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FRACTAL DIMENSIONS OF INTERSTELLAR MEDIUM:
I. THE MOLECULAR CLOUDS IN THE ANTIGALACTIC CENTER

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ABSTRACT

We have estimated the fractal dimension of the molecular clouds in the Antigalactic Center based on the ^{12}CO ($J = 1-0$) and ^{13}CO ($J = 1-0$) database obtained using the 14m telescope at Taeduk Radio Astronomy Observatory. Using a developed code within IRAF, we were able to identify slice-clouds, and determined the dispersions of two spatial coordinates as well as perimeters and areas. The fractal dimension of the target region was estimated to be $D = 1.34$ for low resolution ^{12}CO ($J = 1 - 0$) database, and $D = 1.4$ for higher resolution ^{12}CO ($J = 1 - 0$) and ^{13}CO ($J = 1 - 0$) database, where $P \propto A^{D/2}$. The sampling rate (spatial resolution) of observed data must be an important parameter when estimating fractal dimension. Our database with higher resolution of 1 arcminute, which is corresponding to 0.2 pc at a distance of 1.1 kpc, gives us the same estimate of fractal dimension to that of local dark clouds. Fractal dimension is apparently invariant when varying the threshold temperatures applied to cloud identification. According to the dispersion pattern of longitudes and latitudes of identified slice-clouds, there is no preference of elongation direction.

Key words : ISM: cloud – ISM: structure – ISM: turbulence

I. INTRODUCTION

One way of describing the structure of the molecular clouds as well as other interstellar medium (ISM), especially H I clouds, is based on their fractal dimensions. Fractal dimension D of a molecular cloud boundary can be determined from the perimeter-area relation of a map $P \propto A^{D/2}$. Many studies of the molecular interstellar medium found a similar dimension, $D \sim 1.4$ (Falgarone et al. 1991 and references therein). She observed clouds at two different distances at high angular resolution using several transitions of CO molecule. Cloud edge regions were selected for the study to avoid the spatial crowding of emitting components, which obscured the structure of cloud cores, and found that spatial structure exists on all scales down to linear resolution of 0.02 pc. Her final estimate of the fractal dimension was $D = 1.36$. Recently its best value is known as $D = 1.4$ (Williams et al. 2000). However, these are all for the clouds located in the Inner Galaxy. No studies had been reported on the clouds located outside solar circle, where the environment could be quite different from that of the Inner Galaxy region. In the meanwhile, Kim et al. (2003) also found a fractal dimension $D = 1.47$ for identified H I clouds in Large Magellanic Cloud (LMC). They also claimed that the fractal dimension D found from the relation between area and perimeter is invariant, for clouds identified with different thresholds of the brightness temperature. It is very intriguing that the fractal structure, and its dimension can be a basis to understand morphological study of interstellar medium.

In this paper, we will study the fractal structure of the molecular clouds in the Outer Galaxy, based ^{13}CO as well as ^{12}CO molecular emission cube data with two different spatial resolutions to manifest the universality of fractal structure of the molecular clouds. In section II, we describe the databases, and we define the fractal dimension in Section III. In Section IV, we delineate the results, and discuss the results. We summarize our results in the final section.

