Journal of the Korea Concrete Institute
Vol.16 No.5, pp.719~724, October, 2004

Roles of Bearing Angle in Bond Action of Reinforcing Bars to Concrete

Oan Chul Choi[1)*]

[1)] *Dept. of Architectural Engineering, Soong Sil University, Seoul, Korea*

(Received May 10, 2004; Accepted September 30, 2004)

Abstract

The ribs of deformed bars can split the cover concrete by wedging action or shear off the concrete in front of the ribs. As slip of deformed bars increases, the rib face angle is flattened by the crushed concrete wedge, which reduces the rib face angle to a smaller bearing angle. The roles of bearing angle are explored to simulate this observation.

Analytical expressions to determine bond strength for splitting and pullout failure are derived, where the bearing angle is a key variable. As the bearing angle is reduced, splitting strength decreases and shearing strength increases. When splitting strength becomes larger than shearing strength, the concrete key is supposed to be sheared off and the bearing angle is reduced with decreasing the splitting strength. As bars slip, bearing angle decreases continually so that splitting bond strength is maintained to be less than shearing bond strength. The bearing angle is found to play a key role in controlling the bond failure and determination of bond strength of ribbed reinforcing steel in concrete structures.

Keywords: bearing angle, bond (concrete to reinforcement), rib of bars, pullout, splitting failure

1. Introduction

From the beginning of the past century, reinforced concrete researchers have endeavored to increase the bond characteristics of deformed reinforcing steel. Modern reinforcing bar rib geometries date from the work of Clark in 1949. Since then, knowledge concerning the bond between concrete and deformed reinforcing bars has increased considerably based on studies of the bond cracks, failure modes, in addition to the mechanism in the concrete around ribs.

During the late 1950s and the 1960s, several researchers observed that slip of deformed bars can occur in two ways: the ribs can split the concrete by wedging action and the ribs can crush the concrete[1,2)]. Other researchers have attempted to derive analytical expressions for bond strength by equating the bursting force produced by the wedging action of the bar to the resistance to splitting provided by the concrete cover. The influence of the deformation characteristics of ribbed bars upon the bond mechanisms has been mostly studied for the case of the splitting failure, being the weaker mode and primarily concern for design.[2-4)]

The ribs act as wedges and the concrete in front of the ribs crushes gradually, resulting in a pullout-type failure. Rehm[1)] and Lutz[2)] found that the concrete in front of the ribs undergoes gradual crushing, followed by a pullout mode. Eligehausen et al.[5)] observed the progressive growth of concrete micro-crushing in front of the ribs which reduced the tangent stiffness of bond-slip curves. The high rib face angle on the ribs is flattened by the crushed concrete wedge, which reduces the effective rib face angle to a smaller value (Fig. 1) Geometry of bar deformation pattern governs bond behavior and is instrumental in guaranteeing in an adequate bond resistance. The geometry of ribs, as well as the interfacial properties, have been addressed and explicitly modeled to examine the underlying mechanism that produces the observed bond behavior. Bond strength and bond resistance were observed varying with the bar rib face angle, rib height, rib spacing and relative rib area.[1,6-9)] An analytical study on interfacial bond with key variables, such as bearing angle, friction coefficient, cohesion and confinement force has been executed to predict bond strength of reinforcing bars with rib deformation for the case of splitting bond failure by this author.[10)]

---

* *Corresponding author*
Tel.: +82-2-820-0705 Fax.: +82-2-816-3354
E-mail address: occhoi@ssu.ac.kr

A number of researchers have developed theoretical expressions on bar-concrete bond interaction, however, studies on pullout bond failure have been limited. An empirical relationship between bond stress and the slip for pullout failure mode has been developed by Gambarova and Rosati[11) and they observed that bond strength exhibits an upper bound since high confining pressure leads to bond collapse in shear, owing to the shearing off of the concrete keys between bar ribs. Because the confining reinforcement opposing the bursting force would be provided, a pullout-type failure may develop at any level of confinement. Splitting and pullout modes are completely different in failure mechanics, and thus analysis with different approaches should be performed to have clear pictures of the bond mechanism. With this information as background, this study is intended to analyze further the basic bar-concrete interaction. An analytical study to determine bond strength for pullout failure, in addition to the expression to predict bond strength for splitting, is performed, where the bearing angle is the key variable. The roles of bearing angle are examined to understand bond mechanism of ribbed reinforcing bars to concrete. The behavior of bearing angle will help to understand the nature of the wedging action and the interfacial bond mechanism of ribbed reinforcing steel in concrete structures.

