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LINEAR ANALYSIS OF PARKER-JEANS INSTABILITY WITH COSMIC-RAY

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ABSTRACT

We present the results of the linear analysis for the Parker-Jeans instability in the magnetized gas disks including the effect of cosmic-ray diffusion along the magnetic field lines. We adopted an uniformly rotating two temperature layered disk with a horizontal magnetic fields and solved the perturbed equations numerically. Fragmentation of gases takes place and filamentary structures are formed by the growth of the instability. Nagai et al. (1998) showed that the direction of filaments being formed by the Parker-Jeans instability depends on the strength of pressure outside the unperturbed gas disk. We found that at some range of external pressures the direction of filaments is also governed by the value of the diffusion coefficient of CR along the magnetic field lines $\kappa_{\parallel}$.

Key words : cosmic rays – ISM:clouds – ISM:structure – magnetic fields

I. INTRODUCTION

Cosmic rays (CRs) are formed by supernova remnants. They diffuse throughout our galaxy via hydromagnetic irregularities (e.g., waves, turbulence). Thus although the distribution of supernova remnants is not uniform in our galaxy, cosmic rays (at least for those with energy $< 10^{17} eV$) are distributed rather uniformly. There is a kind of equipartition of energy among turbulent gas motion, magnetic field, thermal gas, and cosmic rays. Their energy densities in our galaxy are estimated as $\sim 0.3\,\mathrm{eV\,cm^{-3}}$, $\sim 0.3\,\mathrm{eV\,cm^{-3}}$, $\sim 1\,\mathrm{eV\,cm^{-3}}$, and $\sim 1\,\mathrm{eV\,cm^{-3}}$ (Gaisser 1990). Therefore, the effects of magnetic field and CR as well are important to the dynamics and evolution of the ISM (e.g., Ferrière 2001).

On the other hand, the distribution of the galactic gases is highly inhomogeneous as can be seen from the many phases of gas clouds, e.g., giant molecular clouds, warm phase gas, hot phase gas, etc. It is thought that Parker instability plays an essential role on the formation of giant molecular clouds (*e.g.,* Parker 1966, Elmegreen 1982, Hanawa *et al.* 1992). Nagai *et al.* (1998) studied the effect of external pressure on the determination of the angle between the magnetic field and the gas filament. They showed that in the high external pressure regime, the filament is formed perpendicular to the magnetic field lines (in which the incompressible mode is dominant). On the other hand, in the low external pressure regime, the filament is formed along the magnetic field lines.

CR effect is usually ignored in the discussions on Parker or Parker-Jeans instability. As mentioned before the dynamic of CR on ISM should be significant, it is highly desirable to include the effect of CRs in a more realistic way. Recently, the effect of cosmic ray diffusion was included in several papers on the Parker instability (*e.g.,* Hanasz & Lesch 1997, Kim *et al.* 1997, Hanasz & Lesch 2000, Hanasz & Lesch 2003, Ryu *et al.* 2003, Kuwabara *et al.* 2004, Hanasz *et al.* 2004). They combined the set of magnetohydrodynamic (MHD) equations and the hydrodynamical approach of cosmic ray propagation (*e.g.,* Drury & Völk 1981, Ko 1992).

In this paper, we present the result of the linear stability analysis for the Parker-Jeans instability with the effect of CRs and show how the CRs have an effect on the formation of the giant molecular clouds.

II. LINEAR STABILITY ANALYSIS

(a) Equations

We take the magnetohydrodynamical (MHD) approach combined with the CR energy equation and the Poisson equation for the gravitational potential. The basic equations are written as follows,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{V}) = 0, \tag{1}$$

$$\frac{\partial}{\partial t}(\rho \boldsymbol{V}) + \nabla \cdot \left[\rho \boldsymbol{V}\boldsymbol{V} + \left(P_{\rm g} + P_{\rm c} + \frac{\boldsymbol{B}^2}{8\pi}\right) I - \frac{\boldsymbol{B}\boldsymbol{B}}{4\pi}\right] - \rho \nabla \phi = 0, \tag{2}$$

$$\nabla^2 \phi = 4\pi G \rho, \tag{3}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{V} \times \boldsymbol{B}), \tag{4}$$

$$\nabla \cdot \boldsymbol{B} = 0, \tag{5}$$

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$$\frac{\partial P_g}{\partial t} + \boldsymbol{V}\cdot\nabla P_g + \gamma_g P_g \nabla\cdot\boldsymbol{V} = 0, \quad (6)$$

$$\frac{\partial}{\partial t}\left(\frac{P_c}{\gamma_c - 1}\right) + \nabla\cdot\left(\frac{\gamma_c}{\gamma_c - 1}\right)\boldsymbol{V} - \boldsymbol{V}\cdot\nabla P_c - \nabla\cdot\left[\kappa_{\parallel}\boldsymbol{bb}\cdot\nabla\left(\frac{P_c}{\gamma_c - 1}\right)\right] = 0, \quad (7)$$

where ϕ, P_g, P_c, γ_g, γ_c, $\boldsymbol{b}$, and $\kappa_{\parallel}$ are the gravitational potential, the gas pressure, the CR pressure, the specific heat ratio of gases, the specific heat ratio of CRs, the unit vector along a magnetic field line, and the diffusion coefficient of CRs along magnetic field lines, respectively. The other variables carry their usual meanings.

