

ON WEAK QUASI-PRESEPARATION AXIOMS

JIN HAN PARK

Abstract. In this paper, some new characterizations of quasi pre- R_0 spaces due to Tapi et al. [The Mathematics Education XXIX(3) (1995) 147] are obtained. A new notion of quasi-pre- R_1 spaces is defined and their properties are studied to some extent.

1. Introduction and preliminaries

Tapi et al. [9] have defined and investigated the notion of quasi-preopen sets in bitopological spaces as a generalization of quasi-open sets due to Dutta [1]. In [8-13], Tapi and coworker used quasi-preopen sets to define the weak quasi-preseparation axioms, namely, quasi pre- T_i ($i = 0, 1, 2$) and quasi pre- R_0 spaces and to define quasi precontinuous and quasi pre-irresolute functions in bitopological spaces. Recently, present author [7] defined and studied generalized quasi-preclosed sets and gqp -closed functions and obtained some characterizations and preservation theorems of quasi p -normal and quasi p -regular spaces due to Tapi et al. [10,13]. The purpose of this paper is to study further properties of quasi pre- T_i ($i = 0, 1, 2$) and quasi pre- R_0 spaces. Further, the notion of quasi-preopen sets is used to introduce a new quasi-preseparation axiom called quasi pre- R_1 spaces and these spaces are studied to some extent.

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Throughout the present paper, a space (X, τ_1, τ_2) stand for bitopological space on which no separation axiom is assumed unless explicitly stated. By τ_i -open set and τ_j -closed set we mean the open set and closed set with respect to the topologies τ_i and τ_j , respectively, where the indices i and j take valued in $\{1, 2\}$ and $i \neq j$. Let A be a subset of (X, τ_1, τ_2) . A subset A is said to be quasi-preopen [9] if for each $x \in A$ there exists either a τ_1 -preopen set U such that $x \in U \subset A$ or a τ_2 -preopen set V such that $x \in V \subset A$. The complement of quasi-preopen set is called quasi-preclosed. The intersection of all quasi-preclosed sets containing A is called the quasi-preclosure [9] of A and is denoted by $\text{qpcl}(A)$. Dually, the quasi-preinterior [7] of A , denoted by $\text{qpint}(A)$, is defined to be the union of all quasi-preopen sets contained in A . A point $x \in X$ is called quasi prelimit point [9] of A if for any quasi-preopen set U containing x , $U \cap (A \setminus \{x\}) \neq \emptyset$ and the set of all quasi prelimit points of A is called quasi prederived set [9] of A and denoted $\text{qpd}(A)$. Then $\text{qpcl}(A) = A \cup \text{qpd}(A)$.

Lemma 1.1 [9]. Let A be a subset of a space (X, τ_1, τ_2) and $x \in X$. Then the following properties hold:

- (a) If $A \subset B$, then $\text{qpcl}(A) \subset \text{qpcl}(B)$.
- (b) $\text{qpcl}(\text{qpcl}(A)) = \text{qpcl}(A)$.
- (c) A is quasi-preclosed if and only if $A = \text{qpcl}(A)$.
- (d) $\text{qpcl}(A)$ is quasi-preclosed.
- (e) $x \in \text{qpcl}(A)$ if and only if $A \cap U \neq \emptyset$ for each quasi-preopen set U of X containing x .

Lemma 1.2 [7]. If A is a bi- α -open set and B is a quasi-preopen set of a space (X, τ_1, τ_2) , then $A \cap B$ is quasi-preopen in X .

Lemma 1.3 [7]. Let $(Z, (\tau_1)_Z, (\tau_2)_Z)$ be a bi- α -open subspace of a space (X, τ_1, τ_2) . If A is quasi-preopen in X , then $A \cap Z$ is quasi-preopen in

Z .

Lemma 1.4 [7]. If $(Z, (\tau_1)_Z, (\tau_2)_Z)$ is a bi- α -open subspace of a space (X, τ_1, τ_2) , then for any subset A of Z , $\text{qpcl}_Z(A) = \text{qpcl}(A) \cap Z$, where $\text{qpcl}_Z(A)$ denotes the quasi-preclosure of A in the subspace $(Z, (\tau_1)_Z, (\tau_2)_Z)$.

