J. Korea Asso. Radiat. Prot.
Vol. 29 No. 4 : 269~273 (2004)

A Study on Counting Statistics of the Hybrid G-M Counter Dead Time Model Using Monte Carlo Simulations

Sang Hoon LEE · Moosung JAE
Innovative Technology Center for Radiation Safety, Dept. of Nuclear Engineering, Hanyang University

몬테칼로 전산모사를 이용한 복합 G-M 계수기 불감시간 모형의 계측 통계 연구

이상훈 · 제무성
한양대학교 원자력공학과 방사선안전신기술연구센터(iTRS)

(2004년 10월 20일 접수, 2004년 12월 9일 채택)

Abstract - The hybrid dead time model adopting paralyzable (or extendable) and non-paralyzable (or non-extendable) dead times has been introduced to extend the usable range of G-M counters in high counting rate environment and the relationship between true and observed counting rates is more accurately expressed in the hybrid model. GMSIM, dead time effects simulator, has been developed to analyze the counting statistics of G-M counters using Monte Carlo simulations. GMSIM accurately described the counting statistics of the paralyzable and non-paralyzable models. For G-M counters that follow the hybrid model, the counting statistics behaved in between two idealized models. In the future, GMSIM may be used in predicting counting statistics of three G-M dead time models, which are paralyzable, non-paralyzable and hybrid models.
Key words : G-M counter, dead time, Monte Carlo simulation, hybrid dead time model

요약 - 고계수율 환경에서의 G-M 계수기의 가용 범위를 확장하기 위하여 두 가지 불감시간(연장가능 및 연장불능)을 채택한 복합 모형이 개발되었으며, 이 복합모형은 참 계수율과 실측 계수율간의 상관관계를 보다 정확히 설명한다. 이 논문에서는 몬테칼로 모사법에 근거한 G-M 계수기 불감효과 분석 프로그램 GMSIM을 개발하여 연장가능 불감시간 모형 및 연장불능 불감시간 모형에 적용하여 그 정확도를 확인하였다. GMSIM을 이용하여 복합 불감시간 모형을 따르는 G-M 계수기의 계수 통계 특성을 분석한 결과, 두 가지 이상적 모형의 중간적 특성을 보였다. 향후 GMSIM은 세 가지 모형의 불감시간 특성을 분석하는데 사용될 수 있다.
중심어 : G-M 계수기, 불감시간, 몬테칼로 모사, 복합 불감 시간 모형

Introduction

The G-M counter is an excellent radiation detector since it is rugged, stable in operation, inexpensive and versatile in detecting radiations. It only lacks the capability of spectrometry and high counting rate measurement. The first disadvantage comes from the basic detection mechanism of the G-M counter which cannot be modified. It is possible that the useful counting rate range of G-M counter systems can be extended if the high counting rate behavior due to dead time effects can be accurately described. The hybrid dead time model has been developed and is widely used in many applications along with two idealized models [1, 2]. However, the counting statistics of the hybrid model is not fully established. In this paper, GMSIM, dead time effects simulator, has been developed to analyze the counting statistics of G-M counters using Monte Carlo simulations.

Dead Time Models

Relatively long dead time of the G-M counter sets major drawbacks on the use of the counter at very high radiation counting rates. The deviation between true and observed counting rates are usually explained in limited ways with the paralyzable or the non-paralyzable model [3].

In the non-paralyzable model, or Type I model as given in Feller (1948, [4]) and Evans (1955, [5]), the dead time is non-extended and radiation events occurring during the detector dead times are not counted. The important relationship between true and observed counting rates is given by:

$$m = \frac{n}{1 + n\tau_{NP}} \qquad (1)$$

where m is the observed counting rate in counts/s, n is the true counting rate in counts/s, and τ_{NP} is the non-paralyzable counter dead time in sec [6].

In the paralyzable model[7-10], or Type II model as given in Feller and Evans, when a radiation event occurs during the detector dead time, the event is not counted but the dead time is extended. The relationship between true and observed counting rates is given by:

$$m = n\exp(-n\tau_P) \qquad (2)$$

where τ_P is the paralyzable counter dead time in s.

To extend the useful range of the G-M detector a new hybrid G-M counter dead time model (Lee and Gardner, 2000, [2]) with two dead times has been introduced by the authors recently. The new combined hybrid model is

$$m = \frac{n\exp(-n\tau_P)}{1 + n\tau_{NP}} \qquad (3)$$

where τ_{NP} and τ_P represent the non-paralyzable and paralyzable dead times.

