*Exploration Geophysics (2004) **35**, 14–18*
Butsuri-Tansa (Vol. 57, No. 1)
Mulli-Tamsa (Vol. 7, No. 1)

Resolving a velocity inversion at the geotechnical scale using the microtremor (passive seismic) survey method

James C. Roberts[1] Michael W. Asten[1]

Key Words: Rayleigh waves, microtremor method, shear velocities, geotechnical studies

ABSTRACT

High levels of ambient noise and safety factors often limit the use of "active-source" seismic methods for geotechnical investigations in urban environments. As an alternative, shear-wave velocity–depth profiles can be obtained by treating the background microtremor wave field as a stochastic process, rather than adopting the traditional approach of calculating velocity based on ray path geometry from a known source. A recent field test in Melbourne demonstrates the ability of the microtremor method, using only Rayleigh waves, to resolve a velocity inversion resulting from the presence of a hard, 12 m thick basalt flow overlying 25 m of softer alluvial sediments and weathered mudstone. Normally the presence of the weaker underlying sediments would lead to an ambiguous or incorrect interpretation with conventional seismic refraction methods. However, this layer of sediments is resolved by the microtremor method, and its inclusion is required in one-dimensional layered-earth modelling in order to reproduce the Rayleigh-wave coherency spectra computed from observed seismic noise records.

Nearby borehole data provided both a guide for interpretation and a confirmation of the usefulness of the passive Rayleigh-wave microtremor method. Sensitivity analyses of resolvable modelling parameters demonstrate that estimates of shear velocities and layer thicknesses are accurate to within approximately 10% to 20% using the spatial autocorrelation (SPAC) technique. Improved accuracy can be obtained by constraining shear velocities and/or layer thicknesses using independent site knowledge. Although there exists potential for ambiguity due to velocity–thickness equivalence, the microtremor method has significant potential as a site investigation tool in situations where the use of traditional seismic methods is limited.

INTRODUCTION

Traditional approaches to shallow seismic testing involve the use of "active source" methods such as seismic reflection and refraction. In geotechnical applications in particular, seismic refraction with surface seismic sources has gained widespread acceptance as a viable investigation tool (Whiteley, 1994). The effectiveness of this approach, especially in urban situations, is limited by the presence of seismic noise and in the choice of a source with sufficient energy to achieve the required depth penetration. Additionally, the seismic refraction method is inherently "blind" to the presence of a velocity inversion (Whiteley and Greenhalgh, 1979).

An alternative approach is to use "natural" microtremors (the "noise" in traditional seismic surveying), as a source of wave energy. The measurement of high-frequency seismic noise, or microtremors, is a well-established method of estimating the seismic resonance characteristics of relatively thick (tens of metres and above) unconsolidated sediments. This approach is described by Nakamura (1989), where the fundamental resonance period (T_S) of a site can be obtained from surface waves and used in the assessment of potential seismic hazard to structures founded in soft soils.

A less well-known application of "natural" seismic noise is based upon the spatial autocorrelation (SPAC) technique first described by Aki (1957), and recently reviewed by Okada (2003), Tokimatsu (1997), and Asten et al. (2003). The value of Aki's array-processing technique is that its estimates of scalar wave velocity are not affected by signals of similar frequency arriving from differing directions. Indeed, the SPAC technique operates more efficiently when signals arrive from a broad range of azimuths (Asten, 2003).

In this paper, the SPAC technique is used to obtain a shear-wave velocity–depth profile at a site displaying a distinct and verifiable velocity inversion.

MICROTREMORS

Microtremors (also called "microseisms") are seismic waves of relatively low energy having amplitudes typically in the range of 10^{-4} to 10^{-2} mm (Okada, 2003). In general, microtremors are an assemblage of body and surface wave motions, although most of the wave energy is transported as surface waves (Toksöz and Lacoss, 1968). To a first approximation, microtremors with a frequency greater than 1 Hz are produced by cultural sources (such as trains, road traffic, and machinery), while frequencies less than 1 Hz are the result of natural phenomena such as wave action at coastlines, wind, and atmospheric variations (Okada, 2003).

