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ARE GALACTIC WARPS INDUCED BY INTERGALACTIC FLOWS?

F. J. SÁNCHEZ-SALCEDO
Instituto de Astronomía, Universidad Nacional Autónoma de México, Ciudad Universitaria, Mexico City, C.P. 04510, Mexico
E-mail: jsanchez@astroscu.unam.mx

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ABSTRACT

The interaction of disk galaxies with intergalactic winds has been invoked as a possible mechanism of the generation of galactic warps. Here we discuss conditions under which intergalactic flows can be relevant for warping field galaxies. Constraints include the heating of the outer disk, the level of asymmetry in the vertical distribution of the volume gas density, the angular frequency of the warp, the symmetry of galactic warps amplitude between the approaching and receding sides of the galaxy, and the speed of the intergalactic flow whether subsonic or supersonic. These constraints are discussed in this paper in reference to the proposal of López-Corredoira et al. that warps can be a natural consequence of accretion flows onto the disk.

Key words : Galaxies: intergalactic medium—Galaxies: kinematics and dynamics—Galaxies: structure

I. INTRODUCTION

Since the first observations of the warps in our Galaxy and in external galaxies (Burke 1957; Kerr 1957; Sancisi 1976), the origin and persistence of galactic warps have been a matter of research for decades. H I observations reveal that over 50% of the spiral galaxies are warped. For instance, in the Local Group the three dominant galaxies present warps. The challenge is to understand how "isolated" galaxies can conserve their warp with a straight line of nodes and avoiding the winding problem as perceived by Kahn & Woltjer (1959).

Hunter & Toomre (1969) showed that no discrete normal modes of vibration exist in disk with soft edges. Discrete modes appear if the dark halo is included but, unfortunately, they damp due to the dynamical friction with the own halo (e.g. Nelson & Tremaine 1995). This fact motivates subsequent researchers to consider that galaxies are not strictly isolated systems but external torques (gravitational or not) ultimately generate the warp.

The most natural hypothesis is to assume that tidal interactions with satellite galaxies are responsible for warping galaxies. However, there are good reasons to believe that this is not the only source of warps. In particular, for the Galactic warp, García-Ruiz, Kuijken & Dubinski (2002) have found that the orientation of the warp is not compatible with this mechanism if the gravitational perturbers are the Magellanic Clouds. Although lopsidedness, U-warps (those for which the gas layer is above or below the plane in both sides), and warps with large amplitudes are present in galaxies that are strongly interacting with nearby companion galaxies, the hypothesis of tidal gravitational interactions cannot explain the finding by García-Ruiz, Sancisi & Kuijken (2002) that galaxies in poor environments are more frequently warped than galaxies in dense environments. After reviewing the different proposals suggested to generate galactic warps (§II), we will concentrate on observable consequences of the interaction of the outer parts of the galaxies with an intergalactic wind. Although the possibility that warps are induced by a wind or by accretion of baryonic matter has been explored in the past, new interest has been revived in the last years. For instance, López-Corredoira et al. (2002) claim to reproduce successfully the amplitude of the Galactic warp assuming that the wind is accreted directly onto the disk.

II. THE PROBLEM OF THE MAINTENANCE OF WARPS AND DRIVING MECHANISMS

The problem of the persistence of galactic warps was first discussed by Kahn & Woltjer (1959). A parcel of gas of the disk rotates around the galactic center with a circular frequency $\Omega(R)$, with R the galactocentric distance. In an oblate potential, the vertical frequency $\nu(R)$ for a vertical displacement of the fluid parcel satisfies $\nu > \Omega$. Thus, the angle at which the parcel crosses the equatorial plane regresses with angular frequency $\omega = \nu - \Omega$, where we have implicitly assumed that $2\Omega > \nu$. Therefore, the line of nodes will wrap halfway around into a leading spiral in a time

$$\tau_{\rm wrap} = \frac{\pi}{\omega(R_1) - \omega(R_2)} \leq 2\,{\rm Gyr}, \qquad (1)$$

where R_1 and R_2 are the inner and outer radii of the warp. The last inequality holds even if the halo is assumed to be spherical (e.g. Binney 1992). This means