2. Some properties of quasi pre- T_i spaces ($i = 0, 1, 2$)

Definition 2.1 [8]. A space (X, τ_1, τ_2) is called:

- (a) quasi-pre- T_0 if for any distinct two points in X , there exists a quasi-preopen set containing one but not the other;
- (b) quasi-pre- T_1 if for each x, y of X such that $x \neq y$, there exist quasi-preopen sets U and V such that $x \in U$, $y \in V$, $x \notin V$ and $y \notin U$;
- (c) quasi-pre- T_2 if for each x, y of X such that $x \neq y$, there exist disjoint quasi-preopen sets U and V in X such that $x \in U$ and $y \in V$.

Theorem 2.2. Every bi- α -open subspace of a quasi pre- T_i space is quasi pre- T_i , for $i = 0, 1, 2$.

Proof. We give a proof for $i = 1$ only; the other cases being similar, are omitted.

Let $(Z, (\tau_1)_Z, (\tau_2)_Z)$ be a bi- α -open subspace of a space (X, τ_1, τ_2) and $x, y \in Z$ such that $x \neq y$. Then there exist quasi-preopen sets U and V of X such that $x \in U$, $y \notin U$, $y \in V$ and $x \notin V$. By Lemma 1.3, $U \cap Z$ and $V \cap Z$ are quasi-preopen in Z . This shows that Z is quasi pre- T_1 . \square

Corollary 2.3 [8]. Every bi-open subspace of a quasi pre- T_i space is quasi pre- T_i , for $i = 0, 1, 2$.

Definition 2.4. A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is said to be point-quasi-preclosure 1-1 if for any $x, y \in X$ such that $\text{qpcl}\{x\} \neq$

$\text{qpcl}\{y\}, \text{qpcl}\{f(x)\} \neq \text{qpcl}\{f(y)\}.$

Theorem 2.5. If $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is point-quasi-preclosure 1-1 and X is quasi-pre- T_0 , then f is injective.

Proof. Let $x, y \in X$ with $x \neq y$. By Theorem 1 of [8], we have $\text{qpcl}\{x\} \neq \text{qpcl}\{y\}$. Since f is point-quasi-preclosure 1-1, $\text{qpcl}\{f(x)\} \neq \text{qpcl}\{f(y)\}$ and so $f(x) \neq f(y)$. Hence f is 1-1. \square

Corollary 2.6. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a function from a quasi-pre- T_0 space X into a quasi-pre- T_0 space Y . Then f is point-quasi-preclosure 1-1 if and only if f is injective.

Tapi et al. [8] proved that if every singleton of (X, τ_1, τ_2) is quasi-preclosed, then X is quasi pre- T_1 . We improve this result as follows:

Theorem 2.7. A space (X, τ_1, τ_2) is quasi-pre- T_1 if and only if every singleton is quasi-preclosed.

Proof. We prove only necessity. To prove that for any $x \in X$, $\{x\}$ is quasi-preclosed, we claim that $X \setminus \{x\}$ is quasi-preopen. Let $y \in X \setminus \{x\}$. Since X is quasi pre- T_1 , there exists a quasi-preopen set V such that $y \in V \subset X \setminus \{x\}$ and hence by Theorem 2.2 of [9], $X \setminus \{x\}$ is quasi-preopen, i.e. $\{x\}$ is quasi-preclosed. \square

Theorem 2.8. Let (X, τ_1, τ_2) be a quasi-pre- T_1 space in which every finite intersection of quasi-preopen sets is quasi-preopen and let $A \subset X$. Then $x \in \text{qpd}(A)$ if and only if every quasi-preopen set containing x contains infinitely many points of A .

Proof. If every quasi-preopen set containing x intersects A in infinitely many points, it certainly intersects A in some point other than x itself, so that $x \in \text{qpd}(A)$. Conversely, let $x \in \text{qpd}(A)$ and suppose some quasi-preopen set U containing x intersects A in only finitely many points. Let $\{x_1, x_2, \dots, x_n\}$ be the points of $U \cap (A \setminus \{x\})$. By assumption and the quasi-pre- T_1 property of X , the set $X \setminus \{x_1, x_2, \dots, x_n\}$ is a quasi-preopen set of (X, τ) and then $U \cap (X \setminus \{x_1, x_2, \dots, x_n\})$ is a quasi-preopen set containing x which intersects the set $A \setminus \{x\}$ not at all. This contradicts $x \in \text{qpd}(A)$. \square

Lemma 2.9. If A is a quasi-preopen set of (X, τ_1, τ_2) and B is a quasi-preopen set of (Y, σ_1, σ_2) , then $A \times B$ is quasi-preopen in the product space $(X \times Y, \tau_1 \times \sigma_1, \tau_2 \times \sigma_2)$.

