions of the cross-section based on the slip theory, and Trahair and Kitipornchai (1972) have used the strain hardening modulus for yielded and strain hardened regions of the cross-section. The tangent modulus theory, as was done by Trahair and Kitipornchai (1972), is adopted in this study. The analysis of columns and beam-columns is tackled using an energy method that requires the incremental and iterative solution of a fourth-order eigenproblem. The displacement and twist of flanges is represented as half sine wave longitudinally with a number of harmonics, while the cubic displacement is assumed for the web. Adequate model of residual stress is not available to suited the Korean I-section member and this study is adopted the simplified residual stress as was done by Miranda and Ojalvo (1965), Fukomoto and Galambos (1966), and Abdel-Sayed and Agaln (1973). Firstly, this study considers the inelastic lateral-torsional buckling of beam-columns subjected to a concentric compressive axial force and uniform bending, and then the reduction of elastic buckling load due to the residual stress is addressed. The research on buckling of behavior of columns is limited to elastic (Bradford 1997) and finally, the inelastic buckling of the columns is analyzed by considering four different I-sections that are manufactured in Korea and these results are compared it with the design method in KSDM (1995).

2. Energy solution

2.1 General

The energy-based method used by Lee (2001) is adopted in this study to analyze the beam-columns and columns with the simplified residual stress.

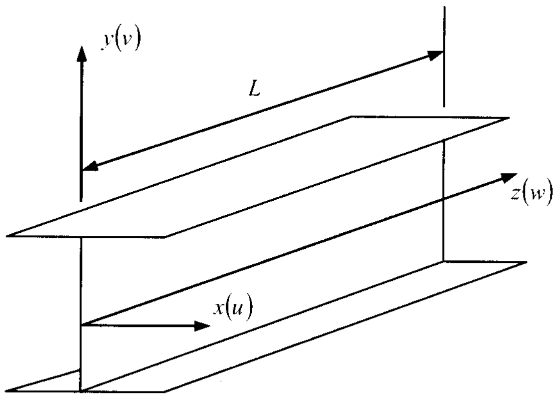

Figure 1(a) Beam-Column

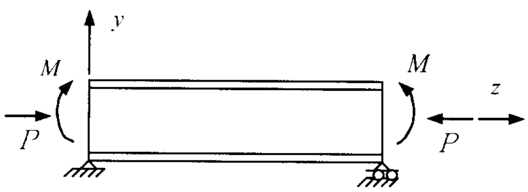

Figure 1(b) Applied axial force and moment

The reference axis positioned at the mid-height of web is shown in Fig. 1(a). Figure 1(b) shows the simply supported I-beam at its end with length L.

which is subjected to a concentric axial compressive force and uniform bending. The flanges are treated as the rigid beam, while the plate theory is used for the web. Thus, the well-known beam and plate theory is deployed for the flange and the web respectively. A detail method of energy method is given in Lee (2001) and Lee and Oh (2004). It has been observed that a pattern of residual stress used in North American is different to those of the residual stress used in Europe. This is attributable to the method of manufacturing the I-sections. The simplified residual stress distribution is shown in Fig. 2. A simplified pattern of residual stress assumes a constant tensile stress in the web and the bilinear residual stress in the flange. In this figure,

$$\sigma_{rc} = 0.3\sigma_y \tag{1}$$

and

$$\sigma_{rt} = \left[\frac{BT}{BT + t_w(D-T)}\right]\sigma_y \tag{2}$$

where B and T are the flange width and the thickness respectively, D is the overall depth of the section and t_w is the web thickness. The simplified residual stress idealization satisfies the static equilibrium, but not that due to torsional equilibrium as