II. DATABASES

Lee et al. (1999) have mapped 17 square-degree section $(l, b) = (178^\circ.0 \sim 186^\circ.0, 3^\circ.5 \sim 6^\circ.0)$ of the Galactic Anticenter region in ^{12}CO $J = 1-0$ (Galactic Anticenter CO Survey: GACCOS) using the 3 mm SIS receiver on the 14 m telescope at Taeduk Radio Astronomy Observatory. The beam size (FWHM) is 47″ and the grid spacing of the map is 3′. They mainly used a 250 KHz filterbank, which covers a velocity range of 170 km s^{-1} with a resolution of 0.65 km s^{-1}. The average rms noise of the data is estimated to be $\Delta T_{rms} \sim 0.25$ K in T_R^* at a velocity resolution of 0.65 km s^{-1}. A total number of about 7,000 positions have been obtained. They found several warm clumps in the main emission region, some of which are matching with regions with strong dust emission. Outside the main emission region, several small patches of molecular gas are also present. Overall, spatial coincidence and close morphological similarity is found between the CO emission and dust far-infrared (FIR) emission as in the case of isolated clouds without background contamination. From this point we name it as Database A. In addition, they identified two molecular clouds with anomalous velocities of about -20 km s^{-1}. At the exact Galactic Anticenter region ($l = 180°$), the velocity is presumably expected to be around 0 km s^{-1}. The velocity of -20 km s^{-1} seems to be very peculiar even considering a possible streaming motion of 5 km s^{-1} (Blitz et al. 1990) at this region. For further study of the distribution and physical condition of molecular gas, Lee et al. (2004) mapped 1 deg^2 section $(l, b) = (180°.0 \sim 181°.0,\ 5°.0 \sim 6°.0)$ in $J = 1 - 0$ transition of ^{12}CO and ^{13}CO using the 3mm SIS receiver on the Taeduk Radio Astronomy Observatory (TRAO) 14 m telescope. The beam size (FWHM) is about 50″ and the grid spacing of the new map is 1′. The same filter-bank system as for Database A was used. The average rms noise of the data is estimated to be $\Delta T_{rms} \sim 0.2$ K for ^{12}CO in T_R^* at a velocity resolution of 0.65 km s^{-1}, and 0.1 K for ^{13}CO in T_R^* at a velocity resolution of 0.68 km s^{-1}. From this point we name it as Database B.

III. DEFINING FRACTAL DIMENSION

To estimate the area and perimeter of each slice-cloud, firstly we should identify it in two dimensional space; we chose to slice into velocity direction from a cube data set in (v, l, b) space. The "slice-cloud" is one part of a cloud sliced through each channel. To effectively identify the clouds at each velocity slice (or channel) from the 3-dimensional cube data set, we developed a code, which is working as a user task within IRAF. Using the ^{12}CO cube data (or any other kinds, such as ^{13}CO or H I), we define a cloud to be an object composed of all pixels in longitude, and latitude that are simply connected and that lie above some threshold temperature. This is the similar method employed by Scoville et al. (1987) and Lee, Snell & Dickman (1990). One merit of making a task within IRAF is that every data can be handled in IRAF image form, which can be transformed to FITS form easily, and can be transported to other reduction packages, if necessary. To make a FORTRAN program using IMFORT into a IRAF task, a CL foreign task interface is required to connect the program to CL callable task. We arbitrarily simplify the CL file for easy handling.

Ideally, one would like to define clouds with a 0 K threshold temperature. However, low threshold temperatures are impractical in view of the noise level in the spectra. On the other hand, with too high a threshold temperature, regions are severely truncated, and it is impossible to obtain a reliable estimate of the size. To define clouds we should choose a reasonable threshold temperature. Above the arbitrary threshold temperature, only those clouds with 3 or more pixels (l, b) space were retained.

We noticed that there could be two different definitions of perimeter-area. Area is simply integration of all the unit pixels, or alternatively, number of pixels above a threshold temperature. Perimeter can be defined in two ways; the first one is the sum of boundaries (real perimeters) of all the pixels. The second one is the number of pixels, which are on the boundaries. For example, for 4-pixel elongated cloud (band- type), the area would be 0, and the perimeter is 4, as all the pixels are located on the boundary. Thus, the cloud with no area could be excluded for estimating fractal dimension. With the first definition, its area would be 4, and its perimeter is 10. For the smaller cloud, the second definition would be extremely erroneous, however, both definition gives us almost the same fractal dimension when clouds become large enough. After identifying the individual slice-clouds, the dispersions of two spatial parameters (l, b) are also calculated.

