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NUMERICAL CALCULATION OF TWO FLUID SOLAR WIND MODEL

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ABSTRACT

We have developed a two fluid solar wind model from the Sun to 1 AU. Its basic equations are mass, momentum and energy conservations. In these equations, we include a wave mechanism of heating the corona and accelerating the wind. The two fluid model takes into account the power spectrum of Alfvénic wave fluctuation. Model computations have been made to fit observational constraints such as electron(T_e) and proton(T_p) temperatures and solar wind speed(V) at 1 AU. As a result, we obtained physical quantities of solar wind as follows : T_e is 7.4×10^5 K and density(n) is 1.7×10^7 cm^{-3} in the corona. At 1 AU T_e is 2.1×10^5 K and n is 0.3 cm^{-3}, and V is 511 km s^{-1}. Our model well explains the heating of protons in the corona and the acceleration of the solar wind.

Key words : Sun: solar wind—Sun: Two-fluid model

I. INTRODUCTION

Solar wind is supersonic outflow of fully ionized plasma from the solar corona. The wind is composed of about 95% of protons and electrons, 4.5% of Helium, and other minor ions. It is well known that there are three types of solar winds : fast wind originating from open magnetic fields in coronal holes, slow wind occurring at streamer belts around solar ecliptic, and transient wind forming of coronal mass ejection. Typical speed of solar wind ranges from 300 to 700 km s^{-1} at 1 AU.

Parker (1958) first presented isothermal solutions of solar wind model and suggested that the corona cannot be in static equilibrium and be continuously expanding outwards. The Helios observations showed that the interplanetary solar wind near the ecliptic plane was far from thermodynamic equilibrium (Marsch 1992). Ulysses has also provided in-situ measurements that showed high proton temperature and fast wind in coronal hole at high latitude of the Sun (McComas et al. 2000).

Since the first two-fluid model suggested by Hartle & Sturrock (1968), there are several two-fluid models to solve the problem of the long standing questions regarding coronal heating and wind acceleration. Low frequency Alfvénic waves are known to be hard to heat the corona and the initial acceleration of the wind because the waves do not easily dissipate within a short distance from the Sun. However, high frequency Alfvénic waves may dissipate in the solar atmosphere near local proton gyro-frequency by ion-cyclotron resonance damping (Marsch et al. 1982). Based on the above ideas, Marsch (1992) suggested that ion cyclotron dissipation of Alfvénic wave supply solar wind energy. Tu & Marsch (1997) further developed a two fluid solar wind model from the Sun to 65 solar radii for coronal heating and wind acceleration by Alfvénic wave fluctuation. Their model showed that the wave effect can result in coronal heating and solar wind acceleration until about 65 solar radii.

On the other hand, in order to explain coronal heating and wind acceleration, somewhat different artificial heating functions were employed; for example, Withbroe (1988), Habbal et al. (1995), and Esser & Habbal (1995). These models showed that a deposition of a suitable amount of energy near the coronal base can produce a hot corona and high speed wind. But it should be noted that their models are not based on real physical processes.

In this paper we have developed a two fluid solar wind model for solar wind from the Sun to 215 solar radii (1 AU). In the model, heating corona and accelerating wind are characterized by high frequency Alfvénic wave fluctuation, on the basis of Tu, Pu, and Wei (1984) who derived the power spectrum from observed Helios observations. We have compared our results of the model with those of solar wind observations at 1 AU. To avoid some numerical difficulty at the critical point where a singularity occurs in the wind equations, we numerically calculate from the critical point to both sides. In section 2, we briefly describe the basic equations of our two fluid model. We explain our results and compare them with typical observations of solar wind at 1 AU in section 3. A brief summary and conclusion are given in section 4.

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TABLE 1
INPUT PARAMETERS AT THE CRITICAL POINT r_c

	f_{max}	n cm^{-3}	T_p K	T_e K	V km s^{-1}	r_c R_s
Case1	1	2.59×10^{-3}	1.7×10^6	0.5×10^6	134	4.28
Case2	2	4.12×10^{-3}	1.5×10^6	0.5×10^6	128	4.98
Case3	4	3.66×10^{-1}	0.9×10^6	0.6×10^6	111	6.89

II. BASIC EQUATIONS

The basic equations of our two fluid model are : (1) the continuity equation for the proton mass density; (2) momentum equation including wave pressure gradient; (3) proton energy equation including a wave heating term; (4) electron energy equation ; and (5) power spectrum equation. These equations were described in detail in Tu & Marsch (1997). Proton heating rate in proton energy equation is described by power spectrum equation of Tu (1987). In order to account for the nonradial expansion in coronal holes, we included a nonradial area function of the flow-tube in the model equations.

