International Journal of CAD/CAM Vol. 3, No. 1, pp. 31~40 (2003)

INTERNATIONAL JOURNAL OF CAD/CAM
www.ijcc.org

Adaptive Slicing with Curvature Considerations

Ashis Gopal Banerjee, Aloke Kumar, Shankar Tejavath and Asimava Roy Choudhury*
Department of Mechanical Engineering, Indian Institute of Technology, Kharagpur, India

Abstract – In this paper, first order slice height calculation in Laminated Object Manufacturing (LOM) of free form surfaces is done with two different considerations: that a) the cutter trajectory is oriented in the direction of local absolute maximum curvature of the surface or b) in the direction of local maximum flatness of the surface. For the former, the slices would be more in number when compared to the case where the cutter trajectory is contained in the normal vertical section (NVS). However, it would help in achieving higher form accuracy of the final part because it would be a form of worst-case check. For the second proposed strategy, least number of slices results, thereby reducing overall build time drastically.

Keywords: **Rapid Prototyping, Slicing, Laminated Object Manufacturing, Adaptive Slicing, Direct Slicing**

1. Introduction

The term *rapid prototyping* (RP) refers to a class of technology that can automatically construct physical models from CAD model data. With the help of appropriate software tools, the CAD model can be directly sliced into a number of thin layers, which are then physically built up one on top of the other. The physical realization of a part through rapid prototyping can be done in several ways, Laminated Object Manufacturing (LOM) being one of them. One way of carrying out LOM is by cutting out the slices from sheet material by a laser / waterjet cutter and attaching them together.

The slices, which are cut out of sheet material, may have vertical or sloping sidewalls. Accordingly, they are referred to as slices with zero order and first order approximation (Fig. 1) respectively. If the sidewalls are vertical, the surface of the built-up part has a 'staircase effect'. In first order approximation, staircase effect is not present as positional continuity exists between corresponding points on successive layers. In first-order, a series of four-sided sloping ruled surface patches are created between successive layer contours (Fig. 2). A ruled surface can be produced by using machines / manipulators that have ideally 5 axes of freedom (x, y, z and rotation about x and y axes of motion).

Further, slices may all be of the same thickness (uniform slicing) or of different thickness values (adaptive slicing). Adaptive slicing is considered to be an improvement over uniform slicing as the number of layers can be drastically reduced for the same degree of accuracy.

1.1. Related work

In conventional practice, slicing of CAD models is carried out after tessellation or triangulation of the CAD model. However, tessellation has some typical difficulties [15,10,11] due to which direct slicing of CAD models is being investigated and gradually finding acceptance. In direct slicing, the intersection profile between the model and a plane is calculated directly without the involvement of tessellation.

Kulkarni and Dutta [12] have carried out research work on the aspect of direct adaptive slicing with zero order approximation. A discrete number of points were selected around a layer contour and slice height was calculated with zero order approximation at every such

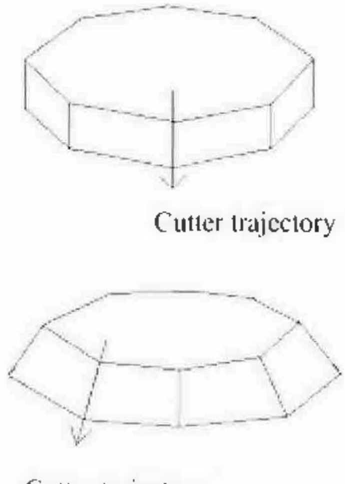


Fig. 1. Zero order (top) and first order (bottom) slices.

*Corresponding author:
Tel: +91-(0)3222-282970
Fax: +91-(0)3222-282278
Homepage: http://www.iitkgp.ernet.in/acads/dept/ME/faculty.php
E-mail: archie@mech.iitkgp.ernet.in