2. Analysis of bond mechanism

2.1 Splitting failure

Wedging action by the rib of deformed bars makes it possible to resolve bond forces into normal stress σ_n and tangential shear stress τ as shown in Fig.2. The resultant of normal component along the bar is what places the surround concrete in tension. When reinforcing bars are in tension P, concrete under the bearing side of a rib is known to be in a state of tri-axial compression with a major principal stress, the bearing stress σ_q on the rib, acting parallel to the bar axis. Normal to the bearing stress, the minor principal stress σ_r acts radially around the bar. As the radial force, the wedging force is applied to the concrete cover and confining bars. The method of analysis (presented here is a slightly revised and condensed form) has previously been used by Choi and Lee[10) to evaluate the bond strength in splitting. Bond force equal to the sum of the bearing stress on a single rib area, T_{Splits}, given by

$$T_{split} = A_r \sigma_q \tag{1}$$

in which A_r = projected area of rib parallel to the bar axis, approximated by $A_r = \pi d_b h_r$ where h_r is the rib height, σ_q = bearing stress on the bar rib acting parallel to the bar axis.

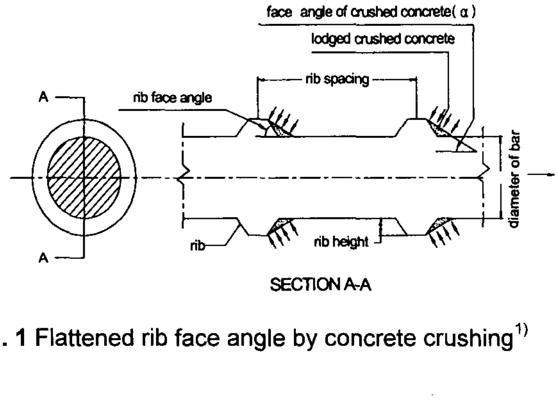

Fig. 1 Flattened rib face angle by concrete crushing[1)

The frictional force between the concrete and the steel (Fig. 3) on the inclined surface of the rib may be represented using the Mohr-Coulomb relation,

$$\tau = c + \mu \sigma_n \tag{2}$$

where c = cohesion, μ = coefficient of friction, σ_n = normal stress.

Suppose the stresses along the interface with an angle of α are in equilibrium with the sliding force by σ_q and the normal force by σ_n. The variable σ_q in Fig. 3 is given by

$$\sigma_q = \left(\sigma_r \frac{(1+\mu\cot\alpha)}{(1-\mu\tan\alpha)} + \frac{c}{\sin\alpha(\cos\alpha - \mu\sin\alpha)} \right) \tag{3}$$

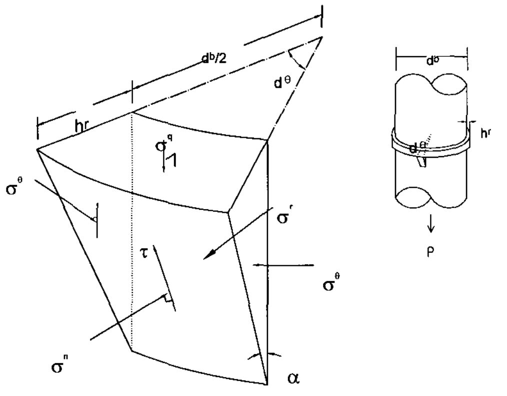

Fig. 2 Stresses acting on rib of bar[4)