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Proceedings of The 2nd Korea-Mexico Joint Workshop on *Physics of the Diffuse Interstellar Medium* that in a short time compared to the life of the disk, the warps will wind up beyond recognition. An external torque to maitain the shape of the warp is required. Besides the anticipated tidal torque, other sources of warps have been discussed in the literature: asymmetric pressure differences in the disk caused by a subsonic wind past the halo of coronal gas of the Milky Way (Kahn & Woltjer 1959), vertical resonances (Binney 1992, and references therein), the cosmic infall (Ostriker & Binney 1989), bending instabilities (e.g., Sellwood 1996), intergalactic accretion flows onto the disk (López-Corredoira et al. 2002), or tension forces by intergalactic magnetic fields (Battaner et al. 1990). All these mechanisms cannot be operating simultaneously with comparable strength in a certain galaxy because each mechanism will drive a $m = 1$ warp with different pattern speeds and, therefore, they will decouple each other leading to the coexistence of multi-modes, i.e. $m > 1$, which are not observed. It cannot be ruled out that warps may be originated by different mechanisms from galaxy to galaxy. However, it is important to discern the role of each potential driver, being aware of the difficulties inherent to each scenario. In this paper we critically review those models based on torques by intergalactic flows. The formation of warped disks is not a problem exclusive of galactic disks. Great effort has also been made in the context of accretion disks around Be stars or compact objects.

III. ACCRETION OF INTERGALACTIC FLOWS

Despite recent progress in Cosmology, very little is still known about the distribution of baryons in the Local Universe. Hydrodynamical simulations of the formation of structures predict that in the present epoch, half of the baryonic matter is in a teneous and relatively warm phase ($T \sim 10^5$–10^7 K). At low redshifts, the typical volume densities range between 10^{-3} cm^{-3} for the intracluster medium to 5×10^{-6} cm^{-3} in small groups of galaxies. For instance, Nicastro (2003) reported a value of 4-6 $\times 10^{-6}$ cm^{-3} in the Local Group, derived from the $z \sim 0$ highly ionized far-ultraviolet and X-ray absorbers, up to densities of 4×10^{-4} cm^{-3} if the diffuse medium is able to condensate in the high Galactic halo when mixing with the ISM. The effects of ram pressure exerted onto the ISM of spiral galaxies are evident for cluster galaxies moving supersonically in the dense intracluster medium. For galaxies in groups, there is no clear observational evidence of the interaction with the intergalactic medium (Bureau et al. 2003).

The effect of this intergalactic material onto the gaseous disks of spiral galaxies may produce observable asymmetries. Kahn & Woltjer (1959) proposed that pressure differences in the outer disk by a subsonic wind past the coronal halo of the Galaxy could be a source of excitation of warps. In fact, these authors model the galaxy as a "rigid" object. On the contrary, in the scenario proposed by López-Corredoira et al. (2002) –see also Mayor & Vigroux (1981)–, warps are induced by the accretion onto the disk of baryons in a wind of $\sim 6 \times 10^{-5}$ cm^{-3} with a velocity at infinity of 100 km s^{-1}. In contrast to previous models, they ignore the coronal gas and assume that all the mass in the wind falls freely until it collides with the disk and is absorbed, resulting in a rate of infall matter ~ 1 M$_\odot$ yr^{-1}, in agreement with the required values to resolve the G-dwarf problem. Throughout this paper we will refer to the former scenario as the 'wind model' and as the 'accretion model' to the latter.

At a first glance, one could think that both models are not so different and, due to the observational uncertainties in the wind parameters, the details of the interaction, either by accretion onto the disk or through pressure differences created by a subsonic wind in a rigid halo (or a combination of these processes), are unimportant. However, this reasoning is not completely correct for the following reason. Suppose, with no loss of generality, that the wind has a velocity at infinity of $(v_x, 0, v_z)$, with v_x, $v_z > 0$. In the wind model, the left side of the galaxy will lie above the plane defined by the inner galaxy and the right side will be placed below this plane (see, e.g. Ikeuchi & Tomisaka 1981). However, in the accretion model, it holds the opposite. Therefore, since they are actually different, it is important to discuss under which conditions either the accretion or the wind model is a better representation of nature.

