New Evaluation on Maximum Ratio Diversity Reception for the Detection of Signals over Correlated Nakagami Fading Channels

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SUMMARY—The performances of M-ary signals using L-branch maximum ratio combining (MRC) diversity reception in correlated Nakagami fading channels are derived theoretically. The coherent reception of M-ary differential phase shift keying (MDPSK), phase shift keying (MPSK), and quadrature amplitude modulation (MQAM) is considered. It is assumed that the fading parameters in each diversity branch are identical. The general formula for evaluating symbol error rate (SER) of M-ary signals in the independent branch diversity system is presented using the integral-form expressions.

Key words—MRC, Correlated Nakagami fading, MDPSK, MPSK, MQAM

I. INTRODUCTION

The statistics for various fading channel models and the resulting communication evaluation have been considerably studied as summarized in [1]. The statistical properties of mobile radio environments can be often specified by the following propagation effects: 1) long-term fading 2) short-term fading [2]. An alternative solution to the problem of obtaining acceptable performances on a fading channel is the diversity technique, which is widely used to combat the fading effects of time-variant channels. When M-ary signals experience the fading channels, diversity schemes can minimize the effects of these fadings since deep fades seldom occur simultaneously during the same time intervals on two or more paths.

In this paper, to model the disturbances it is assumed that the channel model has independent paths with Nakagami fading statistics. We study the effect of correlation on the performance of MRC with an arbitrary number of diversity branches. We can represent the average SER with an exact integral expressions by MRC systems in receiving MDPSK, MPSK, and MQAM signals on this correlated fading channel.

For the case of equal power correlation coefficient ρ between the channels, equal fading parameter m and average SNR γ_o, the conditional probability density function (PDF) of the received instantaneous SNR γ in a MRC diversity system on a Nakagami fading channel is given by [3]

$$f(\gamma)=\frac{\left(\frac{\gamma m}{\gamma_0}\right)^{Lm-1}\exp\left(-\frac{\gamma m}{\gamma_0(1-\rho)}\right){}_1F_1\left(m;Lm;\frac{Lm\rho\gamma}{\gamma_0(1-\rho)(1-\rho+L\rho)}\right)}{\left(\frac{\gamma_0}{m}\right)(1-\rho)^{m(L-1)}(1-\rho+L\rho)^m\Gamma(Lm)}, \tag{1}$$

$$\gamma\geq 0,\quad 0<\rho<1$$

where

$${}_1F_1(\alpha,\beta,\chi)=\sum_{n=0}^{\infty}\frac{\Gamma(\alpha)\Gamma(\beta+n)\chi^n}{\Gamma(\alpha)\Gamma(\beta+n)n!},\qquad \beta\neq 0,-1,-2,\cdots \tag{2}$$

is the Gauss hypergeometric function [4].

In spite of the complexity of MRC compared to other diversity techniques since it requires the knowledge of a fading amplitude in each signal branch, it is worth considering because it has the maximum possible improvement that a diversity system can attain through a fading channel.

II. ERROR RATE ANALYSIS

Once the statistics of the instantaneous SNR are determined as the function of the average SNR, the error performance in a correlated Nakagami fading channel can be evaluated by averaging the conditional probability of error over the PDF of the instantaneous SNR.

When MDPSK signals experience no fading, the expression for the conditional probability of error is given by [5]

$$P_{s,MDPSK}=\frac{\sin\frac{\pi}{M}}{2\pi}\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\frac{\exp\left[-\gamma\left(1-\cos\frac{\pi}{M}\cos\theta\right)\right]}{1-\cos\frac{\pi}{M}\cos\theta}d\theta \tag{3}$$

We can represent the average SER in receiving MDPSK signals under the effect of correlation among the MRC diversity branches by averaging (3) over the PDF of an instantaneous SNR in Nakagami fading channels as follows:

$$P_{e,MDPSK}=\int_0^{\infty}P_{s,MDPSK}\,f(\gamma)\,d\gamma \tag{4}$$

where $P_{e,MDPSK}$ is the average SER of MDPSK signals under a correlated Nakagami fading model.

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Consequently, we can find the symbol error probability under the Nakagami fading model to be

$$P_{e,MDPSK} = \frac{\left(\frac{m}{\gamma_0}\right)^{Lm}}{(1-\rho)^{m(L-1)}(1-\rho+L\rho)^m}\frac{\sin\frac{\pi}{M}}{2\pi} \cdot \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\frac{1}{1-\cos\frac{\pi}{M}\cos\theta} \cdot \left[1-\cos\frac{\pi}{M}\cos\theta+\frac{m}{\gamma_0(1-\rho)}\right]^{-Lm} \cdot \left[1-\frac{1}{1-\cos\frac{\pi}{M}\cos\theta+\frac{m}{\gamma_0(1-\rho)}}\frac{Lm\rho}{\gamma_0(1-\rho)(1-\rho+L\rho)}\right]^{-m} d\theta. \tag{5}$$

which can be written in the integral-form, not in the closed-form.

