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GENERATION OF MAGNETIC FIELDS BY TEMPERATURE GRADIENTS

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ABSTRACT

We showed that magnetic fields are generated in the plasma which have the temperature inhomogeneities. The mechanism is the same as the Weibel instability because the velocity distribution functions are at non-equilibrium and anisotropic under the temperature gradients. The growth timescale is much shorter than the dynamical time of structure formation. The coherence length of magnetic fields at the saturated time is much shorter than kpc scale and then, at nonlinear phase, become longer by inverse-cascade process. We report the application of our results to clusters of galaxies, not including hydrodynamic effects.

Key words : clusters of galaxies – magnetic fields - instabilities - plasmas

I. INTRODUCTION

We have studied the X-ray and radio connection in clusters of galaxies, based on the plasma kinetic theory. We found the magnetic fields are generated in plasmas with temperature gradients. That means that magnetic fields are naturally connected with the thermal plasma. The outline of the generation mechanism is as follows and the details are presented in each section: the electron velocity distribution function should be deviated from the Maxwell-Boltzmann, since the heat flux always flows when the plasma has the temperature gradients (in §2). Then, the low frequency transverse magnetic waves grows even in the absence of an background magnetic field. Since the instability was found by Ramani and Laval (1978), the instability is referred to the Ramani-Laval (RL) instability in this paper. We have shown that the mechanism of the RL instability is identical to the Weibel instability which is well-known as one of the mechanism of the magnetic fields generation (in §3). We found that the saturation level of RL instability is determined by the wave-particle interaction (Ramani & Laval 1978, Gallev & Natanzon 1991). The saturated magnetic fields will evolve into the large scales through an inverse cascade process, as reported in the numerical simulations of the Weibel instability (Lee 1973, Sentoku et al. 2000, Sentoku et al. 2002, Medvedev & Loeb 1999, Medvedev et al. 2004, Medvedev 2004). These nonlinear evolutions of the RL instability are presented in §4. We apply to clusters of galaxies and roughly estimate magnetic field strengths generated by the temperature gradients. We present these results and predictions in §5.

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II. THE VELOCITY DISTRIBUTION FUNCTION IN A PLASMA WITH HEAT FLUX

In this section, how the anisotropic electron velocity distribution function is set up when a temperature inhomogeneity exists in the plasma is discussed from a physical point of view. The absence of a background magnetic field is assumed. Consider the temperature inhomogeneity in a hot electron plasma with a temperature variation scale of L. Since the heat conduction carries the heat flux from the hotter to the cooler regions, the heat flux along the temperature gradient takes a negative finite value; $q \propto \langle v_\parallel v^2 f \rangle < 0$ where the angle brackets denote an integral over velocity space and the subscript $\parallel$ denotes the component parallel to the temperature gradient. Therefore, the electron velocity distribution function f must deviate from the $f_m = n_0(x_\parallel)(\pi v_{\rm th}(x_\parallel)^2)^{-3/2}\exp(-v^2/v_{\rm th}(x_\parallel)^2)$; $\Delta f = f - f_m \neq 0$. Here, $n_0(x_\parallel)$ is the electron number density, $v_{\rm th}(x_\parallel) = (2k_BT(x_\parallel)/m_e)^{1/2}$ is the thermal velocity with the temperature $T(x_\parallel)$ and the electron mass m_e. The above condition together with $\langle \Delta f \rangle = 0$, which is the number density conservation, and $\langle v^2 \Delta f \rangle = 0$, that is the energy conservation, restrict the form of the deviation Δf to be odd function of the velocity component along the temperature gradient, $v_\parallel$. Of course, $\Delta f \to 0$ as $|v_\parallel| \to \infty$. In Figure 1 the possible cases of Δf are shown. From zero current condition, $\langle v_\parallel \Delta f \rangle = 0$, the forms such as type A are rejected, since for $v_\parallel > 0$ and $v_\parallel < 0$, Δf is positive and negative, respectively, and then $v_\parallel \Delta f > 0$ for all velocities, and $\langle v_\parallel \Delta f \rangle > 0$. On the other hand, types B and C satisfy the zero-current condition, since the Δf curve crosses the $v_\parallel$-axis in each positive-and negative-$v_\parallel$ region. The quantity $\langle v_\parallel v^2 f \rangle$ for types B and C has finite values because this is the mean of $\langle v_\parallel \Delta f \rangle$ weighted by v^2 which is larger when $|v_\parallel|$ is larger. The heat flux condition $\langle v_\parallel v^2 f \rangle < 0$ says that Δf should be positive when $v_\parallel \to -\infty$. Thus, type C is the only possible form for the distribution