The assumption that the cold disk ($T \simeq 10^4$ K) is a perfect absorber of the hot gas ($T \simeq 10^6$ K) of the wind, necessarily requires a very efficient cooling mechanism. If the cooling time is not short enough, then the flow is dragged by the bow shock if the wind is supersonic, or it can divert the galaxy if the flow is subsonic. In the subsonic case and in the lack of strong cooling, the wind produces pressure differences that can generate asymmetrical forces between both sides of the galaxy. This situation was discussed by Kahn & Woltjer (1959) half a decade ago and, more recently, by Saar (1979) and Ikeuchi & Tomisaka (1981). If the wind is supersonic, a bow shock is formed and, as a consequence, material leaks out the galaxy laterally (e.g. Mori & Burkert 2000), and the gravitational focusing, which is the main ingredient for warp formation in the accretion model, is suppressed. Therefore, the best situation to have a warp in a model based on accretion is a subsonic wind plus efficient cooling.

The most efficient route of cooling in a line-emitter plasma at say 6×10^5 K is the OVI$_{2s \to 2p}$ doublet. This mechanism is quite inefficient at the quoted intergalactic densities. Even assuming the most generous value of 4×10^{-4} cm^{-3}, the cooling time is about 7×10^8 yr, i.e. comparable to the circular orbital time. Therefore the assumption of perfect accretion of the wind is not justified; the ram pressure should produce heating and could act as a potential ionizing source to explain the Reynolds layer. However, there are serious problems associated with ram pressure as a source of ionization

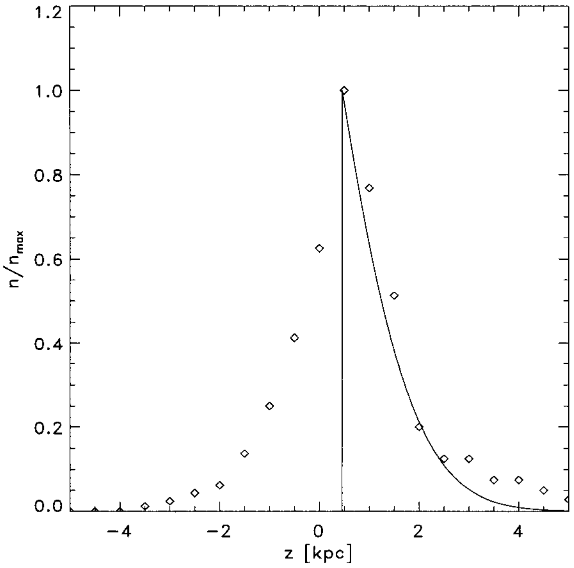

Fig. 1.— Normalized H I number density from Diplas & Savage (1991) at $R = 12$ kpc and $\Theta = 90°$ (diamonds), and the expected profile assuming that the gas layer is kept above the disk by hydrodynamical ram pressure caused by a galactic wind (solid line).

of the Reynolds layer (Mathis 1986; Bland-Hawthorn, Freeman & Quinn 1997). These observations should put stringent upper limits on the accretion rate of a wind onto the disk.

The picture may be somewhat different if the intergalactic medium is not assumed to be homogeneous but clumpy. For certain parameters of the cloudlets, the wind can behave as a collisionless system until it collides onto the ISM disk. The kinetic energy of the cloudlets must be thermalized in the H I disk by shocks and sound waves. For a flow of density $\rho_{\rm w}$ and velocity $v_{\rm w}$, the power of this injection of energy per unit of surface area is $\sim \rho_{\rm w} v_{\rm w}^3$. This injection of energy will heat the gas of the outer disk in a characteristic time of

$$\tau_{\rm heat} = \frac{\Sigma c_s^2}{\rho_w v_w^3} \lesssim 1.5 \times 10^7 \, {\rm yr}, \qquad (2)$$

where $c_s \approx$ 10–15 km s^{-1} is the effective total gas velocity dispersion and Σ the surface density. The estimate was calculated for typical values of $\Sigma \approx 6 \times 10^{20}$ cm^{-2}, $\rho_{\rm w} \approx 6 \times 10^{-5}$ cm^{-3} and $v_{\rm w} \approx 200$ km s^{-1}. This time is too short to be compensated by radiative cooling. The linear dependence of $\tau_{\rm heat}$ with the surface density implies that the turbulent velocity of the ISM should increase with radius dramatically, contrary to observations (Sellwood & Balbus 1999, and references therein). In any event, the assumption that the intergalactic gas is accreted directly onto the disk is problematic and must be taken with extreme caution.