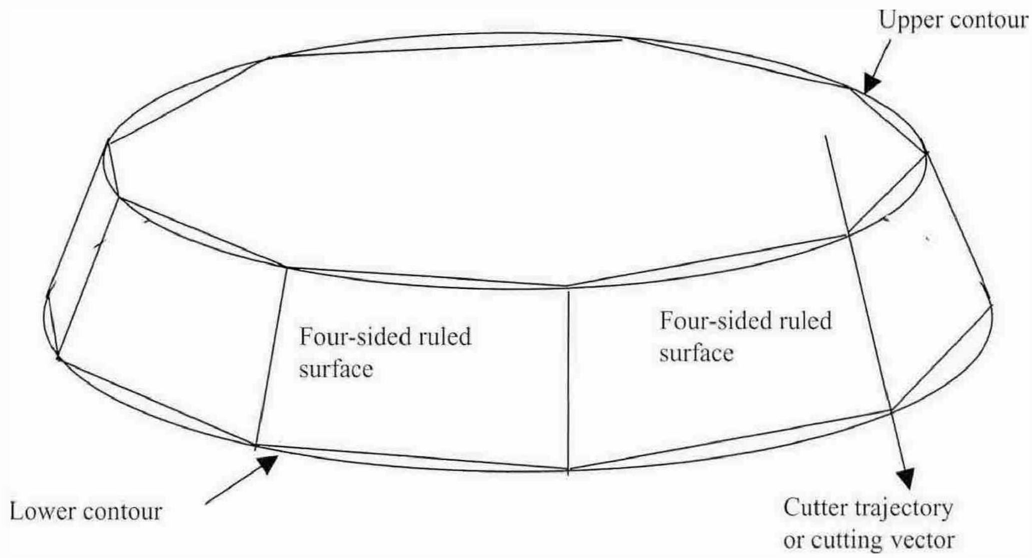


Fig. 2. Sidewalls formed by intersection of four-sided ruled surface patches.

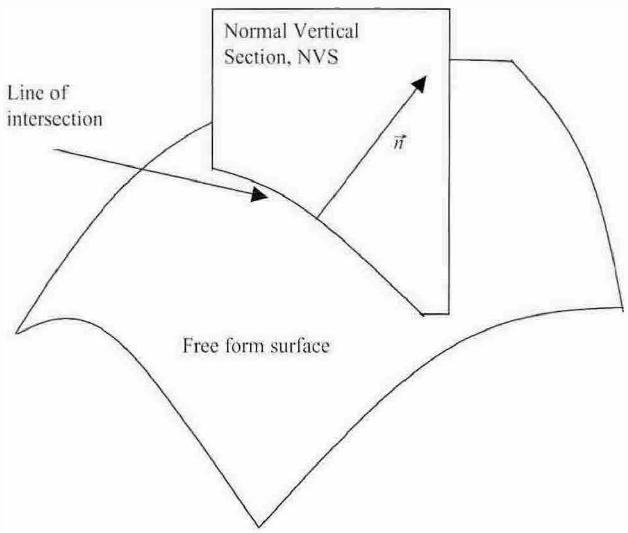


Fig. 3. The location of the normal vertical section or NVS.

point keeping the deviation between the surface and the straight slice walls within a user defined error value (cusp height δ). The thickness of a slice was chosen as the minimum of such heights calculated at these points.

Hope *et al.* [6, 7, 8] and Jager *et al.* [2, 3, 4] have conducted research on the aspect of direct adaptive slicing with first order approximation. The former research group has calculated slice height by employing two different approaches. In the first approach, the cutter trajectory has been considered to be contained in the normal vertical section (NVS) (Figs. 2 and 3). The normal vertical section (NVS) is basically a vertical plane containing the normal $\vec{n}$ to the CAD model at a particular point on the model. The NVS is orthogonal to both the slicing plane as well as the surface at the point in question and thus serves as an appropriate section / plane for error estimation. In the second approach, also suggested by Jager [3, 4], the cutter trajectory is decided by the matching of parametric values between successive layer contours. An additional approach employed by Jager *et al.* [2] is to calculate the 'twist value' in the geometric matching of contour points in adjacent layers. Furthermore, they introduce flexibility for the user in the choice of number of points on a horizontal contour.

As regards the quantification of surface roughness, Hope *et al.* [7] and Novac *et al.* [17] use an additional measure of error - the maximum distance in the layer-plane between the boundaries of the ideal and the built-up part, denoted as ε. Hope *et al.* discuss the significance of surface-finish requirement and volume difference between the ideal and built parts. In their work on 'TruSurf' RP system, which uses sloping layer surfaces cut by a water-jet cutter, they consider both the cusp-height and the volume-difference error ε.