Most seismic prospecting and seismological applications use the traditional "ray-path" approach to estimate seismic velocity based upon the time taken for a distinct non-dispersive seismic "event" to propagate between two (or more) points of observation. However, microtremors are dispersive and form a continuous low-amplitude wave "field" – an assemblage of body and surface waves that originates in space and time from a wide variety of sources, and propagates over a wide frequency band. While the amplitude and frequency content of microtremors can display significant variation in both space and time, they can be assumed stationary when considered over suitably short time intervals. Processing of microtremor data using the SPAC technique is undertaken by treating ground motions as representing a stochastic process (Okada, 2003). Phase velocities are determined by averaging signal coherency between multiple observation points in an array of receivers, with no consideration of the direction (or distance) to the source.

[1] *Monash University, Victoria, Australia*

Email: James.Roberts@sci.monash.edu.au

Manuscript received 24 December, 2003.
Revised manuscript received 2 February, 2004.

SHEAR-WAVE VELOCITY PROFILE BY THE SPAC METHOD

The application described in the following deals only with vertical-component seismic measurements, limiting discussion to (dispersive) Rayleigh-wave motion. It is possible to obtain Rayleigh wave samples propagating from a wide range of azimuthal angles by using a circular array of geophones placed equidistant from a single, central geophone. The signal coherency spectrum may be computed for any pair of geophones in the array using standard spectral analysis techniques (e.g., Koopmans, 1974). Azimuthally averaged coherencies can be computed by averaging the coherencies for all geophone pairs in the array having the same spatial scalar separation. Assuming that, at each frequency, the wave energy propagates with only one (scalar) velocity, it can be shown (Aki, 1957; Asten, 1976; Okada, 2003) that the azimuthally averaged coherency for a circular array is given by:

$$\bar{c}(f) = J_0(kr) = J_0\left(\frac{2\pi f r}{V(f)}\right) \qquad (1)$$

where: f is frequency,
$\bar{c}(f)$ is the azimuthally averaged coherency,
J_0 is the zero-order Bessel function,
k is the scalar wavenumber,
$V(f)$ is the velocity dispersion relationship, and
r is the inter-station distance (station separation) in the circular array.

By comparing azimuthally averaged coherencies calculated from field observations with those derived theoretically from a horizontally layered earth model, an iterative curve matching procedure enables a "best-fit" shear wave velocity–depth profile to be obtained. Theoretical coherencies are determined by first computing dispersion curves of phase velocity vs frequency for Rayleigh waves, using routines by Herrmann (2001), and then computing a model coherency spectrum using equation (1).

FIELD TEST

A field test was undertaken using the SPAC technique on microtremor data collected at an open parkland site located in inner metropolitan Melbourne. Figure 1 shows the array location and the position of nearby boreholes that were used in the analysis.

The site is located to the west of the Yarra River, with surface geology composed of Quaternary basalt known as the Burnley Basalt. The surface of the basalt is weathered to clay to a depth of less than 2 m, on the surface of the site. Underlying the basalt are Pleistocene to Tertiary alluvial clay sediments, representing the flood plain of the Yarra before its course was displaced to its present location by the later basalt flows (Neilson, 1970). Beneath this sediment lies the basement rock, a folded Silurian siltstone formation known as the Melbourne Mudstone.

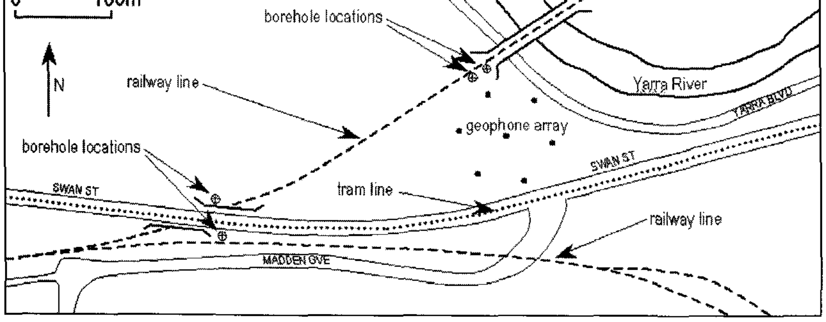

Fig. 1. Field test site in Melbourne, showing array location, boreholes and proximity to sources of seismic noise (roads, railway and tram lines).