$$\int_A \sigma_r dA = \int_A (x^2 + y^2)\sigma_r dA = 0 \tag{3}$$

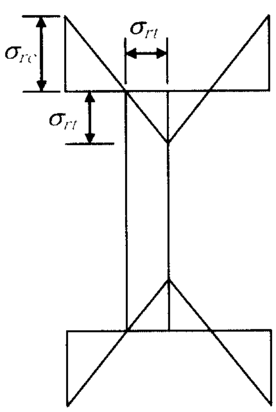

Figure 2. simplified residual stress models

2.2 Moment-curvature relationship for beam-column

The beam-column subjected to constant axial force and uniform bending is considered in this study. The moment-thrust-curvature relationship should be established before the out-of-plane buckling analysis. The applied strain $\varepsilon_a(x,y)$ at any point of the cross section can be expressed as

$$\varepsilon_a(x,y) = \varepsilon_o + (y + \bar{y})\rho + \varepsilon_r(x,y) \tag{4}$$

where $\bar{y}$ is assumed neutral axes. ε_0 is strain due to a constant axial compressive force. ρ is curvature, and the residual strain is given by $\varepsilon_r(x,y) = \sigma_r(x,y)/E$. The regions of elastic, yielded and stain hardened of the cross-sections can be established with Eqn. 4 with initially assumed neutral axes and curvature. With predetermined the elastic and inelastic regions of the cross-section, the stress in the cross-section can be determined with tangent modulus theory as

$$\sigma(x,y) = \int_{\varepsilon_r}^{\varepsilon_a} E_t d\varepsilon_a + E\varepsilon_r \tag{5}$$

The appropriate tangent modulus is shown in Fig. 3.

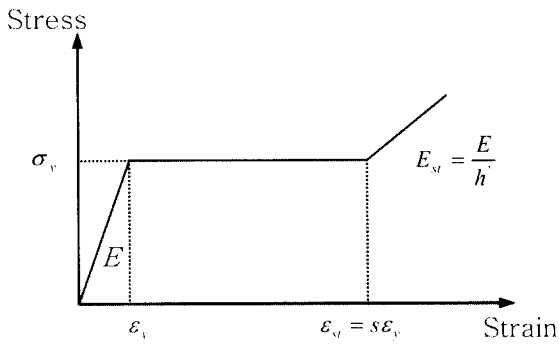

Figure 3. Trilinear elastic-plastic-strain hardening constitutive model

The equilibrium condition of axial force is then used to calculate the position of neutral axis with given curvature, and the maximum moment can be determined. The axial force and moment is given as

$$N = \int_A \sigma(x,y)dA \tag{6}$$

$$M_{max} = \int_A \sigma(x,y)ydx \tag{7}$$

2.3 Determination of end moment for beam-column

The end moment is determined using Newmark (1943) integration technique with known value of the maximum moment and the curvature at the mid span. The flexural rigidity about major axis is determined as

$$\rho = \frac{M^{max}}{(EI)_s} \tag{8}$$

where $(EI)_s$ is the secant flexural modulus of rigidity. The iterative procedure is adopted to determine the applied end moment. Firstly approximate the deflected shape v of the beam-column is assumed and the elastic deflection is assumed for initial approximation (Trahair and Bradford 1998). The beam-column member is divided to four equally spaced stations and compute the new curvature at the each stations using $\rho = \frac{M + Nv}{(EI)_s}$ where moment M is initially guessed. The initially assumed curvatures are corrected with the new curvature. This process is repeated until initially assumed deflections are equal to the calculated deflection. The applied end moment is then $M = M_{max} - Nv_{midspan}$

2.4 Stress-strain relationship for column

The strain due to the bending is eliminated from applied strain given in Eqn. 4 to consider axial force and residual strain. The applied external strain $\varepsilon_a(x,y)$ due to the concentric compressive force on a cross-section is

$$\varepsilon_a(x,y) = \varepsilon_0 + \varepsilon_r(x,y) \tag{9}$$

The applied load N is determined by integrating the stresses over the cross-section and is given in Eqn. 6.

2.5 Buckling analysis

The buckling displacement and the twist of the flange is shown in Fig. 4. The buckling displacement u_T, u_B and the twist ϕ_T, ϕ_B is assumed as a sine curve with a number (n) of harmonics, while the buckling deformation of the web is assumed to be a cubic curve. The buckling deformation of the web can be expresses in terms of flange buckling deformation by enforcing the displacement and slope campatibity between the flange-web junctions.