Another procedure of cusp height determination is to take the maximum deviation between a surface patch and the CAD model as it combines both cusp height error (δ) and the lateral error (ε). Some researchers [14] have attempted this by approximating the outer wall between two successive contours by a series of taut cubic spline patches. However, here also the cutter trajectory is considered to be contained in the NVS.

Some strategies for the determination of cutter trajectory and slice height have been suggested by Im and Walczyk [9], who have put forward two different Profiled Edge Lamination (PEL) cutting trajectory algorithms. One of them, known as Adaptively Vectored Profiles Projection (AVPP) projects the data points from each profile to the opposite side using an adaptive tool vector that does not necessarily lie in NVS. This cutting orientation vector has a maximum inclination of ±30° (lead and lag in the angle of attack) with the normal drawn to the upper contour polyline to eliminate vector-overlapping problems.

1.2. Objectives of the present work

The present study focuses on determination of slice thickness values with cutter trajectory oriented along directions of maximum curvature and maximum flatness. In this respect, it makes a deviation from NVS-based calculations where the cutter trajectory is contained in the NVS.

The main objective of such an endeavor is to either attain maximum accuracy or maximum slice thickness values. In order to improve accuracy, it is proposed in this paper to align the cuter trajectory in the direction of the maximum curvature of the surface at the contour points. In such a case, it will be shown that the difference in volume between the CAD model and the built up part would be the least (among the three methods discussed here). The contribution is thus an increase in accuracy from the point of view of volume difference with the CAD model.

If, on the other hand, the number of slices has to be reduced, it is suggested to orient the cutter trajectory in the directions of the minimum curvature of the surface at the contour points. The contribution, which has been envisaged, is that in the case of minimum curvature, faster building up of the part would be possible as the number of slices is reduced. In addition, accuracy of the part would not be compromised as necessary calculations for cusp height would be formulated and carried out to determine the respective slice heights.

However, it should be mentioned here that for the second part, the maximum slant of the cutter trajectory with vertical was restricted to $\pm 30^{\circ}$ as a number of research papers [2, 9] recommend the same.

2. Theory and Implementation

Many research groups, as mentioned in the previous section, consider the cutter trajectory to be contained in the NVS of the CAD model at the point in question. In this respect, Hope *et al.* [6] suggest that instead, the cutter trajectory could be along the direction of minimum curvature (i.e., maximum flatness) so that least amount of undercut (error) would take place. In the present paper, this is one of the proposed strategies that have been investigated. The other proposed strategy is exactly on the contrary: i.e., the cutter trajectory is taken along the direction of maximum (absolute) curvature of the surface at a point. If the cutter cuts in the direction of maximum curvature, maximum undercutting would occur, which would yield the least value of layer thickness.

This obviously is not what is desired. Research effort has always been aimed at reduction in the number of slices. In spite of this fact, the proposed method has its merits. In this method, the cutter trajectory is oriented along the maximum curved direction and deviation between CAD model and actual part is measured in the section defined by the maximum curved direction and normal to CAD model. It is in this section that the deviation between the CAD model and plane is the maximum. Hence, this method identifies the worst case (i.e., maximum deviation) at every point and uses the data thereof for determination of slice height. In that way it is a kind of worst-case check. In such a case, it can be said with surety that nowhere in that slice is the error more than that in the maximum curvature section, which is being checked.

The question here is of error sampling. If cutter trajectory is taken only in the NVS and error is checked in those very sections, it remains undetermined whether the error is higher than permissible error in some other section or not. If, on the other hand, the cutter trajectory were taken only along the direction of maximum curvature and measurement of error carried out in those sections, the worst conditions would be checked.

Further, the case of the cutter trajectory being in the direction of maximum flatness has also been considered. It is expected to yield highest values of slice thickness as per above discussion. In this respect, it is relevant to discuss the relation between cutter trajectory orientation and surface curvature.