Proof. It follows from [9, Theorem 2.2] and the fact that the product of preopen sets is preopen [2]. \square

Recall that a function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is said to be quasi-preirresolute [11] if $f^{-1}(V)$ is quasi-preopen in X for each quasi-preopen set V of Y .

Theorem 2.10. If $f : X \rightarrow Y$ is a quasi-preirresolute function and (Y, σ_1, σ_2) is a quasi-pre- T_2 space, then the set $\{(x_1, x_2) : f(x_1) = f(x_2)\}$ is quasi-preclosed in the product space $(X \times X, \tau_1 \times \tau_1, \tau_2 \times \tau_2)$.

Proof. Let $A = \{(x_1, x_2) : f(x_1) = f(x_2)\}$. If $(x_1, x_2) \in X \times X \setminus A$, then $f(x_1) \neq f(x_2)$. Since Y is quasi-pre- T_2 , there exist disjoint quasi-preopen sets V and W such that $f(x_1) \in V$ and $f(x_2) \in W$. By the quasi-preirresoluteness of f , $f^{-1}(V)$ and $f^{-1}(W)$ are quasi-preopen set in X such that $x_1 \in f^{-1}(V)$ and $x_2 \in f^{-1}(W)$. Then by Lemma 2.9, $f^{-1}(V) \times f^{-1}(W)$ is quasi-preopen in $X \times X$ such that $(x_1, x_2) \in$

$f^{-1}(V) \times f^{-1}(W) \subset (X \times X) \setminus A$. By Theorem 2.2 of [9], $X \times X \setminus A$ is quasi-preopen and hence A is quasi-preclosed in $X \times X$. \square

Recall that, for a function $f : X \rightarrow Y$, the subset $\{(x, f(x)) : x \in X\} \subset X \times Y$ is called the graph of f and is denoted by $G(f)$.

Theorem 2.11. If $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is a quasi-preirresolute function and Y is a quasi-pre- T_2 space, then $G(f)$ is quasi-preclosed in $X \times Y$.

Proof. Let $(x, y) \notin G(f)$. Then $y \neq f(x)$ and so there exist disjoint quasi-preopen sets V and W such that $f(x) \in W$ and $y \in V$. Since f is quasi-preirresolute, there exists a quasi-preopen set U in X containing x such that $f(U) \subset W$. Then we obtain $(x, y) \in U \times V \subset X \times Y \setminus G(f)$. By Lemma 2.9 and Theorem 2.2 of [9], $X \times Y \setminus G(f)$ is quasi-preopen in $X \times Y$. Hence $G(f)$ is quasi-preclosed in $X \times Y$. \square

3. Quasi-pre- R_0 and quasi-pre- R_1 spaces

Definition 3.1 [12]. A space (X, τ_1, τ_2) is called quasi-pre- R_0 if for each quasi-preopen set G and each $x \in G$ implies $\text{qpcl}\{x\} \subset G$.

To obtain new characterization of quasi-pre- R_0 space we need the following:

Definition 3.2 [12]. In a space (X, τ_1, τ_2) , the quasi-pre-kernel of the point x , denoted by $\text{qpker}\{x\}$, is defined to be the set of all $y \in X$ such that $x \in \text{qpcl}\{y\}$, i.e. $\text{qpker}\{x\} = \{y : x \in \text{qpcl}\{y\}\}$.

Lemma 3.3 [12]. Let (X, τ_1, τ_2) be a space and $x, y \in X$. Then $\text{qpker}\{x\} \neq \text{qpker}\{y\}$ if and only if $\text{qpcl}\{x\} \neq \text{qpcl}\{y\}$.

Theorem 3.4. A space (X, τ_1, τ_2) is quasi-pre- R_0 if and only if for each $x, y \in X$ $\text{qpcl}\{x\} = \text{qpcl}\{y\}$ or $\text{qpcl}\{x\} \cap \text{qpcl}\{y\} = \phi$.