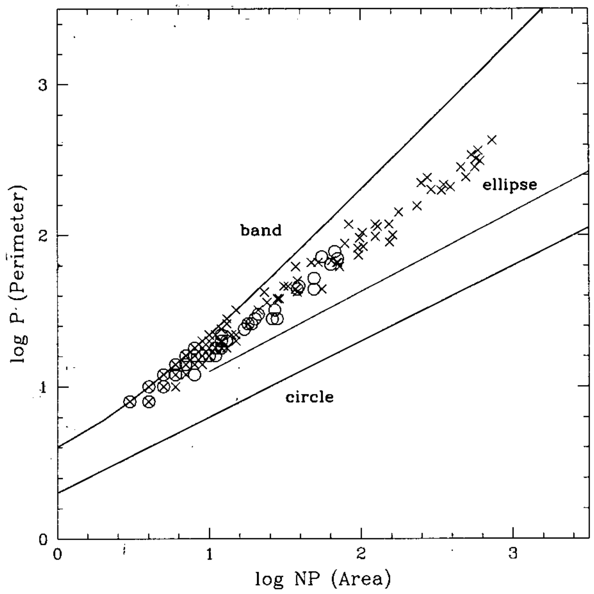

Fig. 1.— Log-log plot of measured perimeter versus area in units of pixels in (l, b) space for the slice-clouds in the Antigalactic Center region. Open circles and crosses represent 3′-arcminute clouds (Database A), and 1′-arcminute clouds (Database B). Three lines are for arbitrary extremely elongated bands, ellipses, and circles (see text).

IV. RESULTS AND DISCUSSION

Using a threshold temperature of 2 σ (0.5 K for Database A, and 0.4 K for Database B), we identified 272 slice-clouds for the GACCOS (Database A), and 318 slice-clouds for the clouds with anomalous velocities (Database B). However, there are many slice-clouds with a small number of pixels; Only 49 slice-clouds are larger than 5 in pixel number for Database A, and 136 slice-clouds for Database B. The number of pixels of the largest slice clouds are 70 for Database A, and 733 for Database B. Figure 1 shows a relationship between the perimeters and the areas of the slice-clouds in log-

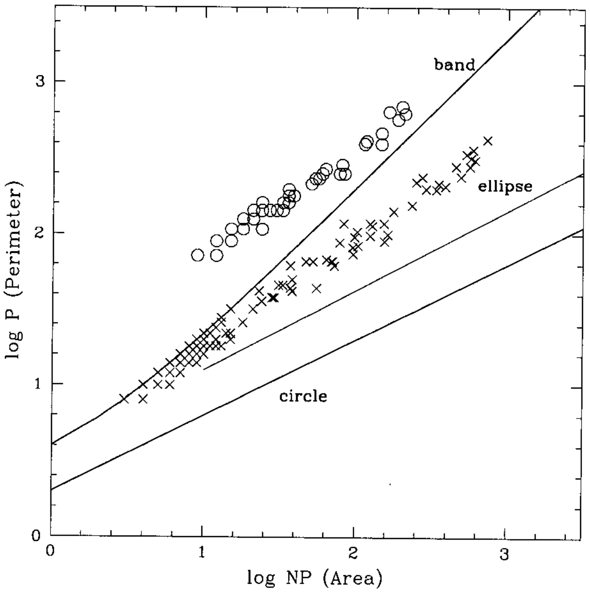

Fig. 2.— Log-log plot of measured perimeter versus area in units of arcminutes in (l, b) space for the slice-clouds in the Antigalactic Center region. Open circles and crosses represent 3′-arcminute clouds (Database A), and 1′-arcminute clouds (Database B). Three lines are the same as in Figure 1.

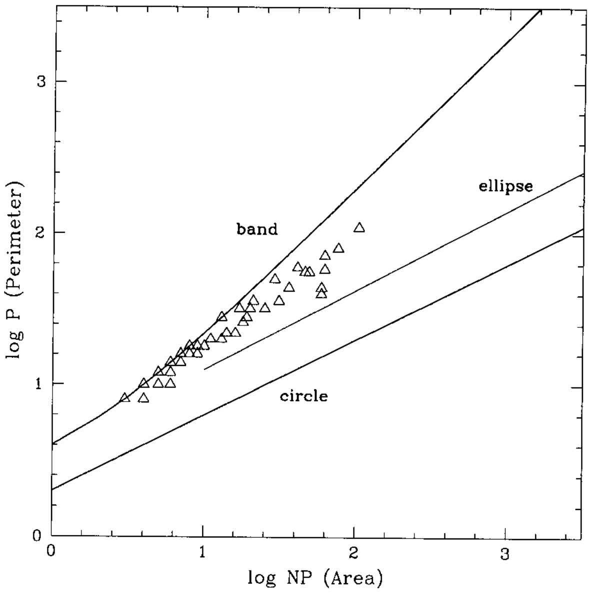

Fig. 3.— Log-log plot of measured perimeter versus area in units of arcminutes in (l, b) space for the ^{13}CO slice-clouds in the Antigalactic Center region. Three lines are the same as in Figure 1.