Monte Carlo Simulations of Dead Time Effects

To numerically simulate the dead time behavior of a G-M counter, the Monte Carlo G-M counter simulator, GMSIM, has been developed. GMSIM requires detector dead times, a true counting rate and a measurement time as input data and produces observed counts as output. Inside the GMSIM code an interval, Δt, between two radiation events are randomly sampled from the well-known interval distribution (p.d.f.), which is

$$f(\Delta t) = n\exp(-n\Delta t) \qquad \Delta t \ge 0 \qquad (4)$$

and its corresponding c.d.f., which is

$$F(\Delta t) = 1 - \exp(-n\Delta t) \qquad \Delta t \ge 0 \qquad (5)$$

Then the sampled interval is compared with the dead times, paralyzable and non-paralyzable, to determine if the radiation event is observed or not and also to determine if the dead time is newly extended or not. The validity of the code has been checked by calculating the observed counting rates for three G-M dead time models, a non-paralyzable model case with a dead time of 300 μsec, a paralyzable model case with a dead time of 300 μsec and a hybrid model case with 250 μsec non-paralyzable dead time and 50 μsec paralyzable dead time. The calculated counting rates were presented and compared

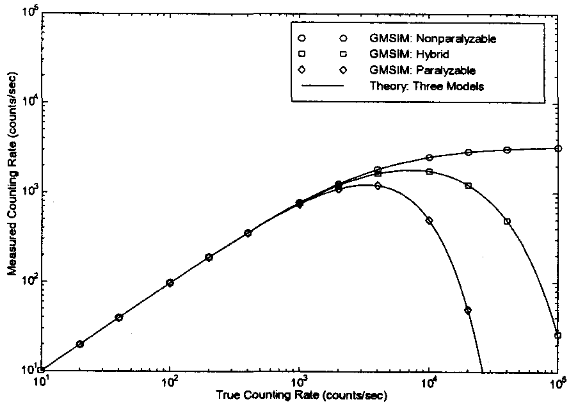

Fig. 1. Observed counting rates as a function of true counting rates for G-M counters with 300 μsec dead time

with the theories given in equations (1) through (3) in Figure 1. According to the comparison results shown in Figure 1 it is clear that GMSIM can accurately simulate three ideal model counters as far as dead time effects on counting rates are concerned.

For the radiation counting statistics, it is well known that, a radiation counter without any dead time follows Poisson statistics and the variance of observed counts ($M=mt$) for measurement time of t should be

$$\sigma^2(mt) = mt \qquad (6)$$

Due to the dead time effects, the actual counting statistics of G-M counters deviate from Poisson statistics. For non-paralyzable dead time model, the counting statistics differ from the Poison statistics and the variance is given by Mueller[6] as follows.

$$\sigma^2(mt) = nt/(1+n\tau)^3 + n^2\tau^2/(1+n\tau)^4 + 2n^3\tau^3/(3(1+n\tau)^4) + n^4\tau^4/(6(1+n\tau)^4) \qquad (7)$$

For paralyzable dead time model, the variance is given by Kosten[8] as follows.

$$\sigma^2(mt) = mt(1-2m\tau+m\tau^2/t) \qquad (8)$$

The theoretical formulae in the equations (7) and (8) have been verified by using Monte Carlo simulations (GMSIM) in which pulses were randomly chosen from pulse time distributions pertinent to the models. For the non-paralyzable and paralyzable models, the GMSIM calculation results were presented and compared with theories in Figures 2 and 3. Each data mark on the variance Figures represents a calculated variance for 10000 sample cases of 1 sec measurement. Three different random number sequences are used at each given counting rate.

It is noted that for non-paralyzable model the relative variance (the ratio of variance observed to the Poisson variance) reduces to zero for very high counting rate cases. In paralyzable model the variance reduces to near zero at intermediate counting rate cases and returns back to Poisson-type variance for very high counting rate cases. In both cases it is clear the variance is strongly related to the counting rates, so important detector information may be extracted from careful observations of variance deviations from Poisson statistics. Also note that the data results shown in the Figures would be unity for a Poisson type variance. The close matches in the Figures 2 and 3 mutually verify the theory and the simulation at the same time.

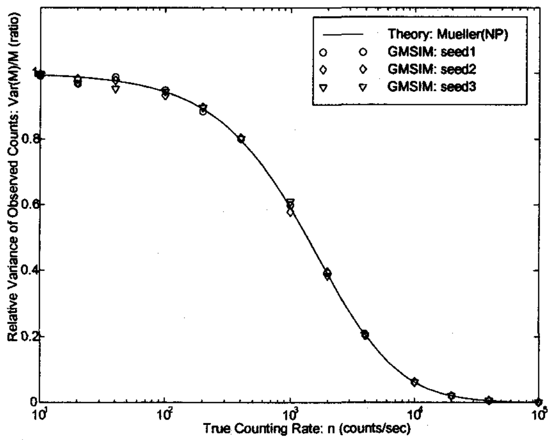

Fig. 2. Relative variance of non-paralyzable model (with 300 μsec non-paralyzable dead time)