It has been observed that lasers and water jet cutters can produce much higher curvature of the slices perpendicular to the ray or jet than along the ray. Hence, following this principle, the cutter trajectory should be oriented in a direction normal to that of the maximum curvature of the surface. In case of smooth surfaces, the directions of the maximum convex and maximum concave curvature are mutually orthogonal [16]. They are also known as the directions of principal normal curvature. If, at a point on a surface, one of the principal normal curvature values be zero (i.e., flat), the surface has curvature only in one direction (like cylindrical surface). In that case, the principle outlined above and the one proposed here as 'minimum curvature' yield the same results. So, for this class of surfaces, there is a clear advantage of employing the minimum curvature technique. If however, the surface has equal values of maximum convex and concave curvatures at one point (a saddle point), the method outlined above and the method of minimum curvature would yield different cutter trajectory orientations. The former would orient the cutter in one of the principal directions while the latter would orient the cutter hafway in-between the directions of principle normal curvature. Both the strategies have their relative merits and demerits. The former does orient the cutter normal to one of the maximum curvature (principle normal curvature) directions but it would result in the cutter being oriented along the other principle normal direction, which is equally important. On the contrary, minimum curvature would make a compromise and strike a fair bargain by being halfway between the two principal directions.

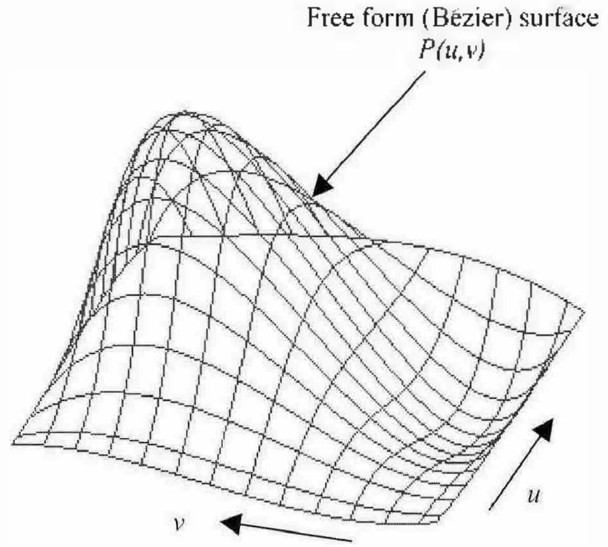


Fig. 4. An example of a free form surface.

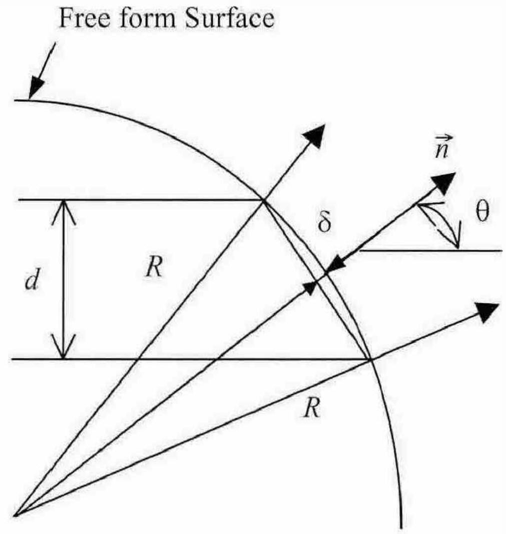


Fig. 5. First order approximation.

2.1. Free form surface

Expression of the Bezier surface is given by (Fig. 4)

$$P(u, v) = \sum_{i=0}^{n} \sum_{j=0}^{m} B_{i,j} J_{q,i}(u) K_{m,j}(v) \tag{1}$$

where $0 \le u \le 1$, $0 \le v \le 1$ and $B_{i,j}$ are the control points $J_{q,i}(u)$ and $K_{m,j}(v)$ are the Bernstein basis functions in the u and v parametric directions and are given by the following equations

$$J_{q,i} = \begin{pmatrix} q \\ i \end{pmatrix} \cdot u^i \cdot (1-u)^{q-i} \tag{2}$$

$$K_{m,j} = \begin{pmatrix} m \\ j \end{pmatrix} \cdot v^j \cdot (1-v)^{m-j} \tag{3}$$

$$\begin{pmatrix} q \\ i \end{pmatrix} = \frac{q!}{i!(q-i)!} \tag{4}$$

$$\begin{pmatrix} m \\ j \end{pmatrix} = \frac{m!}{j!(m-j)!} \tag{5}$$

2.2. Surface curvature and slice thickness

The process of slice thickness determination in case of first order approximation would involve finding of critical points and application of CPI (Curve-plane intersection) and SPI (Surface plane intersection) algorithms. These are quite standard practices and will not be discussed in detail. The reader is referred to Choi [1] where these items are discussed at length.