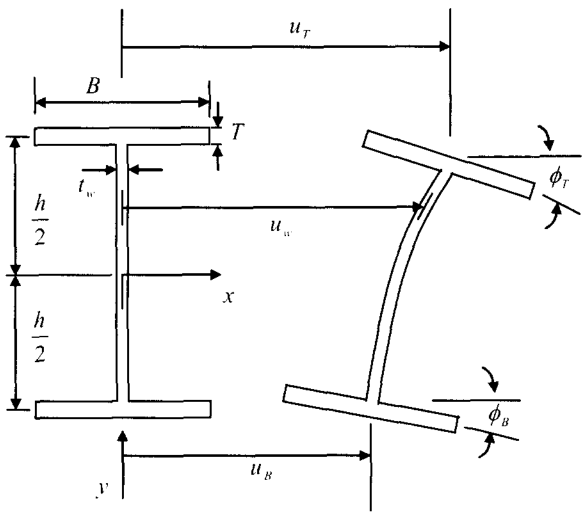

Figure 4. Buckling deformations in the plane of the cross-section

The total strain energy stored in the flange and the web can be expressed as

$$U = U_f + U_w \tag{10}$$

The beam theory (Timoshenko and Gere 1961) is employed to determine the strain energy in the flange with tangent modulus as was done by Trahair and Kitipornchai (1972). The 'well-known' isotropic plate theory (Timoshenko and Woinowsky-Krieger 1959) is used for elastic regions and orthotropic plate theory based on flow theory of plasticity given by Haaijer (1957) and Dawe and Kulak (1984) is used for the yielded and the strain hardened regions. The stiffness matrices $[k_f]$ and $[k_w]$ can be derived from the beam theory for the flange and the plate theory for the web. The total strain energy stored in the flange and the web can be expressed in term of stiffness matrices of the flange and the web.

$$[k]=[k_f]+[k_w] \tag{11}$$

The simplified pattern of residual stress satisfies the static equilibrium but not that with axial torque. In order to satisfy the condition of vanishing axial torque, the torsional rigidity should be changes to $\left((GJ)_t - \int_A \sigma_r (x^2+y^2) dA\right)$ for the flange and the web, as was done by Trahair (1993).

The work done by the flange and the web can be expressed as

$$V = V_f + V_w \tag{12}$$

The work done by the flange and the web can lead to the stability matrices of the flange $[g_f]$ and the web $[g_w]$ and is given as

$$[g]=[g_f]+[g_w] \tag{13}$$

It should be noted that the stiffness and the stability matrices of the flange and the web are depend nonlinearly on the applied curvature.

2.6 Buckling solution

The total potential energy during buckling in the flange and the web can be written as

$$\Pi = \frac{1}{2}\{q\}^T([k]-[g])\{q\} \tag{14}$$

Minimizing Eqn. 14 with respect to $\{q\}$ in the usual way produces the buckling condition

$$\frac{\partial \Pi}{\partial \{q\}} = ([k]-[g])\{q\} = \{0\} \tag{15}$$

As mentioned earlier that the stiffness and stability matrices are function of applied curvature, and therefore the value of applied curvature ρ is adjusted until Eqn. 15 vanishes. The most appropriate iterative method is the method of bisections because ill-behavior of Eqn 15 in the mathematical sense.