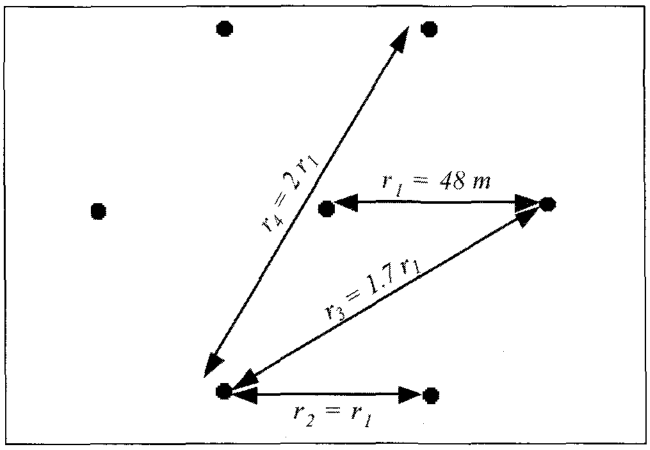

Fig. 2. A hexagonal "circular" array of seven stations, for SPAC processing with four inter-station distances.

Vertical-component ground motions were recorded using seven Mark L4C 1 Hz geophones, placed in a hexagonal array with an array radius of 48 m. The geophones were clamped to aluminium plates, and ground coupling was achieved with three 100 mm spikes on the base of each plate. The geophones were connected to a 6-channel Kelunji seismic recorder and external digitiser set to record at maximum gain. Multiple recordings of ground motion were made for periods of approximately 200 seconds at a sampling rate of 200 samples per second. Because of the presence of nearby trains, trams, and road traffic, which often produce impulsive highly-directional seismic noise which in turn yields poor (noisy) SPAC coherencies, multiple data sets were recorded to enable selection and processing of the "cleanest" averaged coherency spectra possible.

The field recordings resulted in seven records of vertical-component ground motion (one per geophone) for each 200-second recording period. Each of these data sets was transformed to the frequency domain and corrected for the transfer function and gain of each geophone and digitiser channel. Inter-station coherencies were then computed for each pair of spectra in the array. The *average* coherency for all geophone pairs sharing a common array separation (refer to Figure 2) was then computed for the radial station pairs (r_1=48 m), the circumferential pairs (r_2=48 m), the off-diagonals (r_3=83.1 m), and the diameters (r_4=96 m). These coherency curves were compared, and modelling to obtain the shear-wave velocity–depth profile proceeded after selection of the cleanest averaged-coherency spectra obtained from the multiple 200-second records.

The use of a seven-station array as shown in Figure 2 allows coherencies from field measurements to be azimuthally averaged over four station separations (r_1, r_2, r_3, and r_4) as shown in Figure 3. The iterative matching of field and model coherencies is made more robust by fitting the four averaged coherency spectra simultaneously.

ANALYSIS AND INTERPRETATION

Analysis of the field data was undertaken using a horizontally-layered earth model. Each layer was considered to be an isotropic, homogenous unit characterised by four parameters: thickness (h), density (ρ), P-wave velocity (V_P), and shear-wave velocity (V_S). Although there are four model parameters for each layer (h, ρ, V_P and V_S), studies such as those by Mooney and Bolt (1966), Bloch, Hales, and Landisman (1969), and Xia, Miller, and