Proof. Let $x, y \in X$. If $\text{qpcl}\{x\} \neq \text{qpcl}\{y\}$, by Lemma 3.3 $\text{qpker}\{x\} \neq \text{qpker}\{y\}$, i.e. $z \in \text{qpker}\{x\}$ and $z \notin \text{qpker}\{y\}$ for some z . Then $x \in U$ and $y \notin U$ for any quasi-preopen set U containing z . Since X is quasi-pre- R_0 , $\text{qpcl}\{x\} \subset U$ and $\text{qpcl}\{y\} \subset X \setminus U$, which implies that $\text{qpcl}\{x\} \cap \text{qpcl}\{y\} = \phi$.

Conversely, let U be a quasi-preopen set and $x \in U$. If $y \in \text{qpcl}\{x\}$, then by assumption, $x \in \text{qpcl}\{x\} = \text{qpcl}\{y\}$ and so $y \in U$. Then $\text{qpcl}\{x\} \subset U$ and hence X is quasi-pre- R_0 . \square

Theorem 3.5. For a space (X, τ_1, τ_2) , the following are equivalent:

- (a) X is quasi-pre- R_0 ;
- (b) for each $x \in X$, $\text{qpcl}\{x\} \subset \text{qpker}\{x\}$;
- (c) for each $x, y \in X$, $y \notin \text{qpcl}\{x\}$ implies $x \notin \text{qpcl}\{y\}$;
- (d) for each quasi-preclosed set F and each $x \notin F$, there exists a quasi-preopen set U such that $F \subset U$ and $x \notin U$;
- (e) for any quasi-preclosed set F in X and $x \notin F$, $\text{qpcl}\{x\} \cap F = \phi$;
- (f) for each $x, y \in X$, $x \notin \text{qpcl}\{y\}$ implies $\text{qpcl}\{x\} \cap \text{qpcl}\{y\} = \phi$.

Proof. (a) \Leftrightarrow (b): Obvious.

(a) \Rightarrow (c): Since $y \notin \text{qpcl}\{x\}$, $y \in X \setminus \text{qpcl}\{x\}$ and by (a), $\text{qpcl}\{y\} \subset X \setminus \text{qpcl}\{x\} = \text{qpint}(X \setminus \{x\}) \subset X \setminus \{x\}$. Hence $x \notin \text{qpcl}\{y\}$.

(c) \Rightarrow (a): Let G be quasi-preopen in X and $x \in G$. If $y \notin G$, then $x \notin \text{qpcl}\{y\}$ and by (c), $y \notin \text{qpcl}\{x\}$. Thus $\text{qpcl}\{x\} \subset G$ and hence X is quasi-pre- R_0 .

(c) \Rightarrow (d): Let F be quasi-preclosed in X and $x \notin F$. Then $x \notin \text{qpcl}\{y\}$ for any $y \in F$. By (c), $y \notin \text{qpcl}\{x\}$ for any $y \in F$ and hence for any $y \in F$, there exists a quasi-preopen set U_y containing y such that $x \notin U_y$. Put $U = \cup_{y \in F} \{U_y : x \notin U_y\}$. Then U is quasi-preopen in X , $F \subset U$ and $x \notin U$.

(d) \Rightarrow (e): Let F be quasi-preclosed in X and $x \notin F$. Then by (d), there exists a quasi-preopen set U containing F such that $\{x\} \cap F = \phi$, which implies $\text{qpcl}\{x\} \cap F = \phi$.

(e) \Rightarrow (f): Obvious.

(f) \Rightarrow (c): Obvious. \square

Theorem 3.6. A space (X, τ_1, τ_2) is quasi-pre- R_0 if and only if for each $x, y \in X$ with $x \neq y$, if there exists a quasi-preopen set U containing x such that $y \notin U$ then there exists a quasi-preopen set V containing y such that $x \notin V$.

Proof. Let X be a quasi-pre- R_0 space and $x, y \in X$ with $x \neq y$. If there exists a quasi-preopen set U containing x such that $y \notin U$, then $\text{qpcl}\{x\} \subset U$ and we have $X \setminus \text{qpcl}\{x\}$ is quasi-preopen in X , $y \in X \setminus \text{qpcl}\{x\}$ and $x \notin X \setminus \text{qpcl}\{x\}$.