(b) Model

We adopt an uniformly rotating two temperature layered disk with horizontal magnetic fields as the initial equilibrium model. The equilibrium distribution of the physical variables are given by the force balance between the total pressure gradient force and the self-gravity of the gas,

$$\frac{d}{dz}\left[P_g + P_c + \frac{B_x^2}{8\pi}\right] + \rho\frac{d\psi}{dz} = 0, \quad (8)$$

where the magnetic field has only x-component and the gravity has only z-component initially. The system has is nonuniform in z-direction only. By substituting Eq. (3) in Eq. (8), we can obtain the final form of the equation for the balance,

$$\frac{d}{dz}\left[\frac{C_s^2(1+\alpha+\beta)}{\gamma_g P_t}\frac{dP_t}{dz}\right] + \frac{2P_t}{C_s^2(1+\alpha+\beta)} = 0, \quad (9)$$

where α, β, and P_t are the ratio of the magnetic pressure to the gas pressure, the ratio of the CR pressure to the gas pressure, and the total pressure $P_t \equiv (1+\alpha+\beta)P_g$. We set the temperature distribution as follows,

$$T(z) = \begin{cases} T_g, & \text{for } |z| < Z_d \\ \infty, & \text{for } |z| > Z_d, \end{cases} \quad (10)$$

where T_g, and Z_d are the temperature, and the half thickness of the gas disk. The normalization for length, density, and velocity are $H_0 = C_{s,0}/(2\pi G\rho_0\gamma_g)^{1/2} = 200\,\mathrm{pc}$, $\rho_0 = 1.66\times10^{-24}\,\mathrm{g\,cm^{-3}}$, and $C_{s,0} = 5\,\mathrm{km\,s^{-1}}$, where the subscript 0 denotes the value at the mid-plane of the disk. The thickness of the disk Z_d depends on the total pressure outside the disk $P_t(Z_d)$. When $P_t(Z_d)$ is larger, Z_d is smaller. For the rest of the paper, we use thickness of the disk and external pressure interchangably.

(c) Linearization

We linearize Eq. (1)-(7) by assuming the deviation from the equilibrium value for each variable as follows,

$$\begin{bmatrix} \delta\rho \\ \delta V_x \\ \delta V_y \\ \delta V_z \\ \delta P_g \\ \delta P_c \\ \delta B_x \\ \delta B_y \\ \delta B_z \\ \delta\phi \end{bmatrix} = \begin{bmatrix} \delta\bar{\rho} \\ i\delta\bar{V}_x \\ i\delta\bar{V}_y \\ \delta\bar{V}_z \\ \delta\bar{P}_g \\ \delta\bar{P}_c \\ \delta\bar{B}_x \\ \delta\bar{B}_y \\ -i\delta\bar{B}_z \\ \delta\bar{\phi} \end{bmatrix} \exp(\sigma t + ik_x x + ik_y y), \quad (11)$$

where the barred quantities are functions of z only, σ is the growth rate, and k_x, k_y are the wavenumbers in the x and y directions. After some manipulations, we get the following set of four differential equations,

$$\frac{d}{dz}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & 0 \\ A_{21} & A_{22} & A_{23} & A_{24} \\ 0 & 0 & 0 & -1 \\ A_{41} & A_{42} & A_{43} & 0 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}, \quad (12)$$

where $y_1 \equiv -\rho\delta\bar{V}_z/\sigma$, $y_2 \equiv \delta\bar{P}_g + \delta\bar{P}_c + B_x\delta\bar{B}_x/(4\pi)$, $y_3 \equiv \delta\bar{\phi}$, and $y_4 \equiv \delta\bar{g}_z$ ($\delta\bar{g}_z = -d\delta\bar{\phi}/dz$). The components of the coefficient matrix A_{11}, A_{12}, A_{13}, A_{21}, A_{22}, A_{23}, A_{24}, A_{41}, A_{42}, A_{43} are not constant and, in fact, very complicated. We get the dispersion relation by solving Eq. (12) numerically under some suitable boundary condition (see *e.g.*, Hanawa *et al.* 1992). In this paper, we take the symmetric condition at the equatorial plane of the gaseous disk.

III. RESULTS

In this paper, we set $\gamma_g = 1$, $\gamma_c = 4/3$, $\alpha = 1$, $\beta = 1$. We took $\kappa_{\parallel}$ and Z_d as parameters. The realistic value of the CR diffusion coefficient $\kappa_{\parallel}$ in our galaxy is estimated as $3\times10^{28}\,\mathrm{cm^2}\,s^{-1}$ (Berezinskii *et al.* 1990, Ptuskin 2001, Ryu *et al.* 2003). Its normalized value is $\kappa_{\parallel} = 100$ in our unit.