2.1.1. Slice thickness: first order

As per formulation by Faux and Pratt [5], the expression for slice height in NVS can be obtained as

$$d = 2\cos\theta \cdot \sqrt{2R\delta - \delta^2} \tag{6}$$

in which R is radius of curvature in the vertical section (approximating the CAD-model locally as part of a sphere), δ is the user-defined error (cusp height) & θ $[-\pi, \pi]$ is the angle between unit surface normal ($\vec{n}$) and the horizontal plane at half the height of the slice (Fig. 5).

In the present case, the *projection* of the surface normal (at mid-height) on the NVS has been considered for the calculation of this slice height. For this purpose, initially at a contour point, a guess value of the slice height is assumed. Then, intersection points between the CAD Model and the NVS are found out by SPI till half the guess height is attained (Fig. 5). At this point (mid-height), normal to the surface is determined and its projection on the NVS (through the contour point) is calculated. With this projection, a new slice height value is calculated for a user-defined error.

This height now replaces the previous guess value and calculations are repeated to yield another value of slice height. These iterations are carried out till d converges. The final selected slice height for the layer is the minimum of all converged d values around the profile. Once the slice height is determined, intersection between NVS at all the contour points and CAD model is carried out till this height is attained. The points of intersection at this height are the contour points for the next layer. Application of CPI or SPI algorithms is not required.

2.3. Maximum curvature and maximum flatness directions:

Normal curvature can be expressed as [12]:

$$k_n = \frac{L + 2M \cdot h + N \cdot h^2}{E + 2F \cdot h + G \cdot h^2} \tag{7}$$

where

$$L = P^{uu} \cdot \vec{n} \quad E = P^{u} \cdot P^{u}$$

$$M = P^{uv} \cdot \vec{n} \quad F = P^{u} \cdot P^{v} \tag{8a-f}$$

$$N = P^{vv} \cdot \vec{n} \quad G = P^{v} \cdot P^{v}$$

Here, P^i and P^{ij} represent the single and double derivatives of P w.r.t. i and j parameters (which can be u or v) respectively. Also, h $(=dv/du)$ represents the direction along which the curvature has been considered, and $\vec{n}$ is the unit surface normal at the point under consideration. For slice height calculation along a particular section, this value of h has to be determined. In case of NVS, application of SPI between NVS and CAD model would yield the values of dv and du and hence h $(=dv/du)$.

Now, the values of h for the extreme values of normal curvature (namely, principal normal curvatures) can be solved from the following equation [16]:

$$(FN-GM) \cdot h^2 + (EM-GL) \cdot h + EM \cdot FL = 0 \tag{9}$$

The value of h, which corresponds to the larger absolute value of the two, gives the direction of the most curved section. If the cutter trajectory is oriented in this direction, maximum undercutting (error) would occur and hence, thinnest layers would result.

The direction of minimum curvature is obtained by setting the numerator of the curvature expression to zero. If the cutter trajectory is oriented along the direction of minimum curvature (maximum flatness), undercutting will be minimum. However, this does not necessarily mean that the actual slice thickness will be highest as it will be the vertical component of the thickness in the minimum curvature section, which has to be eventually taken into account. This means that the layer thickness in the minimum curvature method is not necessarily thickest/highest. Naturally the question arises, what then is the advantage obtained from this type of orientation of cutter trajectory if layer thickness does not increase as a consequence. It might be stated this way that while the undercutting will be the least, the real advantage would be observed when the direction of minimum curvature bears a very small angle to the vertical.

After obtaining the two principal normal curvature values, the one with maximum absolute (i.e., irrespective of concavity or convexity) value is identified. This curvature will be referred to as 'maximum curvature' in this paper. In case of the proposed method, the slice height needs to be determined in a plane defined by the normal to the surface and the direction of maximum curvature at that point and is called NMCS (Normal Maximum Curvature Section). Fig. 6 shows the NMCS where $\vec{M_c}$ is a unit vector in the direction of maximum