3. Numerical study

The inelastic buckling behavior of the I-sections is investigated in this section. The material property in this study is same as KSDM (1995). The elastic modulus E=2.1×106 kg/cm^2(205.926×103MPa), and the ratio E/E_{st} = 40(which is in range from 35 to 42). The yield stress σ_y is 2400 kg/cm^2(235MPa), and Poisson's ratio = 0.3, and ε_{st} = $10\varepsilon_y$. The energy method presented in this paper is lateral-distortional buckling and it is necessary to suppress the web distortion so that lateral-distortional buckling mode becoming a lateral-torsional one. This can be achieved by expressing the strain energy due to out-of-plane plate flexure of the web U_{wp} as (Bradford and Trahair (1982)).

$$U_{wp} = \frac{1}{2}\gamma_r \int_0^L \int_{-\frac{h}{2}}^{\frac{h}{2}} D_w \left(\frac{\partial^2 U_w}{\partial y^2}\right) dy dz \tag{16}$$

where D_w is plate rigidity factor and γ_r is set to a large value (say 106)

3.1. Inelastic lateral-torsional buckling of besm-columns

The considered cross-sections in this study are 800×300, 400×400, 300×175, and 200×150. The verification of current method is demonstrated by comparing it with inelastic lateral-torsional buckling results obtained in this study and Abdel-Sayed & Aglan (1973). The inelastic lateral-torsional buckling solution obtained by Abdel-Sayed and Aglan is a simply supported North American 8WF31 beam-column and assumed the simplified pattern of residual stress. The material properties and geometry of the cross-section can be found in their paper. Figure 5 shows the comparison between the inelastic buckling load results obtained in this study and Abdel-Sayed and Aglan (1973) buckling solution with constant axial force 0.6N_s, where N_s is the squash loaded. In the figure, the end buckling normalized with respect to the yield moment M_y, while the major axis slenderness ratio L/r_x is used. There are two different value of the tangent modulus E_t is used in yielded regions in this analysis. The tangent modulus E_t assumed by Abdel-Sayed & Aglan's(1973) study was that E_t is equal to elastic modulus for elastic regions and zero is used for yielded regions, and strain hardening modulus is used for strain-hardened regions. Furthermore, Abdel-Sayed & Aglan(1973) assumed that the yielded regions of the cross-section do not affect the torsional rigidity, and therefore Saint Venant torsional rigidity was used for the elastic and yielded regions. This study has adopted same assumption as Abdel-Sayed and Aglan's study and the results agreed very well with independent solutions.

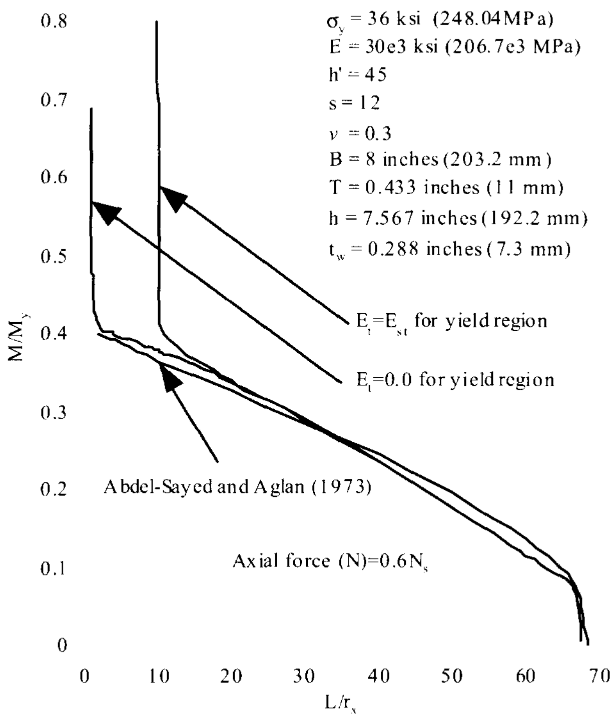

Figure 5. Inelastic lateral-torsional buckling comparison with Abdel-Sayed and Aglan (1973).

Figure 5 also have shown the inelastic buckling solution with $E_t = E_{st}$ in the yielded and strain hardened regions where commences of strain hardening buckling occurs much earlier than conservative assumption of $E_t = 0$ at a higher slenderness ratio.

The results of inelastic lateral-torsional buckling of beam-columns are shown in Figs. 6, 7, 8 and 9 with a constant axial compressive force. In these figure, the inelastic buckling moments M are normalized with respected to yield moment M_y as function of dimensionless slenderness L/r_x, where r_x is radius of gyration about x-axis.