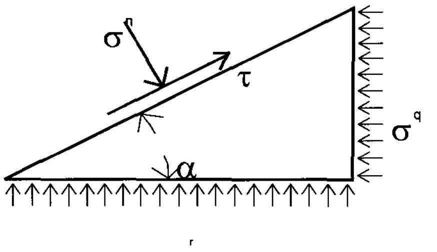

Fig. 3 Stress along interface with angle of α

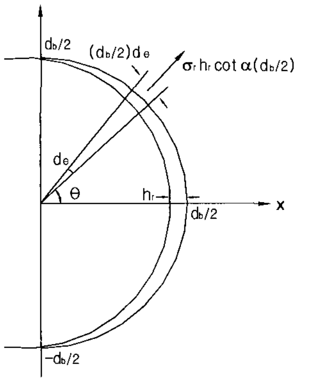

Fig. 4 Radial stress around bar circumference[4)]

Equation (3) is substituted into equation (1) to obtain

$$T_{split} = A_r\left(\sigma_r\frac{(1+\mu\cot\alpha)}{(1-\mu\tan\alpha)}+\frac{c}{\sin\alpha(\cos\alpha-\mu\sin\alpha)}\right) \tag{4}$$

where σ_r acting radially around the bar axis applies to concrete cover as radial stress. The radial stress σ_r acts over a distance of h_r $\cot\alpha$ below the rib, and exerts a bursting force on the concrete around the bar. The component of force in the x-direction (as shown in Fig. 4) is

$$dF_x = \sigma_r h_r \cot\alpha\frac{d_b}{2}d\theta\cos\theta \tag{5}$$

The summation of the component force on the perimeter is given by

$$F_x = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} dF_x = \sigma_r \cot\alpha h_r d_b \tag{6}$$

Eq. (6) is substituted in to Eq. (4) resulting in the final equation to predict bond strength, which is expressed as follows.

$$T_{split} = F_x\pi\tan\alpha\frac{(1+\mu\cot\alpha)}{(1-\mu\tan\alpha)}+A_r\frac{c}{\sin\alpha(\cos\alpha-\mu\sin\alpha)} \tag{7}$$

This simple expression to predict splitting bond resistance agrees well with the test results and empirical equations.[10)] Based on the comparison, ranges of the friction coefficient, cohesion, the effective rib face angle can be determined.

2.2 Pullout failure

A pullout-type bond failure develops with the concrete being sheared on a surface across the tops of the ribs.

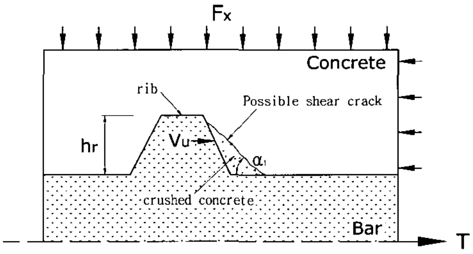

Fig. 5 Shear crack by the concrete key

As deformed bars slip, bars bear against the concrete in front of the ribs, thus increasing the bearing stress on the concrete key. Bearing resistance is obtained by the concrete key between the ribs resisting from crushing or shearing off as shown in Fig. 5.

As in the vicinity of composite connections, direct shear may cause the failure and potential failure plane can be established for such cases along which direct shear stresses are high. The basic approach is to assume that the concrete may crack in an unfavorable manner, or that slip may occur along a predetermined plane of weakness as in shear friction theory. A shear force acts parallel to a possible crack, and the resulting tendency for a concrete block to slip relative to the other block is resisted largely by friction along the concrete interface at the crack. Based on the early study of shear –friction theory[12)], for cracks in monolithic concrete shear strength should not be assumed greater than $0.2f_{ck}A_c$ as

$$V_n = 0.2f_{ck}A_c \tag{8}$$

where A_c is the area of cracked surface.

The area of cracked surface which is sheared off forming a cone with an angle of α as

$$A_c = \frac{2\pi d_b h_r}{\sin\alpha} \tag{9}$$

From Eq. (8) and (9), shearing bond resistance is

$$T_{shear} = \frac{0.2f_{ck}\pi d_b h_r}{\sin\alpha} \tag{10}$$

3. Roles of bearing angle in bond action

The roles of bearing angle in bond action can be explored by examining Eq. (7) and (10). First, let us examine Eq. (7) and the key variables for change of bond resistance in splitting failure. The friction coefficient μ is one of the key variables in determining the bond strength. Bond strength increases as the friction coefficient increases. The friction coefficient is reported to range between 0.40 and 0.50, but should be constant in nature. The confinement force F_x, such as concrete cover or transverse reinforcement, is directly proportional to the bond strength. It is made up of the resistance by concrete cover or by transverse reinforcement. Resistance by cover is related to the splitting tensile strength of concrete, the magnitude of the fracture energy, and the area of failure surface. As slip increases, the splitting concrete continues or/and some amount of the concrete crushing continues, however, the confinement force has a limitation. When confinement has no longer further resistance, the only variable in Eq. (7) is the bearing angle corresponding to the change of bond force as slip increases.