We consider one-dimensional equation of the two fluid model with the parameter as a function of the distance r from the Sun. Solar wind, in which electron fluid and proton fluid possess both the same density(n) and velocity(V), is assumed to be a steady flow with non-spherical expansion.

Mass conservation equation is

$$\frac{1}{A}\frac{\partial}{\partial r}(\rho V A) = 0, \quad (1)$$

where ρ is proton mass density, V is solar wind speed and A is an area function of wind flow tube. Although the solar wind is in a state of quasi-neutral, we assume that the wind is in a state of neutral. Therefore density and velocity of electron and proton are $n_e \simeq n_p = n$, $V_e \simeq V_p = V$.

Momentum equation for solar wind speed is

$$V\frac{\partial V}{\partial r} = \frac{1}{\rho}\frac{\partial}{\partial r}(p + p_A) - \frac{GM_s}{r^2}, \quad (2)$$

where G is gravitational constant and M_s is mass of Sun. Here p_A is the wave pressure obtained by integration over the power spectrum. Thermal gas pressure(p) is

$$p = nk_BT_p + nk_BT_e, \quad (3)$$

where k_B is Boltzmann constant.

The energy equations for protons and electrons are

$$V\frac{\partial T_p}{\partial r} + (\gamma - 1)T_p\frac{1}{A}\frac{\partial(VA)}{\partial r} = \frac{(\gamma-1)}{nk_BA}\frac{\partial}{\partial r}(A\kappa_p\frac{\partial T_p}{\partial r}) - \nu_{ep}(T_p - T_e) + \frac{(\gamma-1)}{nk_B}Q_A \quad (4)$$

and

$$V\frac{\partial T_e}{\partial r} + (\gamma - 1)T_e\frac{1}{A}\frac{\partial(VA)}{\partial r} = \frac{(\gamma-1)}{nk_BA}\frac{\partial}{\partial r}(A\kappa_e\frac{\partial T_e}{\partial r}) + \nu_{ep}(T_p - T_e) - L\frac{(\gamma-1)}{nk_B}, \quad (5)$$

where ν_{ep} is a collisional frequency given by $9 \times 10^{-2} \times nT_e^{3/2}$, κ_p and κ_e is each classical collisional conductivity of proton and electron, and Q_A represents the rate at which wave energy is transferred to dissipation near the local proton cyclotron frequency. γ is the ratio of specific heats. For fully ionized hydrogen, γ is 5/3 but, in general, γ lies between 1 and 5/3. For simplicity, we take $\gamma = 5/3$ in this study. L is the radiative loss function given by Rosner et al. (1978).

By assuming the solar wind originate from coronal holes having open magnetic fields, we consider the non-spherical area function $A(r)$ of coronal hole following Kopp & Holzer (1976). The area function is

$$A(r) = A_0(\frac{r^2}{R_s^2})f(r), \qquad A_0 = 4\pi R_s^2, \quad (6)$$

where

$$f(r) = \frac{f_{max}e^{(r-r_1)/\sigma} + f_1}{e^{(r-r_1)/\sigma} + 1}, \quad (7)$$

where f_{max} is an expansion factor of flow tube area, r_1 is a heliocentric distance of the expansion area, σ is a length of rapid expansion region, and $f_1 = 1 - (f_{max} - 1)e^{(R_s - r_1)/\sigma}$. The input parameters of our model are $r_1 = 1.5R_s$, $\sigma = 0.2R_s$, and f_{max} =1, 2, and 4.

The power spectrum equation made by Tu (1987) is given by

$$\frac{1}{A}\frac{\partial}{\partial r}(A(V + V_A)\frac{P(f,r)}{4\pi}) - (\frac{P(f,r)}{8\pi})\frac{1}{A}\frac{\partial}{\partial r}(AV) = -\frac{\partial}{\partial f}\frac{F(f,r)}{4\pi}, \quad (8)$$

where V is wind velocity, V_A is Alfvénic velocity, $P(f,r)$ is a power spectrum density, and $F(f,r)$ is an energy flux function. This equation was also used in several solar wind model (Tu 1987; Tu & Marsch 1997; Hu et al. 1999). Heating rate by wave, Q_A is obtained by integrating power spectrum equation (8) for frequency range from lowest boundary frequency f_0 to highest boundary frequency f_H, which corresponds to ion-cyclotron frequency at which Alfvénic wave dissipation occurs. Heating rate Q_A and wave pressure p_A are

$$Q_A = \frac{1}{4\pi}F(f_H, r) - (V + V_A cos\phi)\frac{P(f_H, r)}{4\pi}\frac{df_H}{dr} \quad (9)$$

and

$$p_A = \frac{< b^2 >}{8\pi} = \frac{1}{8\pi}\int_{f_0}^{f_H} P(f, r)df, \quad (10)$$

respectively, where

$$f_H = \frac{1}{20\pi}\frac{eB}{m_p c}. \quad (11)$$

The radial component of magnetic field is described by

$$B_r A = B_0 A_0, \quad (12)$$

$$V_A = \frac{B_r}{\sqrt{4\pi\rho}}, \quad (13)$$

where $B_0 = 1.29 \times f_{max}$ and $B_r = Bcos\phi$. Here ϕ is the angle between magnetic field vector and radial direction.