IV. THE VERTICAL DISTRIBUTION IN H I DENSITY

The frequency of the pattern of a warp ω_p that is forced by a constant external force is zero, and therefore the period for the gas to cross along two successive crests of the warp is about 3×10^8 yr, assumed to be similar to the disk rotation speed at a radius of 15 kpc. This time is longer than the sound crossing time over a scaleheight of 1 kpc at the sound speed of 10 km s^{-1}. Thus, hydrostatic equilibrium is a valid approximation at $R =$ 10–16 kpc.

If the force responsible for the warp has a hydrodynamical origin, rather than gravitational as in the cosmic infall scenario, the vertically shifted disk must be supported by a pressure gradient in order to satisfy the condition of hydrostatic equilibrium. Consequently, the gas density in the vertical direction must present a gradient or, conversely, a very asymmetric z-distribution. More quantitatively, the gas must satisfy

$$\frac{\partial P}{\partial z} = \rho g, \qquad (3)$$

where P is the total pressure of the gas and g is the vertical component of the gravitational force. Following Fletcher & Shukurov (2001), the self-gravity of the gas is neglected at the outer galaxy and g can be written as

$$-g = A_1 \frac{z}{\sqrt{z^2 + Z_1^2}} + A_2 \frac{z}{Z_2}, \qquad (4)$$

with $Z_1 = 0.2$ kpc, $Z_2 = 1$ kpc, and A_1 and A_2 are coefficients that depend only on R. Given g, integration of Eq. (3) with $P = (1 + \alpha + \beta)\rho c_s^2$, where α and β represent the pressures arising from magnetic fields and cosmic rays, allow us to derive $\rho(z)$.

For our Galaxy, the z-distribution is available and can provide a test for the accretion scenario by comparing predictions with observations. Diplas & Savage (1991) made use of the data from the northern hemisphere H I survey to study the gas morphology in the outer Galaxy. We will focus on the vertical density distribution of H I at $R = 12$ kpc and azimuthal angle in the Galactic plane $\Theta = 90°$. At this particular radius the amplitude of the northern side warp is 0.5 kpc (we are using $R_\odot = 8$ kpc), and the z-distribution of density is given in Fig. 1. We see that the observed distribution is fairly symmetric in sharp contrast with the predictions of the accretion model. The predicted curve was obtained from Eqs. (3) and (4) with $\alpha = \beta = 1$, $c_s = 15$ km s^{-1}, $A_1 \approx 1.3 \times 10^{-9}$ cm s^{-2} and $A_2 \approx 0.6 \times 10^{-9}$ cm s^{-2} at $R = 12$ kpc. We suggest that the low level of asymmetry in the vertical distribution of the volume density in the Milky Way can be very useful as a diagnostic of warping mechanism.