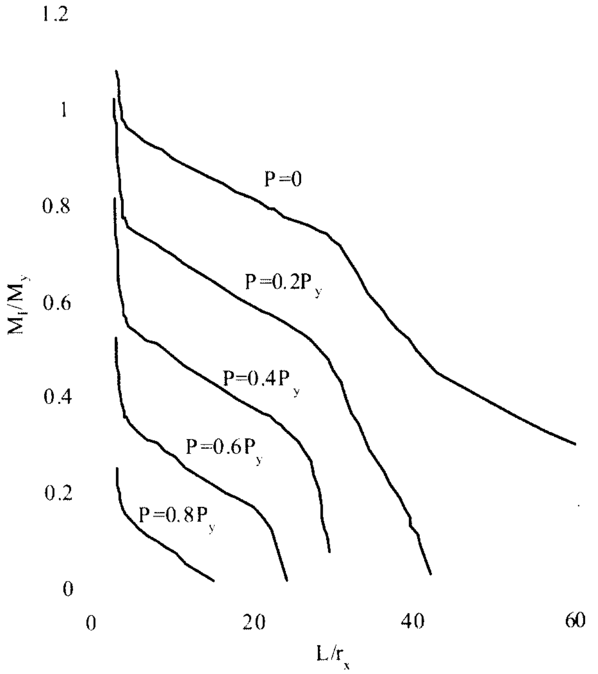

Figure 6. Inelastic lateral-torsional buckling of beam-column 800×300.

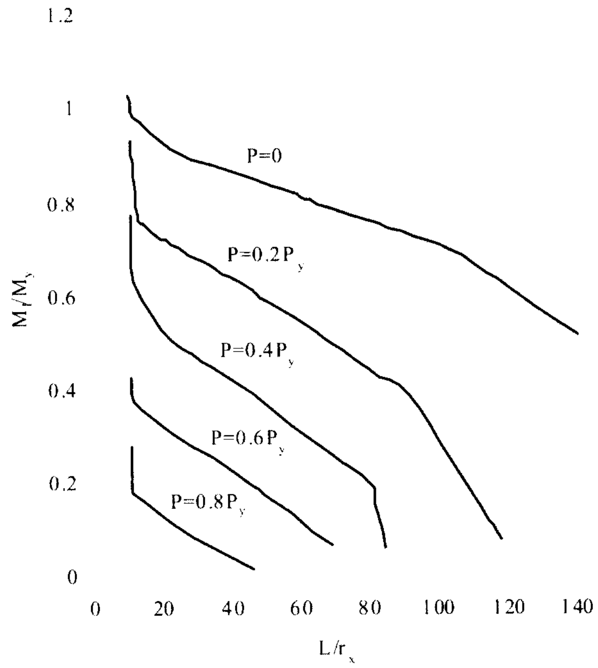

Figure 7. Inelastic lateral-torsional buckling of beam-column 400×400.

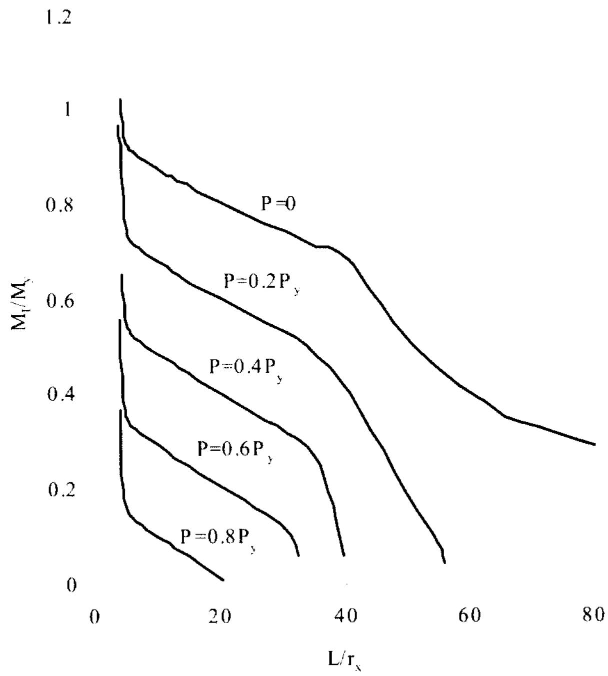

Figure 8. Inelastic lateral-torsional buckling of beam-column 300×175.