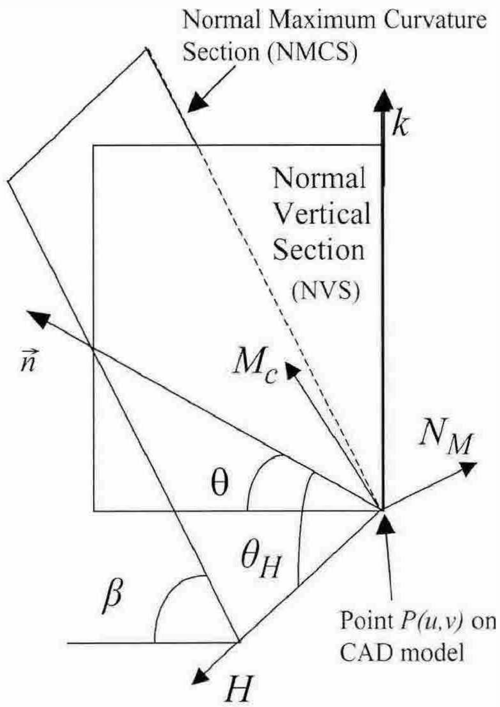


Fig. 6. Normal maximum curvature section.

curvature, θ_H is the angle made by the surface normal with the horizontal in NMCS, $\vec{H}$ is the intersection of NMCS with the horizontal plane through the point $P(u,v)$ on CAD model, $\vec{N_M}$ is the normal to NMCS and β is the angle between the horizontal plane and NMCS, i.e., between $\vec{k}$ and $\vec{N_M}$ [13].

Once $h = dv/du \equiv \Delta v/\Delta u$ for maximum curvature is determined, $\vec{M_c}$ can be found out :

$$\vec{M_c} = P(u+\Delta u, v+\Delta v) - P(u,v) \tag{10}$$

for small values of Δu & Δv.

From the cross product of $\vec{M_c}$ and $\vec{n}$, $\vec{N_M}$ is determined. $\vec{H}$ is obtained from the cross product of $\vec{N_M}$ and $\vec{k}$. θ_H can be calculated from the dot product of $\vec{n}$ and $\vec{H}$. β is obtained from the dot product of $\vec{k}$ and $\vec{N_M}$.

$$\vec{N_M} = \vec{M_c} \times \vec{n} \tag{11}$$

$$\vec{H} = \vec{N_M} \times \vec{k} \tag{12}$$

$$\theta_H = \cos^{-1}\left(\frac{\vec{n} \cdot \vec{H}}{|\vec{H}|}\right), \ \beta = \cos^{-1}\left(\frac{\vec{k} \cdot \vec{N_M}}{|\vec{N_M}|}\right) \tag{13}$$

For calculation of the slice thickness in the NMCS, once again, the projection of the surface normal (at mid-height) on the NMCS has been considered (Fig. 6) and the slice thickness is :

$$d = 2 \cdot \sin\beta \cos\theta_H \cdot \sqrt{(2R\delta - \delta^2)} \tag{14}$$

The same form of expression applies in case of slice thickness in the normal maximum flatness direction. The procedural steps for finding the slice heights in the case of the maximum normal curvature section (NMCS) or Normal maximum flatness section (NMFS) is very much the same as in case of NVS and is not discussed again.

3. Results and Discussions

A CAD model of a free form surface has been prepared and is shown in Fig. 7. Slicing has been performed with first order approximation and with error considerations in the following sections / planes.

1. Normal Vertical section (NVS)
2. Normal maximum curvature section (NMCS)
3. Normal maximum flatness section (NMFS)

Figs. 8-10 shows the slices obtained by applying the above strategies of slicing to the same CAD model (referred to as the first CAD model / first surface) with equal values of cusp height (=0.50 mm). The results of slicing have been tabulated in Table. 1. It is interesting to note that the number of slices in the case of NMCS is the maximum while that in the case of NMFS is the minimum.

As expected, error calculations for the case of cutter trajectory in NMCS yield the maximum number of slices, namely 26. Similarly, error calculations for the cutter trajectory in NMFS give rise to least number of slices, namely 11. The NVS provides a result that lies

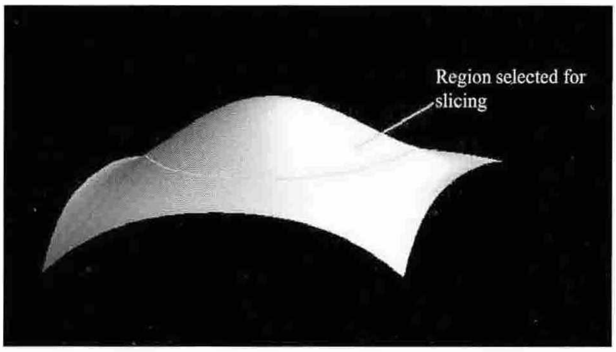


Fig. 7. The free form surface showing region selected (first CAD-model / Surface) for slicing.