Conversely, let G be a quasi-preopen set and $x \in G$. If $y \in \text{qpcl}\{x\}$ such that $y \notin G$, then there exists a quasi-preopen set U containing y such that $x \notin U$, i.e. $x \in X \setminus U$ so that it is impossible that $y \in \text{qpcl}\{x\}$. \square

Theorem 3.7. Every bi- α -open subspace of a quasi pre- R_0 space is quasi pre- R_0 .

Proof. Let $(Z, (\tau_1)_Z, (\tau_2)_Z)$ be a bi- α -open subspace of a space (X, τ_1, τ_2) . Let A be quasi-preclosed in Z and $x \notin A$. Since $Z \setminus A$ is quasi-preopen in Z and Z is bi- α -open in X , $Z \setminus A$ is quasi-preopen in X . Then $(X \setminus Z) \cup A$

is quasi-preclosed in X and $x \notin (X \setminus Z) \cup A$. By Theorem 3.5, there exists a quasi-preopen set U of X such that $x \notin U$ and $(X \setminus Z) \cup A \subset U$. By Lemma 1.3, $Z \cap U$ is quasi-preopen in Z , $A \subset Z \cap U$ and $x \notin Z \cap U$. Hence Z is quasi-pre- R_0 . \square

Corollary 3.8 [12]. Every bi-open subspace of a quasi pre- R_0 space is quasi pre- R_0 .

Definition 3.9. A space (X, τ_1, τ_2) is called quasi-pre- R_1 if for $x, y \in X$ with $\text{qpcl}\{x\} \neq \text{qpcl}\{y\}$, there exist disjoint quasi-preopen sets U and V such that $\text{qpcl}\{x\} \subset U$ and $\text{qpcl}\{y\} \subset V$.

Theorem 3.10. Every quasi-pre- R_1 space is quasi-pre- R_0 .

Proof. Let U be a quasi-preopen set of a space (X, τ_1, τ_2) and $x \in U$. If $y \notin U$, then $\text{qpcl}\{y\} \subset X \setminus U$. This means $x \notin \text{qpcl}\{y\}$ and so there exist disjoint quasi-preopen sets U_x and U_y such that $x \in U_x$ and $y \in U_y$, which implies $y \notin \text{qpcl}\{x\}$. Therefore $\text{qpcl}\{x\} \subset U$ and so X is quasi-pre- R_0 . \square

Corollary 3.11. A space (X, τ_1, τ_2) is quasi-pre- R_1 if and only if for $x, y \in X$, $x \notin \text{qpcl}\{y\}$ implies there exist disjoint quasi-preopen sets U and V such that $x \in U$ and $y \in V$.

Theorem 3.12. Every bi- α -open subspace of a quasi pre- R_1 space is quasi pre- R_1 .

Proof. It follows from Corollary 3.11 and Lemmas 1.3 and 1.4. \square

Theorem 3.13. If a space (X, τ_1, τ_2) is quasi-pre- R_0 and X is finite, then X is quasi-pre- R_1 .

Proof. Let $x \in X$ and $X \setminus \text{qpcl}\{x\} = \{y_1, y_2, \dots, y_n\}$. Then $y_i \notin \text{qpcl}\{x\}$ for any $i = 1, 2, \dots, n$. By Theorem 3.4, $\text{qpcl}\{y_i\} \cap \text{qpcl}\{x\} = \phi$ for any $i = 1, 2, \dots, n$ and hence $\text{qpcl}\{x\} \cap \bigcup_{i=1}^n \text{qpcl}\{y_i\} = \phi$, which implies that $\text{qpcl}\{x\} \subset X \setminus \text{qpcl}(X \setminus \text{qpcl}\{x\}) = \text{qpint}(\text{qpcl}\{x\}) \subset \text{qpcl}\{x\}$ and hence $\text{qpcl}\{x\}$ is quasi-preopen in X . By Corollary 3.11, X is quasi-pre- R_1 . \square

Theorem 3.14. Let (X, τ_1, τ_2) be a space with the following property: for each $x, y \in X$ with $x \neq y$ if there exists a quasi-preopen set U containing x such that $y \notin U$ then there exist disjoint quasi-preopen sets V and W such that $U \subset V$ and $y \in W$. Then X is quasi-pre- R_1 .