arithmic scale. The open circles represent Database A, and the crosses for Database B. The perimeters and areas are in units of pixels. This figure show a good correlationship between two parameters for both Databases. A bisector fit (Isobe *et al.* 1990; Lee, Snell & Dickman 1990) applied to each Database yields a best fit line. For Database A:

$$\log(\mathrm{P}) = 1.338/2(\pm 0.01)\log(\mathrm{NP}) + 0.579(\pm 0.006) \quad (1)$$

and the Database B:

$$\log(\mathrm{P}) = 1.400/2(\pm 0.008)\log(\mathrm{NP}) + 0.567(\pm 0.008) \quad (2)$$

The slopes for two databases are found to be almost the same; $D/2 = 0.7$. We noticed that the intercepts are not important, as they are related to the resolution (pixel size) of the observed data. In addition, we measured the perimeters and areas in the units of arcminutes (arbitrary scale) instead of pixel units. This result is presented in Figure 2. The location of Database A in Figure 2 are moved upward as the perimeters are tripled, and the areas are 9 times bigger. However, the slope of the fitted line is invariant; only the intercepts are changed.

Even though we change the threshold values, such as 3 σ, or 5 σ, there is no big change in fractal dimensions for both databases. In the meanwhile, it is found that if the sampling resolution is coarse, the fractal dimension could be estimated lower. This implies that coarse data can mislead the interpretation of fractal structure, which was expected. However, the fractal dimension of Database B is exactly the same as the results of other studies mentioned above. The sampling rate (spatial resolution) of 1 arcminute, which is corresponding to 0.2 pc at an estimated distance of 1.1 kpc (Kim et al. 2000). There are many kinds of molecular clouds in the interstellar medium, such as quiescent, violent or star forming clouds, shocked gas, and cometary clouds etc. The invariance of the fractal dimension of these clouds (Williams et al. 2000) may imply self-similar structure of the molecular clouds, and turbulent motion within them; A self-similar object is exactly or approximately similar to a part of itself. If the target clouds are fractal, they are called to be self-similar. Most observed molecular clouds has the highly supersonic linewidths, and this probably implies turbulence motion in them, for which one would naturally expect a fractal structure (Mandelbrot 1982). In addition, invariance of fractal dimension D from cloud to cloud might be related to the similarity in mass spectrum index (Kramer et al. 1998). This fact can be applied to the clouds with varying threshold values, though there had been no reports on high density regions of the clouds, such as clumps or cores. This issue remains to be solved in the forthcoming papers.

Three lines indicated are for the elongated bands, ellipses, and circles. The extreme cases for any cloud's structure could be highly elongated (band), or perfectly circular (circle or spheroid). In Figures 1 to 3, we set the ratio of major and minor axes of ellipses as 2 with various sizes, which comprises an ellipsoidal cloud. No specification is introduced for velocity axis. The fractal dimensions of the extremely elongated or highly fractal clouds, and circular clouds are 2 and 1. Thus the range of fractal dimension of any clouds is between 1 and 2. Ellipse-clouds in Figure 1 has almost the same fractal dimension as that of the circular clouds, though their intercepts of both axes are different. However, if the ratio of major and minor axes become higher, the fractal dimension will be getting close to that of the elongated band.

The previous studies on fractal dimensions of the molecular clouds are based on ^{12}CO data (Falgarone et al. 1991; Williams et al. 2000). Here we have ^{13}CO data for the clouds with anomalous velocities. We applied the same method to estimate fractal dimension to this ^{13}CO data cube. In Figure 3 we present its relationship between perimeters and areas. A bisector fit applied to ^{13}CO data cube yields a best fit line as follows:

$$\log(\mathrm{P}) = 1.392/2(\pm 0.12)\log(\mathrm{NP}) + 0.566(\pm 0.024) \quad (3)$$

The trend seems to be almost the same as the ^{12}CO emission (Database B) except a few points. Two points in the middle of the plot is much closer to the ellipse-line than others. These could be some compact dense clumps whose morphology may be more or less circular or ellipsoidal. However, we are not claiming that the fractal dimension could be different for the dense part of molecular clouds yet. As mentioned above, this issue remains to be seen until a large amount of database for the dense cores for large area are collected.