Tu & Marsch (1997) integrated model equations from 1 R_s to 65 R_s. Following their work, we solved these steady-state equations by integrating over radial distance r. The unknown variables of the model are V, n, T_p, T_e, and P. These equations are solved by inputting initial values at the critical point. For effective integration of the equations, we use 4th order Runge-Kutta method with a step of 0.01 R_s. One problem is that there is a singularity at the critical point (r_c) where denominator in wind speed differential equation becomes zero; that is, dv/dr = 0. To avoid the singularity, our model was computed independently for two different regions except for the critical point; one starts from r_c to 1 R_s, and the other, from r_c to 215 R_s. As a result, we can successfully compute the two fluid solar wind model from 1 R_s to 1 AU.

III. RESULTS

By applying the model to three cases having different conditions at the critical point (r_c), we have examined if the model can reproduce the physical parameters in the coronal hole (Mason & Bochsler 1999) and at 1 AU (McComas et al. 2000). Table 1 shows the input parameters for three cases at r_c. We input different f_{max} in each case and the other input parameters are chosen from several trial values within observed physical parameters in order to produce reasonable outputs at 1 AU. In the case 1, $f_{max} = 1$, which indicates narrow open fields in coronal hole. We used $f_{max} = 2$ in the case 2 and $f_{max} = 4$ in the case 3, which correspond to wider open fields than that of the case 1. The case 3 is characterized by low density and large open field structures such as coronal hole. In order to consider proton heating, we input proton temperature higher than electron temperature.

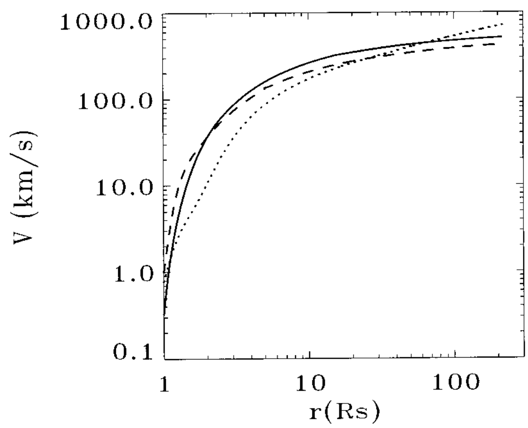

Fig. 1.— Radial variation of the computed wind speed from the Sun to 1 AU. Three curves represent selected three cases(solid - case 1, dashed - case 2, and dotted - case 3).

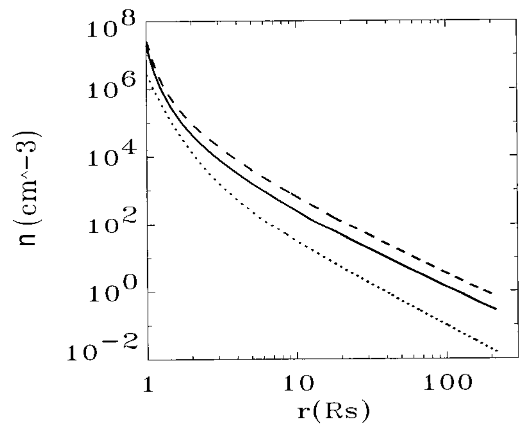

Fig. 2.— Radial variation of the number density n. Other explanation is the same as Fig. 1.

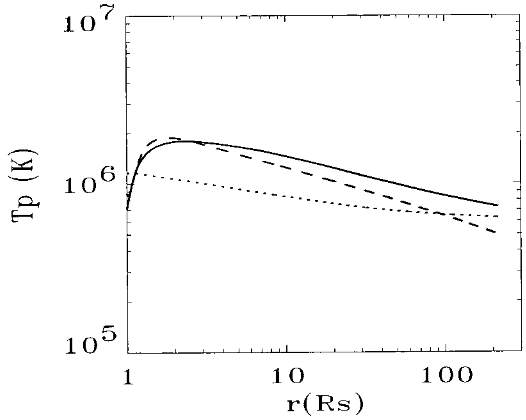

Fig. 3.— Radial variation of the proton temperature T_p. Other explanation is the same as Fig. 1.