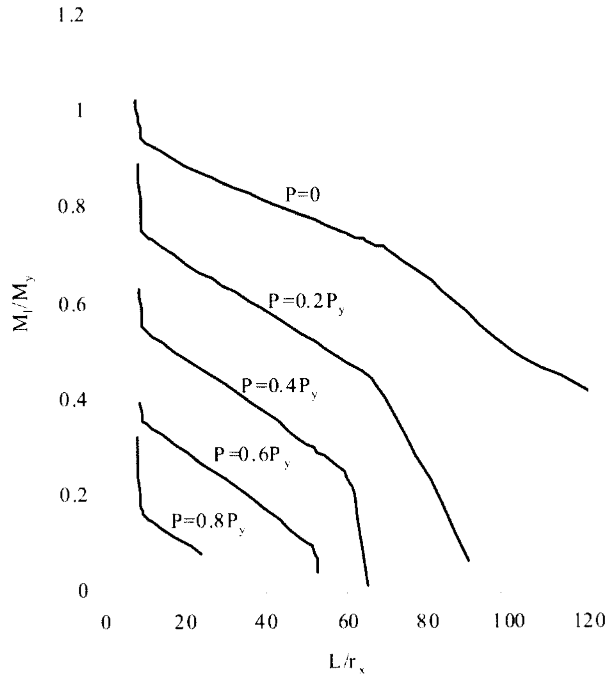

Figure 9. Inelastic lateral-torsional buckling of beam-column 200×150.

As would be expected that the inelastic buckling strength of beam-column is lower than the elastic value. This is due to the yielding of cross-section caused by combination of the residual stress and applied load. The buckling resistance is substantial reduced as axial force increased as the compressive axial force is increased. The buckling results of beam-column have shown that the slender I-section 800×300 and 300×175 has no reserve of bending capacity for L/r_x〉16 and L/r_x〉21 respectively but the compact I-section 400×400 and 200×150 has no reserve of bending capacity for L/r_x〉50 and L/r_x〉38 respectively at a compressive axial force of $0.8P_y$.

3.2. Inelastic flexural buckling of columns

The inelastic buckling of columns subjected to a concentric compressive force is considered in this study. Before introduction of the limit state design method, the columns were designed according to the allowable or working stress design method. The working stress method in British and Australian steel design standard was based on the Perry-Robertson equation and used limiting stress at member with initial curvature only first yields as the basis for determining the strength. The allowable stress method in American steel design standard was different to those of British and Australian standard, it used the tangent modulus critical stress of the straight member with residual stress in the member. The working or allowable stress design method has been modified to the limit state design method or LRFD basis on the experimental studies and included the geometric imperfection so that they are in substantial agreement with experimental results. The current British and Australian steel design standard is based on the multi-curve approach to predict the capacity of the member and Perry-Robertson equation is modified empirically to produce a member capacity curves so that particular members are falls into one of one of these curves, which is depending on the residual stress and geometric imperfection. A regression analysis of the large experimental results were used to produce an only one design curve by American steel design standard.