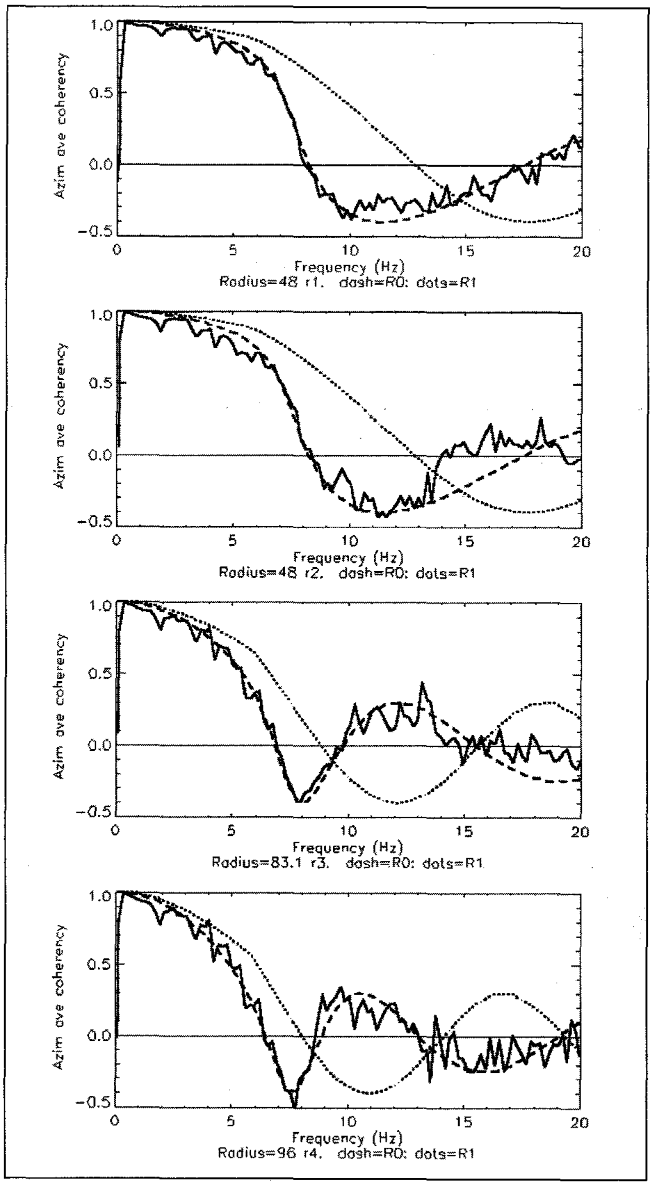

Fig. 3. Best-fit coherency models for field data (separations r_1, r_2, r_3, and r_4). The solid line represents the observed azimuthally averaged coherency spectrum from field data. Dashed line is the theoretical coherency spectrum for fundamental mode Rayleigh waves for the "best-fit" layered earth model. Dotted line is the theoretical coherency spectrum for the first higher mode Rayleigh wave.

Park (1999) show that resulting dispersion curves are primarily sensitive to variations in layer thickness (h) and shear-wave velocity (V_S). Consequently, only these two parameters were varied in the iterative modelling, with the corresponding densities and P-wave velocities set as constants.

A number of boreholes have been drilled for railway works at bridge sites to the north and west of the array (refer to Figure 1). Logs from these boreholes assisted in developing a starting model for computing the theoretical azimuthally averaged coherency spectra for the site. As noted above, the method has a low sensitivity to P-wave velocities and densities of the layers, hence these were fixed at values shown in Figure 5. P-wave velocities for layers of basalt and basement rock were set assuming a Poisson's ratio of 0.25 (i.e., V_P/V_S=1.7). The P-wave velocity of the soft sediments under the basalt was set at an arbitrary value expected for saturated unconsolidated sediments, and typical densities for rock and sediments were assigned from geotechnical data for the Melbourne area.

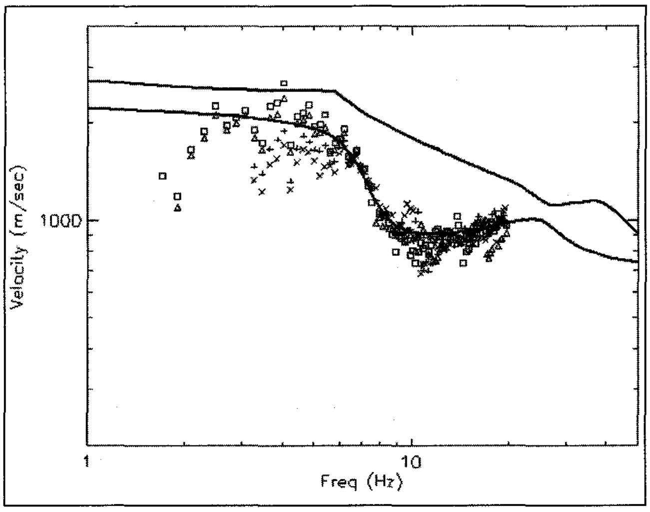

Fig. 4. Shear-wave velocity estimates from the field data shown in Figure 3, together with the theoretical dispersion curves for the fundamental (lower solid line) and first-higher (upper solid line) Rayleigh-wave modes.