TABLE 2
MAIN RESULTS AT 1 R_s AND 1 AU.

	r=1 R_s			r=215 R_s(≈1 AU)			
	$T_p = T_e$ K	V km s^{-1}	n cm^{-3}	T_p	T_e K	V km s^{-1}	n cm^{-3}
Case1	7.4×10^5	0.37	1.69×10^7	7.3×10^5	2.1×10^5	511	0.27
Case2	8.3×10^5	1.03	2.05×10^7	4.9×10^5	3.0×10^5	419	0.67
Case3	11.6×10^5	0.80	0.02×10^7	6.2×10^5	2.4×10^5	710	0.01

TABLE 3
OBSERVATIONAL PROPERTIES OF SOLAR WIND NEAR THE SUN BY SOHO(MASON & BOCHSLER, 1999) AND AT 1 AU BY ULYSSES. ULYSSES OBSERVATION SHOW STATISTICAL MEAN PROPERTIES OF HIGH-LATITUDE SOLAR WIND ABOVE 36 DEGREE HELIOLATITUDE(MCCOMAS ET AL. 2000).

	r=1 R_s		r=215 R_s(≈ 1 AU)		
	T K	n cm^{-3}	T_e K	V_p km s^{-1}	n cm^{-3}
Inter-plume	1.0 × 10^6	0.07 × 10^8	2.0 × 10^5	758	2.7
Coronal hole	0.75 × 10^6	2.0 × 10^8			

Figure 1 shows the profiles of the output flow speed V for the case 1 (solid curve), the case 2 (dashed curve), and the case 3 (dotted curve). In all cases, flow speeds are continuously accelerated and come up to the highest values at 1 AU. In the cases 1 and 2, there are large initial accelerations. Figure 2 shows the profiles of the number density n. Figure 3 shows the profiles of proton temperature T_p. In the cases 1 and 2, there is proton heating caused by Alfvénic wave heating at 2-3 R_s and it makes proton temperature reach about 2×10^6 K. As seen in Figures 1 and 3, wave heating also plays important role in accelerating solar wind in the corona region.

Table 2 summarizes the results of our calculation for each case at 1 R_s and 1 AU. Table 3 shows the observational properties of solar wind near the Sun by SOHO and at 1 AU by Ulysses. The computed number densities at 1 R_s are 1.69×10^7cm^{-3} in the case 1, 2.05×10^7cm^{-3} in the case 2, and 0.02×10^7cm^{-3} in the case 3. The computed temperatures at 1 R_s are 7.4×10^5 K in the case 1, 8.3×10^5 K in the case 2, and 1.2×10^6 K in the case 3, respectively. At 1 R_s, output physical parameters of the cases 1 and 2 are consistent with observational values summarized in Table 3. The computed wind speeds at 1 AU are 511 km s^{-1} in the case 1, 419 km s^{-1} in the case 2, and 710 km s^{-1} in the case 3, which are quite comparable to the observed wind speeds. At 1 AU, the computed electron temperatures are 2.1×10^5 K in the case 1, 3.0×10^5 K in the case 2, and 2.4×10^5 K in the case 3, which are also similar to the observed values at 1 AU (Table 3). In comparison with observed values of number density ranging from a few to about 10 cm $^{-3}$, the cases 1 and 2 give a little bit smaller values than the observations. For the case 3, we obtain quite fast solar wind speed around 700 km s^{-1} at 1 AU. But the number densities near the Sun and at 1 AU are quite lower than observed values (Mason & Bochsler 1999; McComas et al. 2000), which seems to be due to large expansion of flux tubes.

IV. CONCLUSION

We have developed the two fluid solar wind model from the Sun to 1 AU. For effective calculation of solar wind equations, we simultaneously solved the differential equations by integrating from the critical point to both sides. As a result, we obtained steady-state solutions of the solar winds with two fluids. By taking into account the Alfvénic wave heating to plasma, we successfully reproduced proton heating and wind acceleration.

Our results show that the physical parameters near the Sun is consistent with observation. We can also reproduce normal solar wind speed of about 400 ~ 500 km s^{-1} at 1 AU. In the case of large expansion rate of flux tubes, we can obtain high speed solar wind over 700 km s^{-1} at 1 AU but much lower number density than observed values. From the results it is found that there is systematic tendencies among the solar properties; that is, the low number density and large expansion rate in the corona can generate high speed wind, and solar wind speed increases as the decrease rate of proton temperature near 1 AU decreases. Such tendencies are consistent with observational characteristics. In our model, wave heating is small relative to the other terms such as conductive cooling term (Kim 2002). If more effective wave heating or other heating terms are included, then we can obtain higher proton temperature near the Sun and faster solar wind speed. Some multi-fluid models(e.g., Tu & Marsch 2001) which include ion effect for heating and acceleration may be needed to explain all observed physical quantities of the solar wind.

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