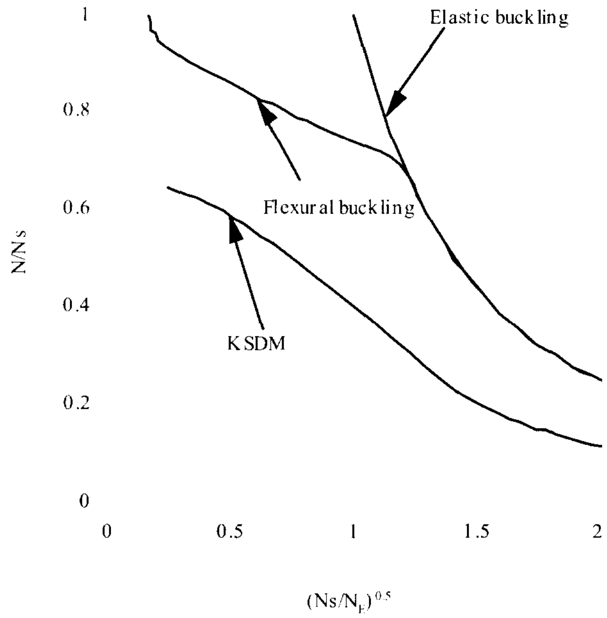

Figure 10. Inelastic flexural buckling of column 800×300

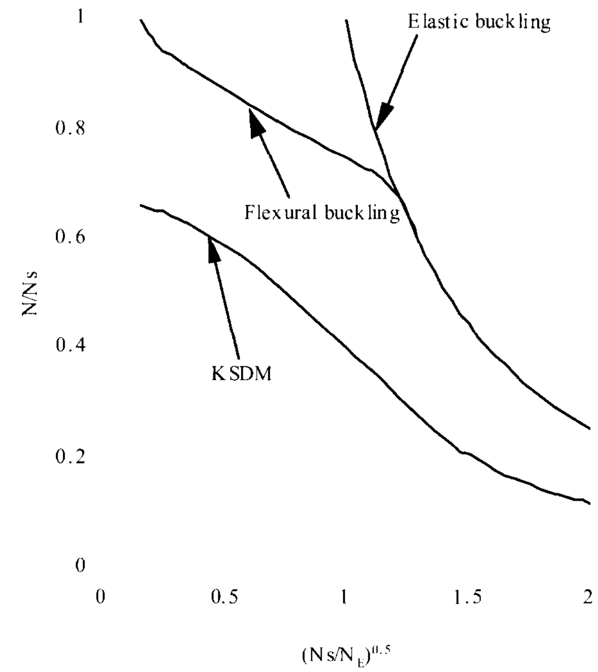

Figure 11. Inelastic flexural buckling of column 400×400

The design method KSDM (1995) is based on the allowable stress method. Therefore, the purpose of this study is to examine the accuracy of the KSDM (1995). The considered cross-sections considered in this section are same as previous sub-section. Figure from 10 to 13 show the inelastic buckling load obtained in this study with the simplified residual stress model. In the figures, the inelastic buckling load N is normalized with respect to the squash load $N_s(=A\sigma_y)$ and the modified column slenderness is represented as $\sqrt{N_S/N_E}$, where N_E is the Euler load of the column. These figures is also shown column curve derived from KSDM(1995), and is given as .

$\lambda \le \lambda_p$

$$f_c = \frac{\left\{1-0.4\left(\frac{\lambda}{\lambda_p}\right)^2\right\}\sigma_y}{\frac{3}{2}+\frac{2}{3}\left(\frac{\lambda}{\lambda_p}\right)^2} \qquad (17)$$

$\lambda > \lambda_p$

$$f_c = \frac{0.277\sigma_y}{\left(\frac{\lambda}{\lambda_p}\right)^2} \qquad (18)$$