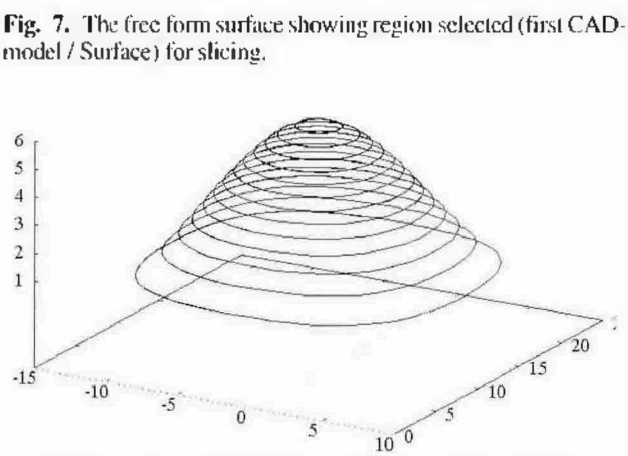


Fig. 8. Slices for cutter trajectory in Normal Vertical Section (NVS) for first surface.

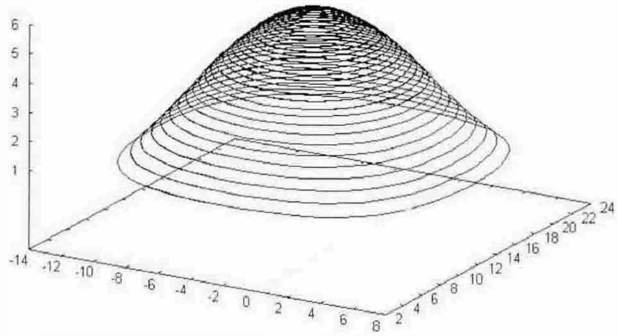


Fig. 9. Slices for Cutter trajectory in Normal Maximum Curvature Section (NMCS) for first surface.

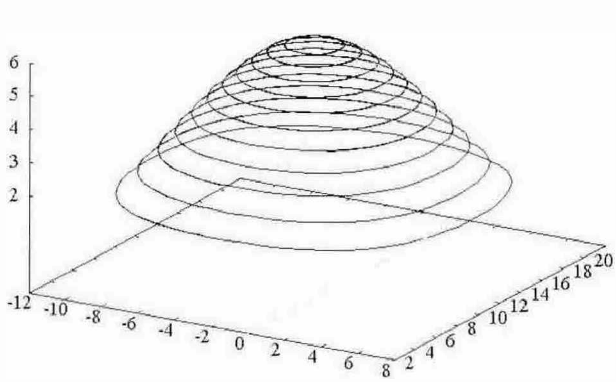


Fig. 10. Slices for Cutter trajectory in Normal Maximum Flatness section (NMFS) for first surface.

Table 1. Total slice numbers employing different strategies for surfaces considered

Surface ↓	Number of Slices as per following strategy		
	NVS	NMCS	NMFS
First Surface	13	26	11
Second Surface	27	51	19

predictably in-between the two with 13 slices.

Figs.11-14 show the slices obtained by applying the above three strategies of slicing to a second CAD model, again with the equal values of cusp height (=0.50 mm). The results of slicing have been tabulated in Table 1.

Here it should be mentioned again (as previously in subsection 1.2 Objectives of the present work) that the maximum slant of cutter trajectory with vertical is restricted to $\pm 30^{0}$ as per recommended practice [2,9].

Here too, error calculations for the case of cutter trajectory in NMCS yield the maximum number of slices, namely 51. Similarly, error calculations for the cutter trajectory in NMFS give rise to least number of slices, namely 19. The NVS provides a result that lies in-between the two with 27 slices.