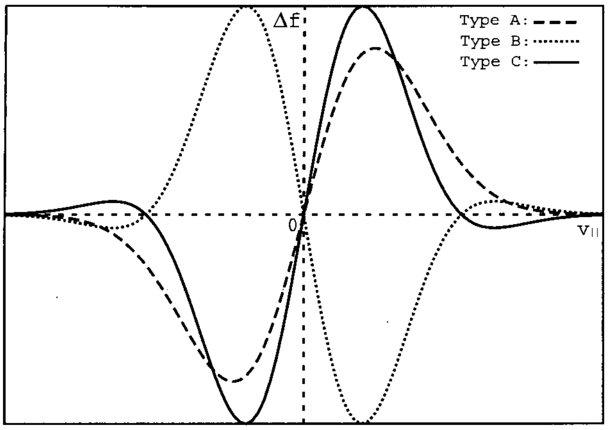

Fig. 1.— Possible candidates for the deviation of the velocity distribution function from Maxwell-Boltzmann.

function deviation of a plasma where a finite heat flow exists.

The deviation Δf can also be deduced analytically. The Boltzmann equation is

$$\frac{\partial f}{\partial t} + v_{\parallel}\frac{\partial f}{\partial x_{\parallel}} - \frac{e}{m}E_{\parallel}\frac{\partial f}{\partial v_{\parallel}} = -\nu(f - f_m),$$

where $E_{\parallel}$ is the zeroth-order electric field along the temperature gradient and the right-hand side has the Krook operator for the collision term, where ν is the Coulomb collision frequency. For simplicity, pressure balance is assumed. Then, $E_{\parallel} = 0$ (Ramani & Laval 1978). If perturbative treatment is appropriate for describing the system, the distribution function can be expanded in ϵ (Chapman & Cowling 1960) as

$$f = f_m + \epsilon f^{(1)} + \epsilon^2 f^{(2)} \ldots,$$

where $f^{(j)}$ $(j = 1, 2 \cdots)$ describes the deviation of the distribution function from the Maxwell-Boltzmann to the order of ϵ^j. This expansion is known as the Chapman-Enskog expansion. Now, we set the heat conduction $q = \langle \frac{1}{2} m_e v^2 v_{\parallel} f \rangle = -\kappa_{\rm SP}\nabla T$, where $\kappa_{sp} \simeq 1.31 n_e \lambda_e k_B \left(k_B T/m_e\right)^{1/2}$ is the Spitzer conductivity with Coulomb mean free path λ_e (Sarazin 1988). Therefore, the electron distribution function up to the first order in ϵ is obtained as

$$f = f_m \left[1 + \epsilon \frac{v_{\parallel}}{v_{\rm th}} \left(\frac{5}{2} - \frac{v^2}{v_{\rm th}^2}\right)\right],$$

$$\epsilon = \frac{8}{5 n_e m_e v_{\rm th,e}^3} \kappa_{sp} \nabla T \simeq 0.74 \lambda_e \nabla \ln T.$$

The form of the deviation is essentially the same as type C shown in Figure 1. We would like to note that adopting the Krook operator is not essential for determining the form of the deviation.