After several iterations, a "best-fit" horizontally layered earth model was achieved by varying the shear-velocity and thickness parameters for the layers. Figure 3 shows the coherency plots for the "best-fit" model. The plots in Figure 3 indicate a close fit between the theoretical and measured coherency, particularly in the 6–10 Hz range. Although the coherencies at higher frequencies (>10 Hz) are more "noisy", the theoretical coherency curves still follow the apparent trend in the data. The corresponding phase velocity vs frequency dispersion curve for this model is shown in

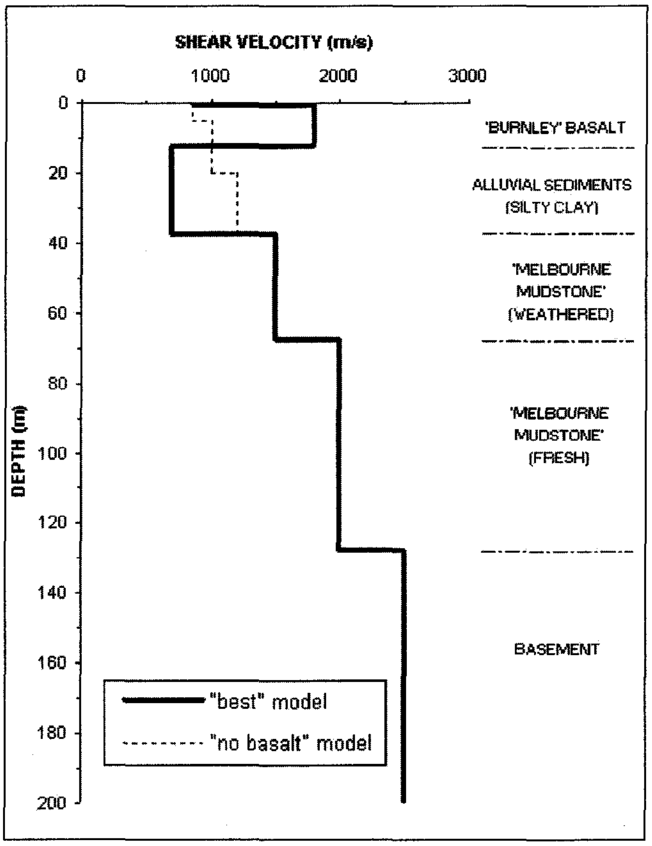

Fig. 5. Shear wave velocity profiles for horizontally layered earth models obtained by iterative modelling. Assumed densities for basalt, sediments, weathered mudstone, and fresh mudstone are 2.4, 1.9, 2.2, and 2.5 t/m^3 respectively. Corresponding assumed P-wave velocities are 3100, 1800, 2600, and 3500 m/s respectively.

Figure 4. The resultant profile of shear-wave velocity vs depth is plotted in Figure 5.

For comparison purposes, the theoretical coherency for a "no-basalt" model was also computed. This "no-basalt" model has the corresponding shear-wave velocity–depth profile also plotted in Figure 5, consisting of a gradual velocity increase with depth in place of the velocity inversion resulting from the presence of the basalt layer. Note that the basement depth and velocities are the same for both the "best-fit" and "no-basalt" layers. Plots of the modelled coherency spectra for the "no-basalt" model (Figure 6) clearly show a poorer fit to the field data when compared with models incorporating a velocity inversion (Figure 3).

Some equivalence in the model response was observed when comparing the effects of varying the velocity and thickness of single layers, particularly when making relatively small adjustments to model parameters. Despite this, the "best-fit" model clearly provides a superior fit compared with all models lacking a high velocity shallow layer overlying softer, lower-velocity material at depth.