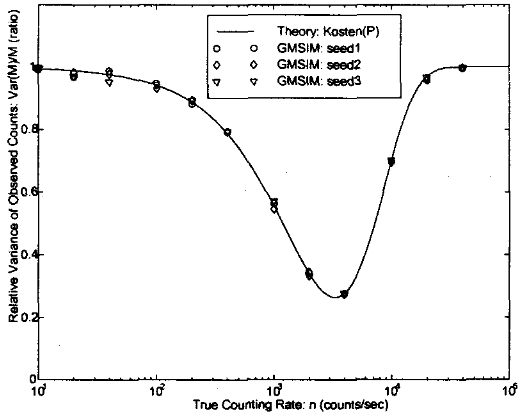

Fig. 3. Relative variance of paralyzable model (with 300 μsec paralyzable dead time)

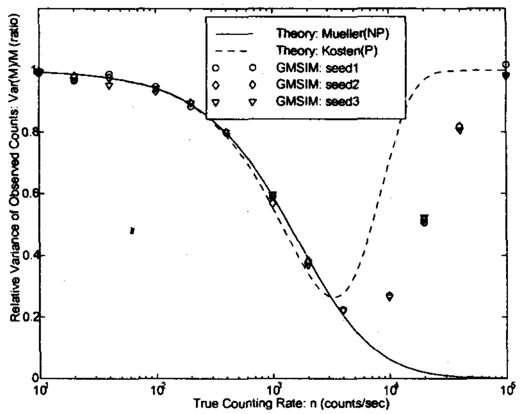

Fig. 4. Relative variance of hybrid model (with 250 μsec non-paralyzable dead time and 50 μsec paralyzable dead time)

Simulation results for a hybrid model case with 250 μsec non-paralyzable dead time & 50 μsec paralyzable dead time were presented with corresponding non-paralyzable and paralyzable model theories in Figure 4. The variance of the hybrid model behaves in between two idealized models. Up to now, no satisfactory variance model has been suggested for the hybrid model. But the results suggest that some combining approaches of non-paralyzable and paralyzable models may work for the variance of the hybrid model. For the hybrid model, the variance also depends on the counting rates significantly.

Conclusions and Discussions

To analyze the counting statistics of G-M counters with Monte Carlo simulations, GMSIM, dead time effects simulator, has been developed. The observed counting rates as a function of true counting rates, calculated through GMSIM, showed good agreements with already established theories for non-paralyzable, paralyzable and hybrid models. GMSIM was used in analyzing the variances of observed counts, which deviate from well-known Poisson statistic because of dead time effects. For non-paralyzable model, the relative variance (the ratio of variance observed to the Poisson variance) reduces to zero for very high counting rate cases. In paralyzable model the variance reduces to near zero at intermediate counting rates and returns back to Poisson-type variance for very high counting rate cases. GMSIM results and theoretical variance models by Kosten and Mueller match with each other very closely.

The variance of the hybrid model behaves in between two idealized models as shown in Figure 4. Since no satisfactory variance models are available for the hybrid model, GMSIM may be used in analyzing counting statistics of hybrid model along with two idealized models. Through these analysis, in the near future, it is hoped that variance theories be established for the hybrid dead time model and the theory reveal certain ways to take advantage of the information hidden in the counting statistics deviation from Poisson statistics.

Acknowledgement

This research has been financially supported by KOSEF through iTRS (Innovative Technology Center for Radiation Safety), Hanyang University.

References

1. Lee, S. H., Jae, M. and Gardner, R. P, *Journal of Nuclear Science and Technology*, supplement 4, pp. 156-159(2004)
2. Lee, S. H. and Gardner, R. P, A new G-M counter dead time model, *Appl. Radiat. Isot.*, vol. 53, pp. 731-737(2000)
3. Knoll, G, *Radiation Detection and Measurement*, John Wiley & Sons, New York(1989)
4. Feller, W, On probability problems in the theory of counters, in R. Courant Anniversary Volume, *Studies and Essays*, Interscience, New York(1948)
5. Evans, R. D, *The Atomic Nucleus*, McGraw-Hill, New York(1995)
6. Mueller, J. W, Dead-time problems, *Nuclear Instruments and Methods*, vol. 112, pp. 47-57(1973)
7. Levert, C. and Scheen, W. L, Probability fluctuation of discharges in a Geiger-Mueller counter produced by cosmic radiation, *Physica*, vol. 10, pp.225-238(1943)
8. Kosten, L, On the frequency distribution of the number of discharges counted by a Geiger-Mueller counter in a constant interval, *Physica*, vol. 10, pp. 749-756(1943)
9. Libert, J, *Nuclear Instruments and Methods*, vol. 121, pp. 589-590(1975)
10. Libert, J, Statistique de comptage avec distribution uniforme de temps mort, *Nuclear Instruments and Methods*, vol. 151, pp. 555-561(1978)