We can observe that the result of (5) for $\rho=0$ is perfectly equivalent to the result of [6, Eq. (5.2.13)].

The exact SER of coherent MPSK under a nonfading channel can be represented as [5]

$$P_{s,exact,MPSK} = \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}-\frac{\pi}{M}}\exp\left[-\gamma\sin^2\left(\frac{\pi}{M}\right)\sec^2\theta\right]d\theta. \tag{6}$$

Next, we can find the integral-form performance of MPSK signals under the effect of MRC diversity in a correlated Nakagami fading channel to be [7]

$$P_{e,MPSK}=\int_0^\infty P_{s,MPSK}f(\gamma)\,d\gamma = \frac{\left(\frac{m}{\gamma_0}\right)^{Lm}}{(1-\rho)^{m(L-1)}(1-\rho+L\rho)^m}\frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}-\frac{\pi}{M}}\left[\sin^2\left(\frac{\pi}{M}\right)\sec^2\theta+\frac{m}{\gamma_0(1-\rho)}\right]^{-Lm} \cdot \left[1-\frac{1}{\sin^2\left(\frac{\pi}{M}\right)\sec^2\theta+\frac{m}{\gamma_0(1-\rho)}}\frac{Lm\rho}{\gamma_0(1-\rho)(1-\rho+L\rho)}\right]^{-m} d\theta. \tag{7}$$

For the special case of $m=1$, we can thus find that the result of (7) for $\rho=0$ corresponds to that of [8, Eq. (A.5)] in a Rayleigh fading channel.

Next, we analyze the performance of MRC diversity reception of MQAM signals in a correlated Nakagami fading channel. We can derive the integral-form performance exact for $m=2$, when j is even, by averaging the conditional probability of error over the PDF of an instantaneous SNR under correlated Nakagami fading channels.

MQAM is, in practice, frequently used technique which requires less average transmitted power to achieve the same performance as MPSK signals. Thus, it may be valuable to evaluate the performance of MQAM with MRC diversity receiver over the Nakagami fading channels.

Now, to derive the integral-form performance in a correlated Nakagami fading channel, given that j is even, we introduce the exact SER in the presence of AWGN channel, represented as [9]

$$P_{s,MQAM} = \frac{4B}{\pi}\int_0^{\frac{\pi}{2}}\exp\left(-\frac{g\gamma}{\sin^2\theta}\right)d\theta-\frac{4B^2}{\pi}\exp\left(-\frac{g\gamma}{\sin^2\theta}\right)d\theta \tag{8}$$

where $g=\frac{3}{2(M-1)}$ and $B=\frac{\sqrt{M}-1}{\sqrt{M}}$.

Average SER for MQAM with the correlation between the branches under the effect of MRC diversity can be shown to be given by

$$P_{e,MQAM} = \int_0^\infty P_{s,MQAM}\quad f(\gamma)\,d\gamma \equiv P_{e1,MQAM}-P_{e2,MQAM} \tag{9}$$

Then, $P_{e1,MQAM}$ and $P_{e2,MQAM}$ can be expressed as

$$P_{e1,MQAM} = \int_0^\infty P_{s1,MQAM}\,f(\gamma)\,d\gamma = \frac{\left(\frac{m}{\gamma_0}\right)^{Lm}}{(1-\rho)^{m(L-1)}(1-\rho+L\rho)^m}\frac{4B}{\pi}\int_0^{\frac{\pi}{2}}\left[\frac{g}{\sin^2\theta}+\frac{m}{\gamma_0(1-\rho)}\right]^{-Lm} \cdot \left[1-\frac{1}{\frac{g}{\sin^2\theta}+\frac{m}{\gamma_0(1-\rho)}}\frac{Lm\rho}{\gamma_0(1-\rho)(1-\rho+L\rho)}\right]^{-m} d\theta \tag{10}$$

and

$$P_{e2,MQAM} = \int_0^\infty P_{s2,MQAM}\,f(\gamma)\,d\gamma = \frac{\left(\frac{m}{\gamma_0}\right)^{Lm}}{(1-\rho)^{m(L-1)}(1-\rho+L\rho)^m}\frac{4B^2}{\pi}\int_0^{\frac{\pi}{4}}\left[\frac{g}{\sin^2\theta}+\frac{m}{\gamma_0(1-\rho)}\right]^{-Lm} \cdot \left[1-\frac{1}{\frac{g}{\sin^2\theta}+\frac{m}{\gamma_0(1-\rho)}}\frac{Lm\rho}{\gamma_0(1-\rho)(1-\rho+L\rho)}\right]^{-m} d\theta \tag{11}$$

where

$$P_{s1,MQAM} = \frac{4B}{\pi}\int_0^{\frac{\pi}{2}}\exp\left(-\frac{g\gamma}{\sin^2\theta}\right)d\theta \tag{12}$$

and

$$P_{s2,MQAM} = \frac{4B^2}{\pi}\int_0^{\frac{\pi}{4}}\exp\left(-\frac{g\gamma}{\sin^2\theta}\right)d\theta \tag{13}$$

It is clear that given the number of diversity branches, the performance of MQAM for $M=4$ is perfectly equal to that of MPSK in Fig. 4 [10].