The bearing angle may be decreased to a less value from the previous bearing angle such as the initial effective rib face angle. While the ribs undergo gradual crushing from the rigid body motion of the steel bars, the high rib face angle is flattened by the crushed concrete wedge, which decreases the effective rib face angle to a smaller angle. The bearing angle may be continually reduced in any circumstance such as bars are confined by heavy transverse reinforcement. Reduction of bearing angle definitely changes the bond resistance in splitting. As the bearing angle is reduced, the bond resistance in splitting decreases. The change of the splitting bond strength versus bearing angle is illustrated in Fig. 6. The splitting resistance increase, as the level of confinement becomes higher, but bond resistance by splitting decrease as the bearing angle is decreased.

Let us also examine Eq. (10) and the key variables of bond resistance in pullout failure. Bearing resistance is obtained by the concrete key between the ribs resisting from crushing or shearing off. As in Eq. (10), the bearing resistance is attained by the concrete key which would be sheared off forming a cone with length of several times the rib height. The length of the cone is function of the bearing angle α.

Reducing of shearing angle definitely increase the area of the cone increasing the bond resistance. When the high rib face angle is flattened by the crushed concrete wedge, which reduces the possible shearing angle to a smaller angle, the shearing bond resistance increases. Thus, the bearing angle can be assumed to be continually reduced to a smaller value so that the shearing strength would reach the maximum bond resistance as much as possible. The shearing bond strengths increases with decrease in the bearing angle as illustrated in Fig.7.

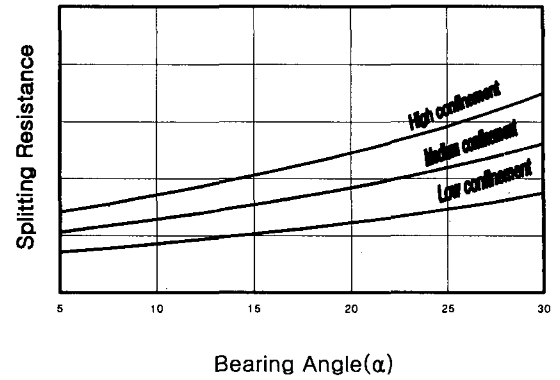

Fig. 6 Splitting bond resistance versus bearing angle

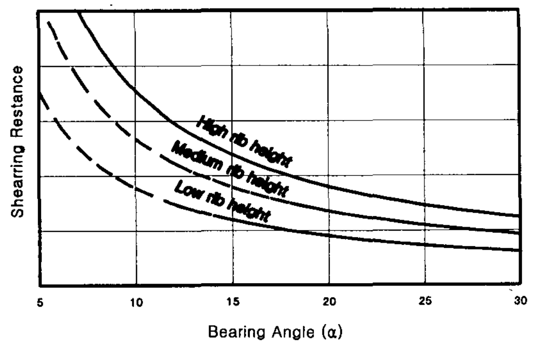

Fig. 7 Shearing bond resistance versus bearing angle

As in the figure, the shearing resistance also increases as h_r increases. Bond strength is determined along the interface at a state of forces in equilibrium under any failure conditions. The weaker of the two mechanisms is considered to control bond strength. If the bond strength of splitting is higher than that of shearing strength, the concrete key should be sheared off the concrete key resulting in rotating bearing angle and bond strength. As bars slip or bond force increases due to increase in confinement force, and if the bond strength of splitting becomes higher than that of shearing strength, the concrete key should be sheared off resulting in decreasing the bearing angle and bond strength, until a limiting resistance of the concrete is reached. Splitting strength is maintained to be less than the shearing strength. Thus,

$$T_{split} \le T_{shear} \tag{11}$$

As the concrete key is sheared off, the bearing angle is reduced decreasing splitting bond strength and increasing possible shearing bond strength. The bearing angle at the interface rotates so that splitting strength is maintained to be less than the shearing strength. The bond strength is determined from the bearing angle and the schematic determination of the bond strength with decreasing bearing angle is shown in Fig. 8.