III. THE PHYSICAL MECHANISM OF MAGNETIC FIELD GENERATION

The physical mechanism of magnetic field generation can be understood from the nature of the velocity distribution function when a finite heat flow exists, as illustrated in Figure 2. In the discussion below, strict numerical accuracy is ignored. The peak position is shifted toward the positive $v_{\parallel}$-direction, and the amount of the shift is $v_{\parallel} \sim \epsilon v_{\rm th}$. The phase velocity of any low-frequency magnetic transverse wave must be close to the velocity at which the distribution function has a peak value; otherwise, a finite net electric current is induced by the wave magnetic fields. This explains why the waves excited by this instability have a phase velocity of $\epsilon v_{\rm th}$ and travel in one direction, rising up the temperature gradient, and why the waves cannot travel in the direction perpendicular to the temperature gradient. The reason why the waves excited by the Weibel instability are standing waves is simply because there is no shift in the peak position of the velocity distribution function in the Weibel case. The peak value is increased by $\sim 1 + \epsilon^2$ from that of the pure Maxwell-Boltzmann case, since the amplitude of the deviation takes a value of $\epsilon v_{\parallel} \sim \epsilon^2 v_{\rm th}$ relative to the Maxwell-Boltzmann, and the value of the Maxwell-Boltzmann part is nearly same as the peak value at $v_{\parallel} \sim \epsilon v_{\rm th} \ll v_{\rm th}$. Since the total electron number density must be unchanged, for an observer comoving with the waves this can be interpreted as a decrease of the effective electron temperature in the direction of the temperature gradient, $T_{\parallel}$, by $\sim 1 - \epsilon^2$ relative to the temperature perpendicular to the temperature gradient, $T_{\perp}$ (Fig. 2). The growth of the magnetic waves in this instability is therefore due to essentially the same mechanism as in the Weibel instability (Weibel 1959; Fried 1959), in which the temperature anisotropy is the driving force of the instability. Consider the waves traveling nearly parallel to the temperature gradient. In this case, $T_{\perp,\mathbf{k}} \sim T_{\perp}$ and $T_{\parallel,\mathbf{k}} \sim T_{\parallel}$, where $T_{\perp,\mathbf{k}}$ and $T_{\parallel,\mathbf{k}}$ are the temperature components perpendicular and parallel to the wavevector for the observers comoving with the waves, respectively. Since $T_{\perp,\mathbf{k}} > T_{\parallel,\mathbf{k}}$, the waves can grow. As a result, the direction of the magnetic field generated by the instability is almost perpendicular to the temperature gradient. The growth rate of the mode that travels in the direction of the temperature gradient with wavenumber k is obtained from that of the Weibel instability (Krall & Trivelpiece 1973),

$$\gamma \sim v_{\rm th}\left[\left(\frac{T_{\perp}}{T_{\parallel}} - 1\right)k - \left(\frac{ck}{\omega_{\rm p}}\right)^2 k\right]$$

$$\sim v_{\rm th}\left[\epsilon^2 k - \left(\frac{ck}{\omega_{\rm p}}\right)^2 k\right].$$

The growth rate gets the maximum value of $\gamma_{\max} \sim \epsilon^3 (v_{\rm th}/c)\omega_{\rm p}$ at $k = k_{\max} \sim \epsilon\omega_{\rm p}/c$. When the direction of the wavevector is perpendicular to the temperature

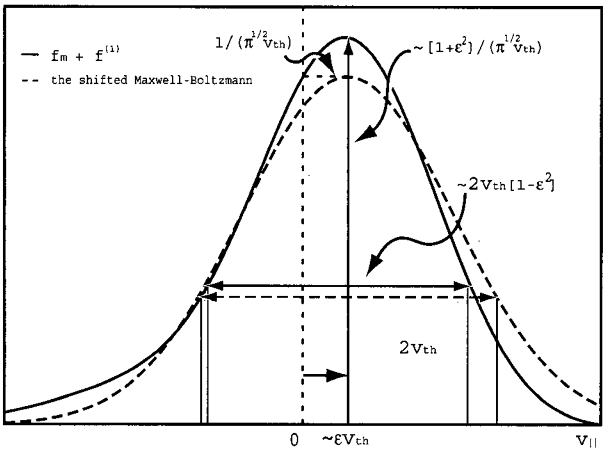

Fig. 2.— The $v_{\|}$ section (solid line) of the total velocity distribution function $f_m + f^{(1)}$ in a plasma with a temperature gradient. The peak position is shifted by $\sim \epsilon v_{\rm th}$ from Maxwell-Boltzmann. The peak value is increased by $\sim 1+\epsilon^2$ compared to Maxwell-Boltzmann. For comparison, the Maxwell-Boltzmann velocity distribution function shifted by $\sim \epsilon v_{\rm th}$ is drawn with the dashed line. The velocity distribution function gets thinner in the $v_{\|}$-direction. This can be interpreted as the decrease of the effective temperature by $\sim 1-\epsilon^2$ in the direction of the temperature gradient.

gradient, $T_{\perp,\vec{k}} = T_{\|}$ and $T_{\|,\vec{k}} = T_{\perp}$.Since $T_{\perp,\vec{k}} < T_{\|,\vec{k}}$ in this case, the wave cannot grow. These results are exactly the same as the results found from plasma kinetic theory, except for numerical factors (Ramani & Laval 1978; Okabe & Hattori 2003a).