Proof. If the conditions of Theorem 3.14 are fulfilled so are the conditions of Theorem 3.6, and then the space X is quasi-pre- R_0 . Let us show that it is quasi-pre- R_1 as well. If $\text{qpcl}\{x\} \cap \text{qpcl}\{y\} = \phi$, then $X \setminus \text{qpcl}\{x\}$ is quasi-preopen in X , $y \in X \setminus \text{qpcl}\{x\}$ and $x \notin \text{qpcl}\{x\}$ and by assumption, there exist disjoint quasi-preopen sets U and V such that $x \in U$, $y \in V$ and $V \subset X \setminus \text{qpcl}\{x\}$. Since X is quasi-pre- R_0 , $\text{qpcl}\{x\} \subset U$ and $\text{qpcl}\{y\} \subset V$. Hence X is quasi-pre- R_1 . \square

Tapi et al. [12] showed that a space is quasi-pre- T_1 if and only if it is quasi-pre- R_0 and quasi-pre- T_0 . We obtain similar result as follows:

Theorem 3.15. A space (X, τ_1, τ_2) is quasi-pre- T_2 if and only if it is quasi-pre- R_1 and quasi-pre- T_0 .

Proof. Let $x, y \in X$ with $x \notin \text{qpcl}\{y\}$. Then $x \neq y$ and there exist disjoint quasi-preopen sets U and V such that $x \in U$ and $y \in V$. By Corollary 3.11, X is quasi-pre- R_1 .

Conversely, let $x, y \in X$ with $x \neq y$. Since X is quasi-pre- T_0 , there exists a quasi-preopen set U_x containing x such that $y \notin U_x$ or there exists a quasi-preopen set U_y containing y such that $x \notin U_y$, i.e. $x \notin \text{qpcl}\{y\}$ or $y \notin \text{qpcl}\{x\}$. Since X is quasi-pre- R_1 , there exist disjoint quasi-preopen sets V and W such that $x \in V$ and $y \in W$. Hence X is quasi-pre- T_2 . \square

References

- [1] M. C. Dutta, Contribution to the theory of bitopological spaces, Ph. D. Thesis, B. I. T. Pilani, India, (1971)
- [2] S. El-Deeb, I. Hasanein, A. S. Mashhour and T. Noiri, On p -regular spaces, Bull. Math. Soc. Sci. Math. R. S. Roumanie, 27, (1983), 4, 311–315
- [3] J. C. Kelly, Bitopological spaces, Proc. London Math. Soc. 13, (1963), 71–80
- [4] A. S. Mashhour, M. Abd El-Monsef and S. El-Deeb, On precontinuous and weak precontinuous mappings, Proc. Math. Phys. Soc. Egypt, 53, (1982), 47–53
- [5] A. S. Mashhour, F. H. Khder and S. N. El-Desh, Separation axioms in bitopological spaces, Bulletin of the faculty of Science, Assint University, 1, (1982), 53–67
- [6] T. Noiri, H. Maki and J. Umehara, Generalized preclosed functions, Mer. Fac. Sci. Kochi Univ. (Math.), 19, (1998), 13–20
- [7] J. H. Park and Y. S. Pyo, On generalized quasi-preclosed sets and quasi presep-
aration axioms, Honam Math. J. 25, (2003), 1, 141–152
- [8] U. D. Tapi, S. S. Thakur and A. Sonwalkar, Quasi pre T_0 , quasi pre T_1 and quasi
pre T_2 bitopological spaces, U. Scientist Phyl. Sciences, 6, (1994), 1, 122–125
- [9] —, Quasi-preopen sets, Indian Acad. Math. 17, (1995), 8–12
- [10] —, Quasi P -normal bitopological spaces, Bulletin of Pure and Applied Sciences,
14, (1995), 2, 147–150
- [11] —, On quasi pre continuous and quasi pre-irresolute mappings, Acta Ciencia
Indica, 21, (1995), 2, 235–237

- [12] ., Quasi pre- R_0 bitopological spaces, The Mathematics Education, XXIX, (1995), 3, 147–149
- [13] ., Quasi P -regular spaces, (under communication) (1995)

Jin Han Park
Division of Mathematical Sciences,
Pukyong National University,
Pusan 608–739, Korea
E-mail : jihpark@pknu.ac.kr