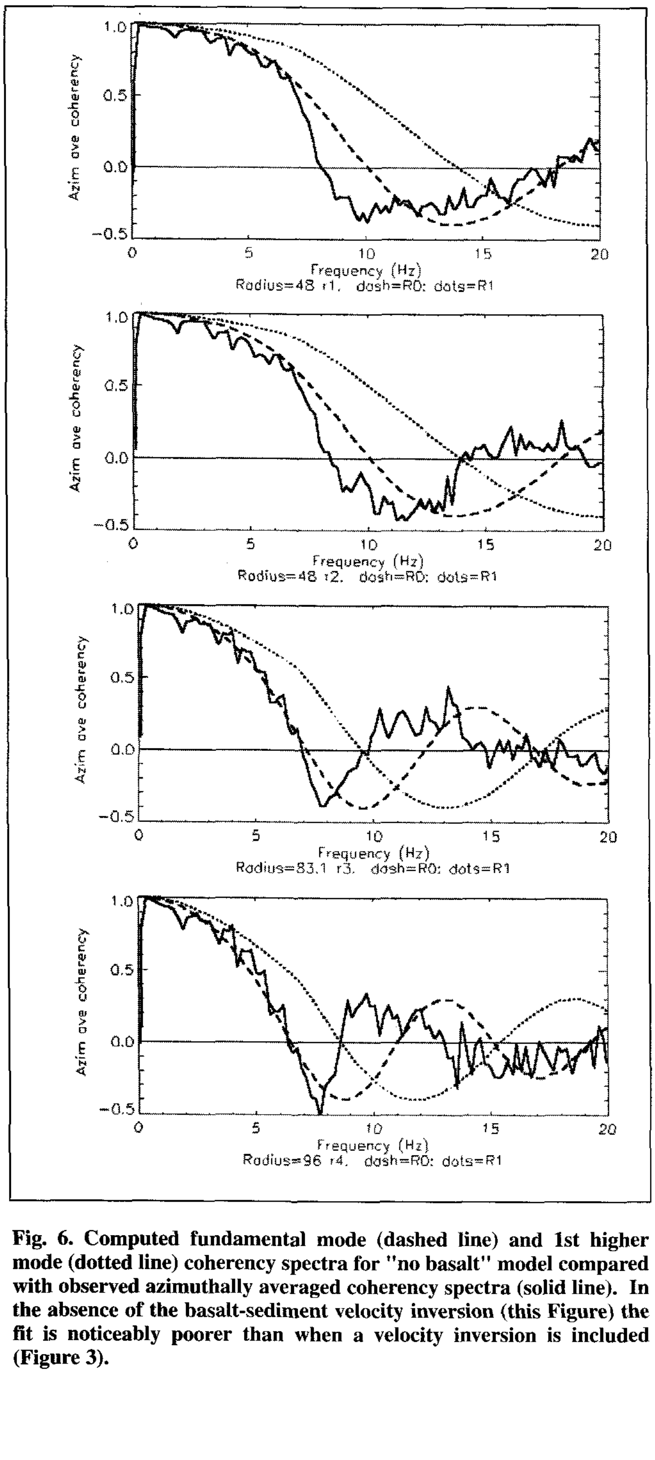

Fig. 6. Computed fundamental mode (dashed line) and 1st higher mode (dotted line) coherency spectra for "no basalt" model compared with observed azimuthally averaged coherency spectra (solid line). In the absence of the basalt-sediment velocity inversion (this Figure) the fit is noticeably poorer than when a velocity inversion is included (Figure 3).