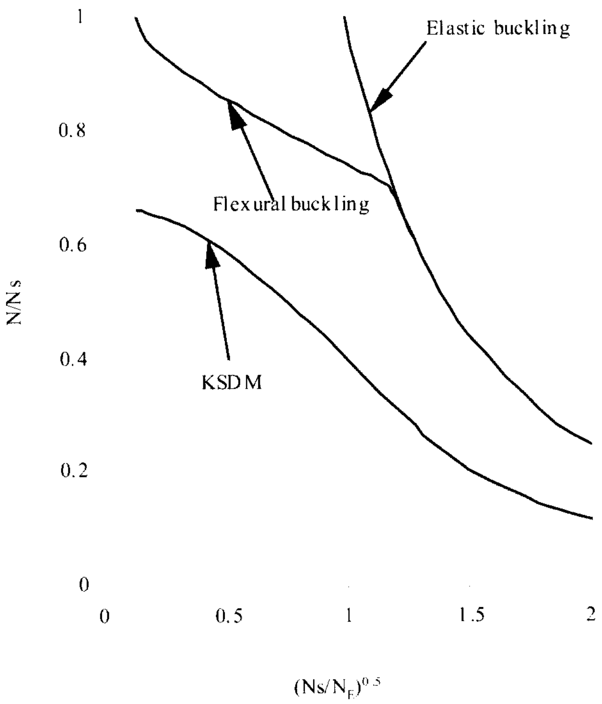

Figure 12. Inelastic flexural buckling of column 300×175

It can be seen in the figures that the inelastic buckling load curve derived from this study and KSDM(1995) are not agreed. The disparity between the KSDM(1995) and the numerical solution is not surprising, since the KSDM(1995) are based on the allowable stress design method while those of the numerical results obtained in this study are based on inelastic bifurcation. The discrepancy is increased as the length of columns is decreased.

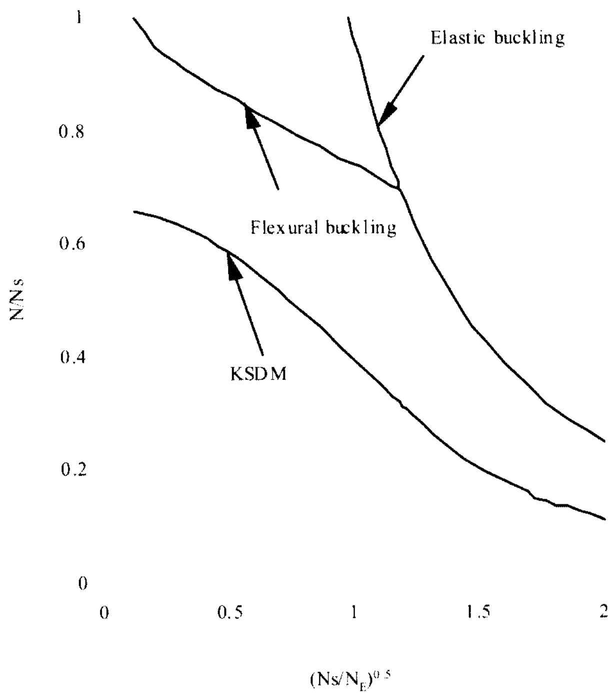

Figure 13. Inelastic flexural buckling of column 200×150

Table 1. Percentage difference of buckling load at $\sqrt{N_S/N_E}$ is approximately equal 1.

Cross-Section	% Difference
200×150	47.7
300×175	48.8
400×400	46.5
800×300	46.3

Table 1 shows the percentage between the current method and KSDM(1995) at $\sqrt{N_S/N_E}$ approximately equal to 1. It can be seen that the percentage difference between these two methods is approximately 48. The percentage difference is accentuated for intermediate and short span of the columns. This comparison study has demonstrated that the design method in KSDM(1995) is excessively conservative.

4. Conclusion

An energy-based method is deployed to analysis the inelastic buckling behavior of the beam-columns and the columns. The simplified pattern of residual stress is incorporated with energy-based method which is a lead to iterative solution of a fourth order linear eigenproblem. A finding of this study is that the buckling behavior of the beam-columns is strongly influence by the residual stress. The compact I-sections have more bending capacity than the slender I-section at a given value of the dimensionless length L/r_x as the axial force is increased. The reduction of elastic buckling load is signification in the inelastic region due to the presence of the residual stress, whereas the effect of the residual stress in the elastic region is negligible. This study is then extended to the inelastic buckling of the columns subjected to a concentric axial compressive force and these results are compared with design method in KSDM (1995). It is found that KSDM is excessively conservative.

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