Fig. 1 (*top*) shows the dependence of the growth rate σ_x on $\kappa_{\parallel}$ in the case of $k_y = 0$. As $\kappa_{\parallel}$ increases the short wavelength perturbations become more and more unstable. Fig. 1 (*bottom*) shows the dependence of the maximum growth rate on $\kappa_{\parallel}$. When $\kappa_{\parallel} < 10$, $\sigma_{x,\max}$ increases rapidly with $\kappa_{\parallel}$, then it levels off to almost a constant when $\kappa_{\parallel} > 100$. This characteristics of the maximum growth rate also can be seen in the case without self-gravity. The only difference is $\sigma_{x,\max}$ does not become zero when $\kappa_{\parallel} = 0$ in the case with self-gravity, while it becomes zero in the case without self-gravity.

Fig. 2 shows the dependence of the growth rate σ_y on $\kappa_{\parallel}$ in the case of $k_x = 0$. In this case, the growth rate does not depend on $\kappa_{\parallel}$. This is reasonable because we

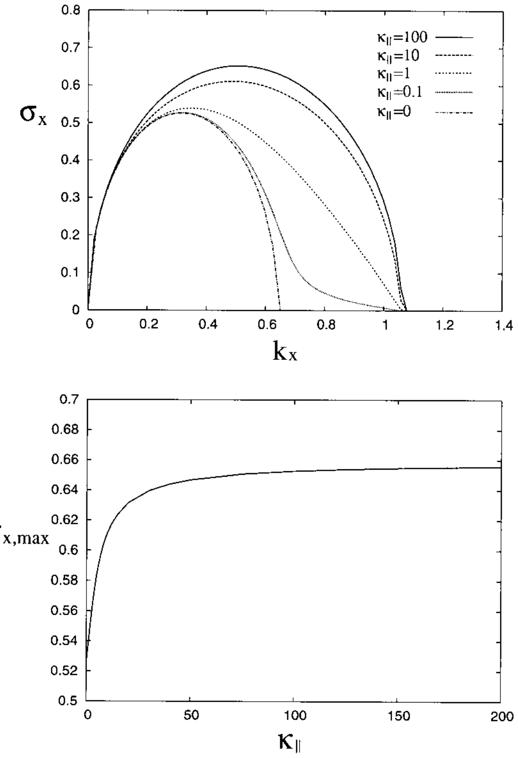

Fig. 1.— *Top*: Dependence of the dispersion relation on the CR diffusion coefficient, $\kappa_{\parallel}$, in the case of $k_y = 0$. *Bottom*: Dependence of the maximum growth rate on the CR diffusion coefficient, $\kappa_{\parallel}$, in the case of $k_y = 0$.

assume CR diffusion along the magnetic field lines only. Thus, even if $\kappa_{\parallel}$ changed, the instability perpendicular to the magnetic fields was not influenced.

Fig. 3 shows the dependence of the growth rate on the half thickness of the gas disk, Z_d (which represent the strength of external pressure). The gray region shows the range of Z_d where the angle between the longitudinal axis of the gas filament and the magnetic field lines becomes larger than 45°. This is the effect of CR diffusion. When $\sigma_{x,\max} \lessgtr \sigma_{y,\max}$, the angle becomes $\lessgtr 45°$. In Fig. 3 we take $\kappa_{\parallel} = 100$ as the upper limit because the maximum growth rate is almost constant when $\kappa_{\parallel} \geq 100$. ¿From this result, we notice the direction of the gas filament (formed by Parker-Jeans instability) with respect to the unperturbed magnetic field may be governed by CR diffusion in some range of Z_d.

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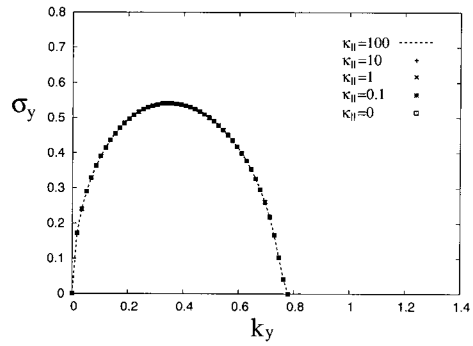

Fig. 2.— Dependence of the dispersion relation on the CR diffusion coefficient, $\kappa_{\parallel}$ in the case of $k_x = 0$.

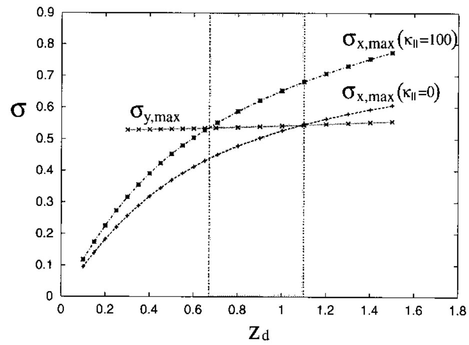

Fig. 3.— Dependence of the dispersion relation on the half thickness of gas disk, Z_d, in the case of $k_y = 0(\sigma_{x,max})$ and $k_x = 0(\sigma_{y,max})$.

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