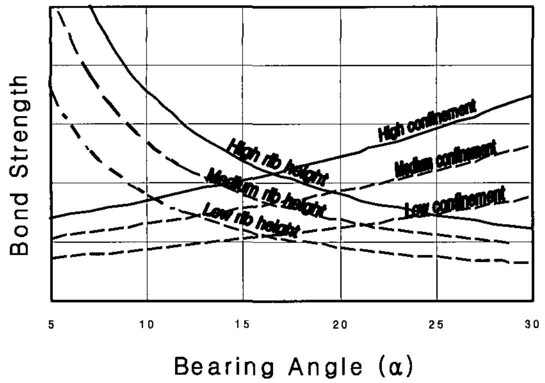

Fig. 8 Determination of bond strength with decreasing bearing angle

4. Discussion

There should be several stages of structural damage in concrete depending on levels of bond stress and confinement, as bond force increases. The concrete damage may be the adhesion breaking, the transverse micro-cracks, the splitting of concrete cover, and the crushing or shearing of the concrete producing by the ribs. In the case of light to medium transverse reinforcement, through splitting is mobilized at increasing slip values. Splitting failure occurs remote and results in structural damage, but simultaneously, concrete crushing by micro-cracking occurs at the bar-concrete interface rotating the bearing angle.

Bond strength reaches a peak and then starts decreasing. Bond behavior tends to become of the dry-friction type, as the concrete keys between ribs are crushed or sheared off. The bearing angle is reduced and rotates decreasing splitting bond strength with shearing off the concrete key. The observed behavior from decreasing bearing angle closely matches the experimental observation that high rib face angle is flattened by the crushed concrete wedge that decreases the effective rib face angle to a smaller angle.

Fig. 8 also shows that bars with higher rib height possessing higher shearing resistance can maintain larger bearing angle α , and consequently higher bond force, than bars with lower rib height. This behavior matches the finding by the previous study $^{6)}$ that lower crushed angles were observed on the bars with lower ribs, while higher angles were observed for the bars with higher ribs producing higher bond strengths.

A similar expression to predict bond strength has been developed by Cairns and Jones$^{4)}$ as,

$$\tau_{bu} = (\sigma_{sp} + 2 f_{coh} f_R) \cot \varphi_b \qquad (12)$$

where τ_{bu} is the bond strength, σ_{sp} is the confining stress, f_{coh} is the cohesion, f_R is the relative rib area and φ_b is the angle of internal friction. Equation (12) suggests that bond strength consists of two contributions, one related to the splitting resistance and the other to a non-splitting resistance component dependent on the friction at bar-concrete interface, which is observed in this study. The angle of friction in Eq. (12) seems to be related to the bearing angle in this study, however, the role of the bearing angle on the bond strength in Eq. (7) and (10) becomes much clear.

Fig. 8 offers an explanation that when the shearing resistance is constant, the bearing angle decreases as confinement increases and; when the splitting resistance is constant the bearing angle increases, as the rib height increases. The analysis using decreasing bearing angle will help to illustrate the beneficial effects of high rib area bars on bond performance.

The role of bearing angle is examined and found to play the key role for the nature of the wedging action by the ribs of deformed bars to concrete. The mechanism of the flattened rib face angle becomes clear in this study. The bearing angle at the interface decreases so that splitting strength is maintained to be less than the shearing strength. Further research related to the decreasing bearing angle will help to understand several key behaviors of bar-concrete interaction.

5. Conclusions

Analytical expressions to determine the bond resistance for splitting and shearing failures are derived and finally used to predict bond strength. It is found that the bearing angle is the major variable in the expressions. Following is the summary of findings in this study.