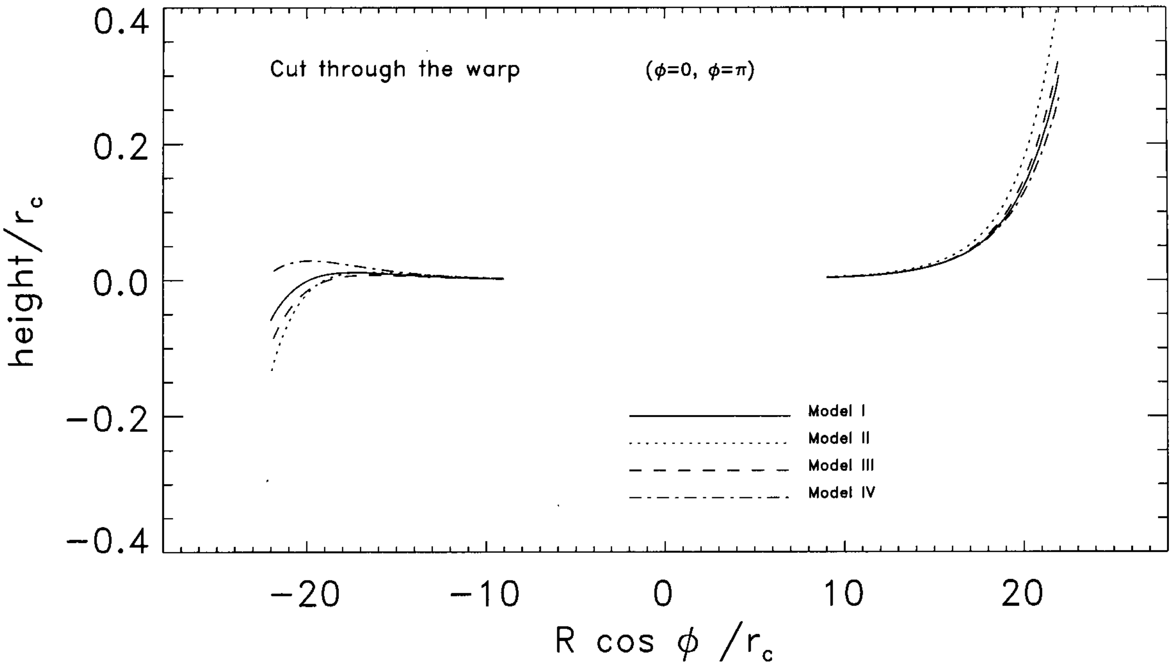

Fig. 2.— Shape of the warp along a cut through the (straight) line of maximum heights for the different models of Table 1.

TABLE 1
MODEL PARAMETERS, AMPLITUDE RATIOS, AND SHAPES

Model	q[1]	θ_0(deg)[1]	γ_l[2] at $R = 22r_{\rm c}$	Shape[3]
I	0.75	45	5.1	S[4], type I[6]
II	0.9	45	3.0	S, type I
III	0.75	30	3.4	S, type I
IV	0.75	60	26.8	L[5], type I

[1] Parameters for four models.
[2] The amplitude ratio between the two sides of the warp at $R = 22r_{\rm c}$.
[3] The shape of the warp.
[4] Type S denotes integral-sign warps.
[5] Type L stands for one-sided warps (see Fig. 2).
[6] Type I refers to warps with increasing amplitudes towards the edge of the disk.

V. SHAPES AND AMPLITUDES OF THE WARPS: PREDICTIONS OF THE ACCRETION MODEL

In the previous sections we have discussed a list of perceived difficulties associated with the accretion model as a source of excitation of warps. Let us disregard all these potential problems and estimate the resulting displacement of the gas layer of galaxies in this highly idealized scenario. As discussed already in §III, a combined model including the halo gas plus accretion would lead to smaller warp amplitudes than in the pure accretion model. For this reason, we will consider the same physical scenario described in López-Corredoira et al. (2002), except that we also include the presence of a slightly oblate dark halo as cosmological simulations of galaxy formation suggest. Let θ_0 be the angle between the wind velocity at infinity v_∞ and the plane of the galaxy. $\theta_0 = 90°$ means that the wind is face-on. Our starting point is the classical vertical equation of motion for vertical displacements (e.g., Sparke & Casertano 1988), adding the time-independent external force due to accretion. This procedure differs from the adopted one by López-Corredoira et al. (2002); the latter method is numerically not very robust, since it allows ω_p to be nonzero. Indeed, they found that the $m = 1$ mode is dominant over the $m = 0$ mode for angles of the wind $3° < \theta_0 < 80°$, which is rather counterintuitive. Hence, we decided to carry out our own calculations treating both modes ($m = 0$ and $m = 1$) in a self-consistent approach. The details of the calculations can be found in Sánchez-Salcedo (2004).

We adopted a truncated exponential profile for the surface density with scalelength $R_{\rm d}$, i.e. $\Sigma(R) = \Sigma_0 \exp(-R/R_{\rm d})$, whenever $R \leq R_{\rm t}$, with $R_{\rm t}$ the truncation radius. The dark halo is assumed to be frozen, i.e. it behaves as a fixed potential, and flattened along the symmetry axis of the disk with axis ratio q, typically $q \approx 0.75$, with a density profile:

$$\rho_{\rm h}(R,z) = \begin{cases} \rho_0 \frac{\exp(-a/r_{\rm t})}{1+a^2/r_{\rm c}^2} & \text{if } a \leq r_\infty; \\ 0 & \text{otherwise,} \end{cases} \qquad (5)$$

with $a^2 = R^2 + z^2/q^2$, r_c the core radius of the halo, $r_t = 50r_c$ and $r_\infty = 65r_c$.