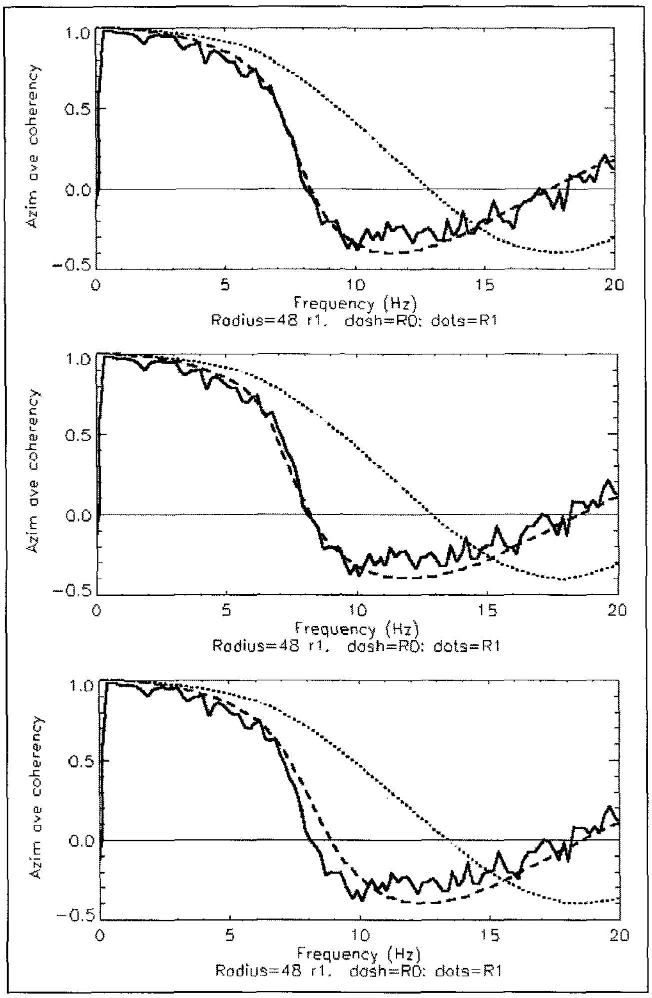

Fig. 7. Examples from sensitivity analysis. Top: "best-fit" coherency model using parameters from Fig. 5. Centre: Perturbation when the model has a 2 m increase in basalt thickness (12 m to 14 m). Bottom: Perturbation when the model has a 100 m/s increase in shear velocity of the underlying sediments (700 m/s to 800 m/s).

SENSITIVITY ANALYSIS

The Rayleigh wave coherency spectra derived from phase velocity curves for a given layered earth model represent the effect of waves propagating at a range of frequencies. Phase velocity at a particular Rayleigh frequency is affected by physical parameters (principally shear-wave velocity) within approximately one wavelength of the surface. The relative sensitivity of phase velocity to physical parameters varies over this range, with sensitivity to variations in shear velocity being greatest in the depth range approximately 0.25 to 0.4λ (Asten, 1976). Since phase velocity is determined by unequally weighted contributions of shear velocity of layers between the surface and a depth of one wavelength, the response of relatively thin layers may be poorly resolved due to this "smearing" effect – particularly for those layers located at depth.

In order to gain an understanding of the accuracy associated with the "best-fit" model a simple qualitative sensitivity analysis was undertaken. The modelled coherency spectrum was recomputed for a series of models, in which a single model parameter (h or V_S) was varied in a single layer. Each parameter (basalt velocity, basalt thickness, sediment velocity, sediment thickness, and basement velocity) was varied individually in the range of ±10%.

Overall, parameter variations of order ±10% produced observable changes in the model coherency curves and poorer fits to the observed data, with the effect being more pronounced with thicker layers. However, for some model parameters, variations closer to ±20% were required to provide an unambiguously poorer fit to the observed data. To illustrate this, coherency plots (fundamental mode R1 coherencies only) for a 2 m increase in basalt thickness and a 100 m/s increase in sediment velocity are presented in Figure 7. Although the 2 m increase in basalt thickness represents a greater relative change in the model parameter (+16.7%), the effect on the coherency plot is far less pronounced than that of a 100 m/s increase in sediment velocity (+14.3%). Note that while the "+2 m basalt" model provides a close fit to the observed data, on close inspection it is a poorer fit than the "best-fit" model, particularly in the 6–7 Hz range where the field data is relatively smooth.

Parameters for layers below the weathered mudstone are insensitive to variation, and so are poorly resolved by this data. Sensitivity to shear velocity and layer thicknesses at greater depths requires use of a larger array; see for example the use of an array of effective radius 300 m to acquire shear-velocity data to a depth of order 1000 m in the Santa Clara Valley, California (Asten, 2004).

As mentioned in the previous section, some equivalence in varying shear-velocity and layer thickness is observed in the coherency curve, with an increase in layer thickness having a similar effect to a decrease in layer shear-velocity. This ambiguity can in part be overcome using a priori site knowledge. Information on the basic geologic structure can assist in providing constraints on layer thicknesses and likely range of physical properties of the subsurface units.

CONCLUSIONS

Use of the passive seismic technique with SPAC processing at this site illustrates the potential for the method to define and differentiate geological structure at a geotechnical scale at a site that might otherwise be deemed "seismically hostile" due to its geology (a high-velocity surface layer) and its location (between two railway lines and adjacent to an arterial road). The SPAC technique for microtremor analysis has the potential to unlock substantial information from seemingly random "natural" ground motion. Furthermore, the data obtained can be indicative of subsurface conditions over a wide depth range of order $0.1r$ to $2r$, where r is the array radius.

Sensitivity analyses undertaken as part of this study indicate that under favourable conditions, layer thickness and velocity can be resolved to accuracy on the order of 10% to 20% with some knowledge as to the subsurface geology. Although only data recorded with a 48-m radius array (the maximum permitted by the size of the open space) has been presented here, data collected from a smaller array radius would (as demonstrated by Asten and Dhu, 2003) provide additional resolution in the near surface. Likewise, a larger array radius allows better sampling of lower frequencies and hence better resolution at depth. Lateral resolution is restricted by the requirement to perform layered-earth (one-dimensional) interpretations for data from each array.