IV. ON THE NONLINEAR EVOLUTION OF THE RL INSTABILITY

The nonlinear saturation level of the excited wave is estimated assuming that the wave-particle interaction determines the saturation level. The fundamentals are illustrated in Figure 3 (Ramani & Laval 1978; Gallev & Natanzon 1991). Once the Larmor radius of an electron gets shorter than the wavelength of the growing mode, the electron is trapped by the magnetic field of the wave, and the magnetic flux enclosed by its orbit becomes finite. Then, the kinetic energy of the trapped electron starts to monotonically increase with the growth of the magnetic field strength, since the increase of the magnetic flux enclosed by the electron orbit causes induction electric fields that accelerate the electron like a betatron accelerator. Once the Larmor radius of a typical thermal electron, $r_L \sim v_{\rm th}\omega_c^{-1}$, gets shorter than the wavelength of the fastest growing mode, that is, $r_L k_{\rm max} < 1$, the increase of the kinetic energy of the electron system becomes significant if the waves still continue to grow. Since this ultimately violates energy conservation, the growth of the magnetic field strength must be saturated when $r_L k_{\rm max} \sim 1$. The evolution of the magnetic fields after the nonlinear saturation could be described as follows: Some numeri-

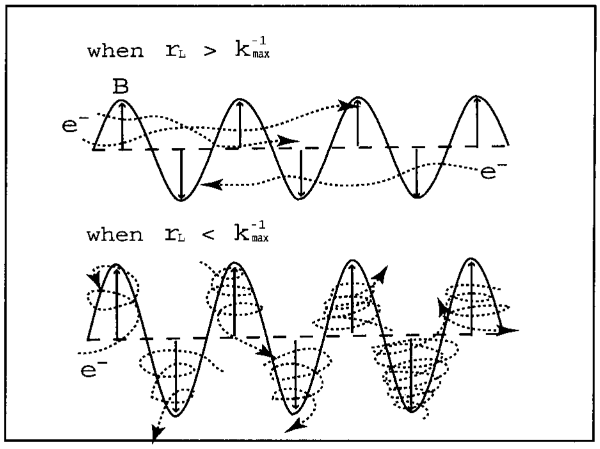

Fig. 3.— Nonlinear saturation by wave-particle interaction. Top: For $r_L > k_{\rm max}^{-1}$. The thermal electrons travel throughout the waves, but their orbits are randomly disturbed by the wavy magnetic fields. Bottom: For $r_L < k_{\rm max}^{-1}$. The thermal electrons are trapped by the fields and feel net nonzero fields.

cal simulations that follow the evolution of the Weibel instability showed that the strength of the magnetic field driven by the Weibel instability decreases after it reaches the maximum value (Morse & Nielson 1971). This can be understood as follows: As the magnetic field grows, the electron velocity distribution becomes isotropic, and it becomes difficult to maintain the electric current field that supports the magnetic field of the waves. In these simulations, the system is assumed to be isolated, and the initial anisotropic velocity distribution function is let free to evolve to an isotropic one.