In Figure 4, dispersion of longitude and latitude of each slice-cloud is plotted in the units of the arcminutes. As the pixel size of Database A is 3 arcminutes, and that of Database B is 1 arcminute, the distribution of smaller size clouds of Database B is more widely spreaded than Database A. There is no significant difference for the middle size clouds for two databases. There is no preference of latitude or longitude in the elongation direction (the angles of the major axes) for the slice-clouds, though the original integrated intensity map of the Galactic Anticenter molecular clouds is quite elongated (see Figure 1 of Lee et al. 1999) along the Galactic Longitude.

As we have discussed above, the fractal dimensions of ^{12}CO and ^{13}CO clouds are the same, $D = 1.4$, sort of self-similar structure. Moreover, the highly supersonic linewidths of cloud ensembles may imply well-know turbulent motions, for which one would naturally expect a fractal structure (Mandelbrot 1982). Again this issues should be applied to the dense parts of the cloud core and clumps for further study.

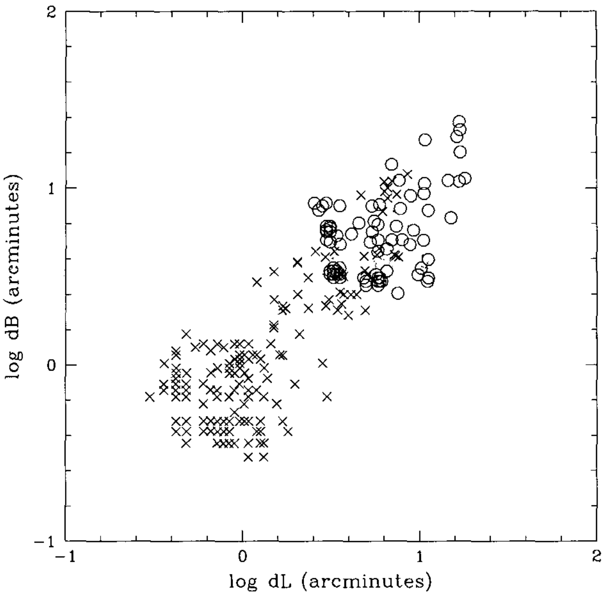

Fig. 4.— Log-log plot of dispersions of longitudes and latitudes for the slice-clouds in the units of arcminutes. Open circles and crosses are the same as in Figure 1.

V. SUMMARY

We have estimated the fractal dimension of the molecular clouds in the Antigalactic Center based on the ^{12}CO ($J = 1 - 0$) and ^{13}CO ($J = 1 - 0$) database obtained using the 14m telescope at Taeduk Radio Astronomy Observatory. Using a developed code within IRAF, we were able to identify slice-clouds, and determined the dispersions of two spatial coordinate as well as perimeters and areas. Using a threshold temperature of 2 σ (0.5 K for Database A, and 0.4 K for Database B), we identified 272 slice-clouds for the GAC-COS (Database A), and 318 slice-clouds for the clouds with anomalous velocities (Database B). We found that there are many slice-clouds with a small number of pixels; Only 49 slice-clouds are larger than 5 in pixel number for Database A, and 136 slice-clouds larger than 5 in pixel number for Database B. The number of pixels of the largest slice clouds are 70 for Database A, and 733 for Database B. Fractal dimension D of a molecular cloud boundary can be determined from the perimeter-area relation of a map. The fractal dimension of Database B, was estimated to be $D = 1.4$, where $P \propto A^{D/2}$, which is very similar to that of the local dark clouds. It is the same trend for the ^{13}CO database. The fractal dimension of the target region was estimated to be $D = 1.34$ for low resolution database. The sampling rate (spatial resolution) of observed data must be an important parameter when estimating fractal dimension. Fractal dimension is apparently invariant when varying the threshold temperatures applied to cloud identification. According to the dispersion pattern of longitudes and latitudes of identified slice-clouds, there is no preference in elongation direction for the identified clouds, though the original CO emission distribution extends along the Galactic Longitude. Fractal dimension of dense parts (cores or clumps) of molecular clouds seems to be intriguing issue to understand the structure and structure. This can be conducted when a large amount of database with high-spatial dynamic range observations of the dense cores are collected.

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