We note that the microtremor survey method could in future be used to create profiles by consecutive placements of a circular array, although the logistics of such a survey would be less efficient than for linear profiles obtained using conventional seismic techniques.

Although the SPAC technique is relatively untested when compared with conventional seismic methods, it has the potential to provide an alternative approach to geotechnical-scale site investigation. While the practicality of placing a large array may be restricted in some urban settings, the SPAC microtremor technique has the advantage of being less intrusive and less demanding in terms of permits required than active-source seismic methods. These advantages offer the likelihood of being more cost effective in many urban situations.

ACKNOWLEDGEMENTS

James Roberts is supported by a Monash University PhD scholarship. The authors thank Dr R.J. Whiteley for review and constructive comments on the paper.

REFERENCES

Aki, K., 1957, Space and time spectra of stationary stochastic waves, with special reference to microtremors: *Bulletin of the Earthquake Research Institute*, **35**, 415–456.

Asten, M.W., (1976), *The use of microseisms in geophysical exploration*: PhD Thesis, Macquarie University, Australia.

Asten, M.W., 2003, Lessons from alternative array design used for high-frequency microtremor array studies: in Wilson, J.L., Lam, N.K., Gibson, G., and Butler, B., (eds.), *Earthquake Risk Mitigation: Proceedings of a Conference of the Australian Earthquake Engineering Soc.*, Melbourne, Paper 14.

Asten, M.W., Dhu, T., Jones, A., and Jones, T., 2003, Comparison of shear-velocities measured from microtremor array studies and SCPT data acquired for earthquake site hazard classification in the northern suburbs of Perth W.A.: in Wilson, J.L., Lam, N.K., Gibson, G., and Butler, B., (eds.) *Earthquake Risk Mitigation: Proceedings of a Conference of the Australian Earthquake Engineering Soc.*, Melbourne, Paper 12.

Asten, M.W., 2004, Method for site hazard zonation, Santa Clara valley: Thickness and shear-velocity mapping of Holocene-Pleistocene sediments by array studies of microtremors: *Proceedings of First Annual Northern California Earthquake Hazards Workshop*, USGS, Menlo Park.

Bloch, S., Hales, A.L., and Landisman, M., 1969, Velocities in the crust and upper-mantle of southern Africa, from multimode surface-wave dispersion: *Bulletin of the Seismological Society of America*, **59**, 1599–1629.

Herrmann, R.B., 2001, *Computer programs in seismology - an overview of synthetic seismogram computation Version 3.1*: Department of Earth and Planetary Sciences, St Louis University.

Koopmans, L.H., 1974, *Spectral analysis of time series*: Academic Press.

Mooney, H.M., and Bolt, B.A., 1966, Dispersive characteristics of the first three Rayleigh modes for a single surface layer: *Bulletin of the Seismological Society of America*, **56**, 43–67.

Nakamura, Y., 1989, A method for dynamic characteristics estimation of subsurface using microtremors on the ground surface: *Quarterly Reports of the Railway Technical Research Institute, Tokyo*, **30**, 25–33.

Neilson, J.L., 1970, *Geological report on the western portion of the proposed additional span for the railway bridge over the Yarra River between Hawthorn and Burnley*: Unpublished report. Department of Mines, Victoria, Australia.

Okada, H., 2003, *The Microseismic Survey Method*: Society of Exploration Geophysicists of Japan. Translated by Koya Suto, Geophysical Monograph Series No. 12, Society of Exploration Geophysicists.

Tokimatsu, K., 1997, Geotechnical site characterization using surface waves: in Ishihara (ed.), *Earthquake Geotechnical Engineering*: Balkema.

Toksöz, M.N. and Lacoss, R.T., 1968, Microseisms: mode structures and sources: *Science*, **159**, 872–873.

Whiteley, R.J., and Greenhalgh, S.A., 1979, Velocity inversion and the shallow seismic refraction method: *Geoexploration*, **17**, 125–141.

Whiteley, R.J., 1994, Seismic refraction testing – a tutorial: in Woods, R.C. (ed.), *Geophysical Characterisation of Sites*: Balkema, 45–47.

Xia, J., Miller, R.D., and Park, C.B., 1999, Estimation of shear-wave velocity by inversion of Rayleigh waves: *Geophysics*, **64**, 691–700.