III. NUMERICAL RESULTS

Figs. 1-3 show the performance of MDPSK, MPSK, MQAM signals on the MRC detection under the effect of correlation in the Nakagami fading channel, respectively. It is expected result as the power correlation coefficients decrease, the fading depth decreases. These figures illustrate that, by increasing the number of M, the SNR per symbol increases to achieve an equal SER for the given values of m, L, and ρ. The results indicate that given the fading parameter m and the order of diversity L, the discrepancy for the error performance of MQAM signals between the correlation coefficient becomes more apparent than that of MDPSK and MPSK signals as the alphabet size M grows. The performance of MRC diversity branches with increasing M is improved very restrictedly in correlated Nakagami fading conditions for the given values of the fading parameter m and the diversity branches L regardless of the power correlation coefficient.

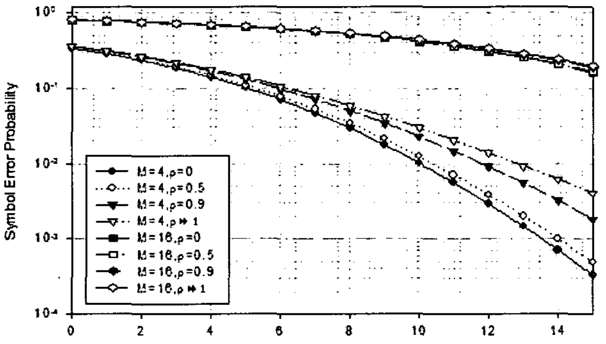

Fig. 1 Error performance comparisons of MDPSK signals adopting correlated MRC diversity technique for m=2 and L=2.

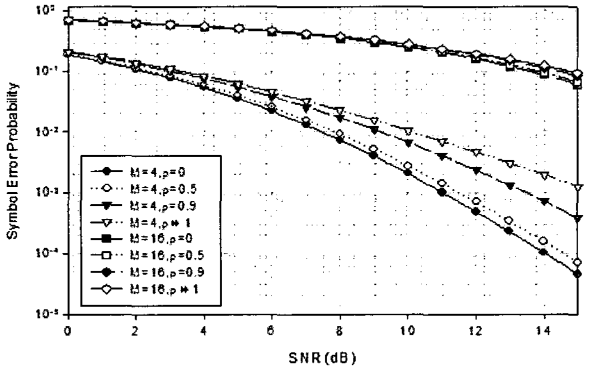

Fig. 2 Error performance comparisons of MPSK signals adopting correlated MRC diversity technique for m=2 and L=2.

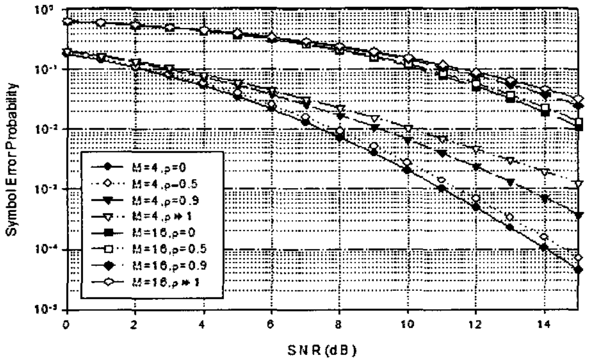

Fig. 3 Error performance comparisons of MQAM signals adopting correlated MRC diversity technique for m=2 and L=2.

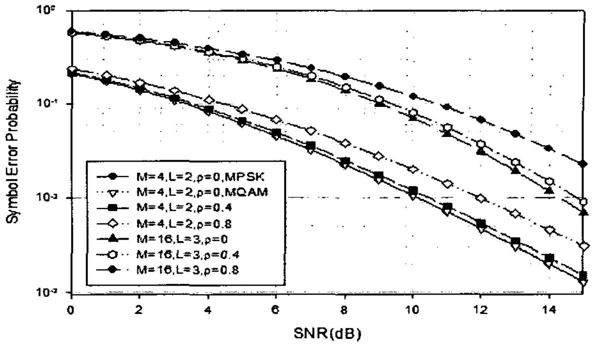

Fig. 4 Error performance comparisons of MQAM signals adopting correlated MRC diversity technique for the Rayleigh fading.

IV. CONCLUSION

The particular case of interest for the correlation between branches in the MRC diversity case under Nakagami fadings has been considered. Average SER formulae of MDPSK, MPSK, and MQAM signals under the effect of correlation have been derived in terms of integral expressions. These results are sufficiently general to offer a convenient method to evaluate the performance of several current M-ary modulation systems that operate on channels with a wide variety of fading conditions in wireless personal communications.

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