1) As the concrete key is crushed or sheared off, the bearing angle is reduced decreasing splitting bond strength and increasing possible shearing bond strength;
2) The weaker of the two mechanisms controls bond strength. As the splitting strength is higher than the shearing strength, the concrete key is crushed or sheared off and the splitting strength decreases with decreasing the bearing angle;
3) The bearing angle at the interface decreases so that splitting strength is maintained to be less than the shearing strength as bars slip;
4) The bearing angle plays a key role in controlling bond mechanism and determination of the bond strength of reinforcing bars to concrete.

The behaviors of bearing angle in this analytical study agree well with the findings by the previous studies. This study will provide a rationale to develop high rib area bars possessing apparently better bond performance and help to understand the major aspects of ribbed reinforcing bar-concrete interaction.

Notation

A_c = area of cone with an angle of α
A_r = projected area of rib parallel to bar axis
c = cohesion
d_b = bar diameter
F_x = total bursting resistance on bonded length in the x direction
f_{coh} = cohesion
f_{ck} = compressive strength of concrete
f_R = relative rib area
h_r = rib height
T_{split} = splitting bond resistance
T_{shear} = shearing bond resistance
α = bearing angle
μ = coefficient of friction
σ_n = normal stress on interface
σ_q = sliding stress on bar rib acting parallel to bar axis
σ_r = radial stress on bar rib acting perpendicular to bar axis
σ_{sp} = confining stress
τ = shear stress on interface
τ_{bu} = bond strength
φ_b = angle of internal friction

Acknowledgement

This research was supported by Soongsil University Research Program. The financial support is gratefully acknowledged.

References

1. Rehm, G., 1957, "*The fundamental Law of Bond,*" RILEM-Symposium on Bond and Crack Formation in Reinforced Concrete, Vol.II, Stockholm, 491pp.
2. Lutz, L. A. and Gergely, P., 1967, "Mechanics of Bond and Slip of Deformed Bars in Concrete," *ACI Journal*, Proceedings, Vol.64, No.11, pp.711~721.
3. Tepfers, R., 1979, "Cracking of Concrete Cover Along Anchored Deformed Reinforcing Bars," *Magazine of Concrete Research*, Vol.31, No.106, pp.3~12.
4. Cairns, J. and Jones, K., 1996, "An Evaluation of the Bond-Splitting Action of Ribbed Bars," *ACI Material Journal*, Vol.93, No.1, pp.10~19.
5. Eligehausen, R., Popov, E. P., and Bertero, V. V., "*Local Bond Stress-slip Relationships of Deformed Bars under Generalized Excitations,*" Earthquake Engineering Research Center, Report No. UCB/EERC-83/23, University of California, Berkeley, 1983, 169pp.
6. Darwin, D. and Graham, E. K., "Effect of Deformation Height and Spacing on Bond Strength of Reinforcing Bars," *ACI Structural Journal*, Vol.90, No.6, 1993, pp.646~657.
7. Hamad, B. S., "Bond Strength Improvement of Reinforcing Bars with Specially Designed Rib Geometries," *ACI Structural Journal*, Vol.92, No.1, 1995, pp.3~13.
8. Tholen, M. L. and Darwin, D., "*Effects of Deformation Properties on the Bond of Reinforcing Bars,*" SM Report No. 42, University of Kansas, 1996, 370pp.
9. Cox, J. V. and Yu, H., "Radial Elastic Stiffness Associated with Bond between Steel Bars and Concrete," *ACI Structural Journal*, Vol. 98, No.1, 2001, pp.16~28.
10. Choi, O. C. and Lee, W. S., "Interfacial Bond Analysis of Deformed Bars to Concrete," *ACI Structural Journal*, Vol.99, No.6, 2002, pp.750~756.
11. Gambarova, P. G. and Rosati, G. P., "Bond and Splitting in Bar Pull-out: Behavioral Laws and Concrete Cover Role," *Magazine of Concrete Research*, 49, No.179, 1997, pp.99~110.
12. Birkeland, P. W. and Birkeland, H. W., "Connections in Precast Concrete Construction," *Journal of American Concrete Institute*, Vol.63, No.3, 1966, pp.345~368.