Figure 2 displays the height of the disk respect to the symmetry plane of the halo versus the radius of the disk along a cross-section cut through the warp for the models specified in Table IV. In all the models, a velocity of $v_\infty = v_d/1.6$ was adopted, where v_d is the peak value of the circular speed by the disk itself, $v_d \approx 0.64\sqrt{GM_d/R_d}$ (Binney & Tremaine 1987), and a density of the wind $\rho_\infty = 2.2 \times 10^{-3} M_d (4\pi R_t^3/3)^{-1}$. For a galaxy like the Milky Way, for which $R_d = 3$ kpc and $v_d = 200$ km s^{-1}, this would correspond to $v_\infty = 125$ km s^{-1} and $\rho_\infty = 3.4 \times 10^{-4}$ cm^{-3}. This density is likely a very generous assumption (see the discussion in §III). Even so, the inferred Galactic warp amplitude is of the order of 0.2 kpc at a distance of 14.5 kpc, too small compared to the observed height of 1 kpc at that radius in the northern side (Burton 1992). Consequently, keeping ρ_∞ constant, velocities of v_∞ *significantly larger* than ~ 300 km s^{-1} are required to explain amplitudes comparable to the warp of the Milky Way. Note that our amplitudes are much smaller than those obtained by López-Corredoira et al. (2002).

In contrast to other theories that predict $\omega_p > 0$ (Binney 1992), this model predicts $\omega_p = 0$. Still, $\omega_p = 0$ is not sufficient to explain the unexpected stellar velocity distribution in the warped Galactic disk, which requires $\omega_p < -20$ km s^{-1}kpc^{-1} (Smart et al. 1998).

In the accretion model, warps should appear very asymmetric as it becomes apparent in Fig. 2. In order to quantify the level of asymmetry, we also give in Table IV the amplitude ratio between the two sides at the end of the warp, γ_l. Smaller the angle θ_0 is, more antisymmetric (i.e., they are closer to a integral-sign shape) the warp becomes, but even for $\theta_0 = 30°$, the amplitudes between the two sides in the warp are significantly different (see also Table IV). For angles $\theta_0 > 50°$ warps bent with a L-shape (see model IV) or even a bowl-shape. Hence, we expect a percentage of occurrence of $\sim 23\%$ of these types of warps. This predicted number of highly asymmetric warped galaxies is too high to be compatible with the observations. According to this model, those optical observations that cannot go beyond $R = (19\text{–}20)r_c$ would detect only U-warps or L-warps (see Fig. 2). Contrary to this prediction, only $\sim 12\%$ of the warped galaxies of the sample of Sánchez-Saavedra et al. (2003) are of these types.

An important feature of this model is that all the warps with a shape of integral-sign must be of type I in the terminology of Sparke & Casertano (1988). Certainly, the shape of the warp of our Galaxy cannot be explained by the accretion model alone. Note that within a certain radius typically of $19\,r_c$–$20\,r_c$, the four models show a warp only on one side (L shape warps) because the amplitude of the $m = 0$ mode is comparable to that of the $m = 1$ mode within that region. In general, the gas layer starts to depart from the plane of the central disk at different galactocentric radii at each side. The accretion model also predicts that asymmetric warps with one side falling back to the disk-plane at a certain radius, must present a bowl-shape (U-warp) at least out to that radius, as occurs for model IV.

VI. SUMMARY

The dynamical morphology of the outer extended H I gas in galaxies may give information about the last stage of galaxy formation. In principle, galactic warps could be a consequence of cosmic infall of matter in the outer galaxy. In a *different scenario*, it has been suggested that galaxy disks bent due to the ram pressure exerted onto the disk by the accretion of the teneous intergalactic gas. Here we have discussed under which conditions intergalactic flows could be relevant as a source to induce warps. Major results from the present study can be summarized as follows. The ram pressure would be significantly high to produce excessive heating of the H I gas if the cold disk is assumed to be a perfect absorber of the infalling baryonic matter. On the other hand, the inclusion of a halo of gas surrounding the disk would reduce not only the heating but also the warp amplitude. In addition, the hydrodynamical pressure should have observable consequences in the vertical distribution of the gas density, in contrast to the fairly symmetric distribution observed in the Milky Way. In this paper we have also derived the shape and amplitude of the warps under the optimistic assumption that the momentum of the wind is transferred to the gas disk. We conclude that the present day mean density of the intergalactic medium seems to be too low to reproduce the amplitude of the observed warps. Moreover, the predicted warps would appear excessively asymmetrical. This model would lead only to warps of type I. We estimate that $\sim 23\%$ of the warped galaxies should be of type L or U. Other asymmetries in the projected H I surface density should be observable in edge-on galaxies as a signature of the existence of the $m = 0$ mode, for which there is no evidence.

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