On the other hand, in the case of the RL instability there is a driving force that maintains the anisotropy of the velocity distribution function. As long as the temperature gradient does not disappear, the finite heat flux transports the heat from the hot to the cold region, and the anisotropic velocity distribution function discussed in §2 is maintained. Therefore, the decrease of the magnetic field strength after the nonlinear saturation as found in the Weibel instability may not occur in the RL instability. It is expected that the generated magnetic field strength will be kept at the saturated value for the lifetime of the temperature gradient. Wallace & Epperlein (1991) performed numerical simulations to follow the evolution of the Weibel instability when an initial anisotropic distribution function is maintained by an external source. They showed that the magnetic field strength is kept at a constant value after the saturation. Their results support the above expectation for the RL instability. There have been several indicative numerical simulations concerning the organization of globally connected magnetic fields evolved from the wavy fields generated by the instability. The wavy magnetic fields generated by the Weibel instability evolve into longer wavelength modes after saturation (Lee & Lampe 1973; Sentoku et al. 2000, 2002; Medvedev 2004; Medvedev et al 2004). This result indicates that an excited wavy magnetic field automatically evolves into globally connected fields. This can be understood as follows: After the magnetic field strength reaches the saturated level, the electric current field starts to act as individual electric beams every half-wavelength. Each beam is surrounded by the azimuthal magnetic field generated by the current beam itself. The electric beams interact with each other via the Ampere force (Sentoku et al. 2000, 2002). Beams in the same direction are attracted to each other and automatically gather. Finally, they merge into one larger beam. Since the physical mechanism of the growth of the RL instability is the same as for the Weibel instability, as shown in Figure 2, the same evolution is expected even in the RL case. Although the reduction of the heat conductivity was originally considered to be due to electrons scattered by the waves generated by the RL instability (Ramani & Laval 1978; Hattori & Umetsu 2000), this may not be the case. As discussed above, the wavy magnetic field generated by the instability could tend to form a global magnetic field automatically. Therefore, the suppression of the heat conductivity may be determined by the trapping of the electrons by the organized magnetic field. To estimate the suppression of the heat conductivity quantitatively, we have to know the final structure of the magnetic field due to the self-organization. Detailed nonlinear studies, numerical simulations for example, are desired to answer these questions.

V. APPLICATIONS TO CLUSTERS OF GALAXIES

The Chandra X-ray observatory and XMM-Newton unveiled the various temperature structures in clusters of galaxies: global gradients, fluctuations and sharp changes across cold fronts (Markevitch et al. 2000). Based on our proposed mechanism, self-organized magnetic fields can be generated in clusters with temperature gradients, since the timescale of inverse-cascading of the magnetic fields is expected to be short compared with the dynamical timescale. Therefore, we can estimate the generated magnetic field strengths from the observational data. Since the generated magnetic fields are ubiquitous in the temperature inhomogeneities, the saturated magnetic fields can be compared with volume-averaged strengths estimated by radio halos and relics, or inverse Compton hard-X ray emissions. However, we have not yet known the structure of generated fields, including the coherent length. Therefore, it is difficult to compare with results of Faraday rotation measurement and we now focus on four clusters with cold fronts and diffuse radio emissions. we shall roughly estimate the saturated strength from the X-ray data. We note that, though we should use number densities and temperatures of plasma with no magnetic fields, using the data of clusters which have magnetic fields does not significantly change the estimation since the generated magnetic energy is much smaller than the thermal energy. The predicted strengths of magnetic fields are in a good agreement with the observed strengths. We summarize the results and prediction of this mechanism.

- Our proposed mechanism can predict that $\sim 5\mu$G magnetic fields along cold fronts in A3667 exist all over fronts (Okabe & Hattori 2003a), while the existence of fields required to suppress the KH instability is not indicated all over fronts (Vikhlinin, Markevitch & Murray 2001).
- Magnetic fields required from a viewpoint of hydrodynamics can be naturally explained by the plasma kinetic theory.
- Our proposed mechanism can predict $0.1 - 1\ \mu$G magnetic fields derived by studies of radio halos and inverse Compton hard X-ray emissions (Okabe & Hattori 2003b).
- Magnetic fields are ubiquitous in any space plasma with temperature inhomogeneities.
- The magnetic fields are naturally connected with thermal plasma via this mechanism.
- Spatial distributions of Magnetic fields are associated with the X-ray temperature map, because magnetic fields are generated by temperature gradients.
- The steep correlation between the radio power P_ν and the temperature k_BT (Liang et al. 2000) can also explained (Okabe & Hattori 2004).
- Magnetic field strengths don't depend on the redshift because the evolution timescale of magnetic fields is short (Okabe & Hattori 2003a).
- There is the possibility that the heat conduction is self-regulated, since magnetic fields are generated by the plasma itself.

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