The directions of the cutting vectors for the different strategies have also been shown in Fig. 14 (in a single figure, to save space) at different points on the surface. Dashed arrows mean those cutter trajectory orientations

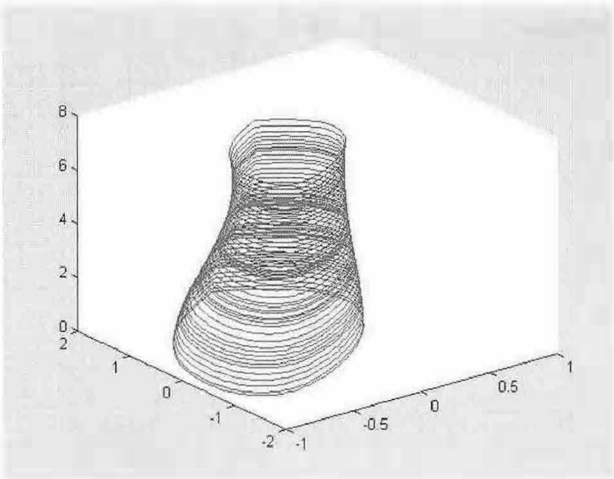


Fig. 11. Slices for Cutter trajectory in Normal Maximum Curvature Section (NMCS) for second surface.

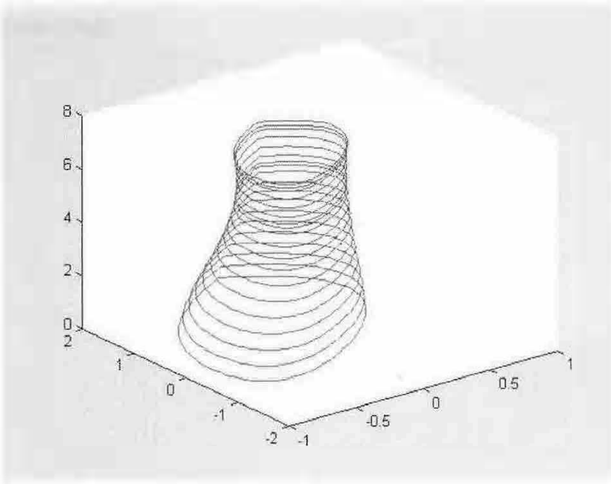


Fig. 13. Slices for Cutter trajectory in Normal Maximum Flatness section (NMFS) for second surface.

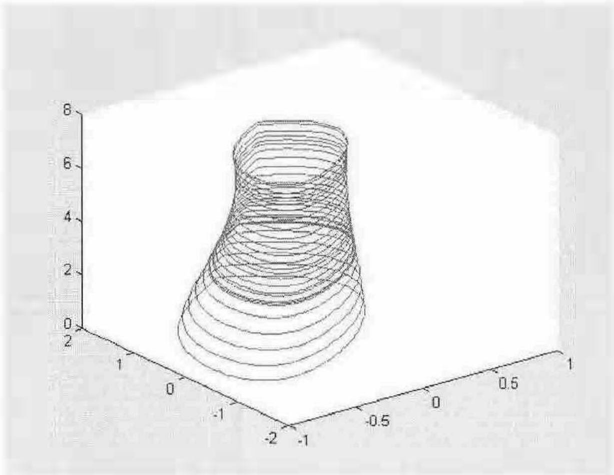


Fig. 12. Slices for cutter trajectory in Normal Vertical Section (NVS) for second surface.

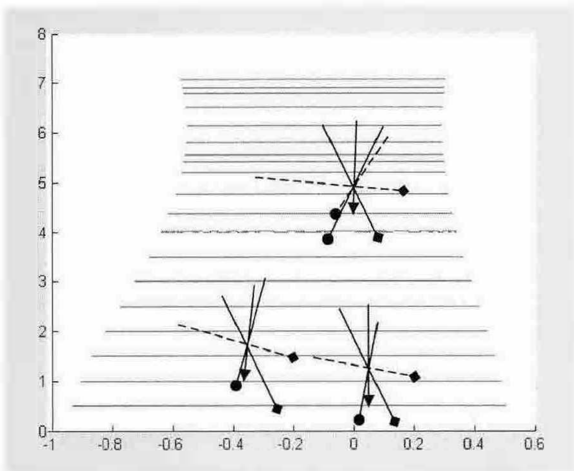


Fig. 14. Slices for NMFS for second surface (side view) with orientations of cutting vectors in NVS (solid arrows), in NMFS (solid or dashed oval arrows) and NMCS (solid or dashed diamond arrows).

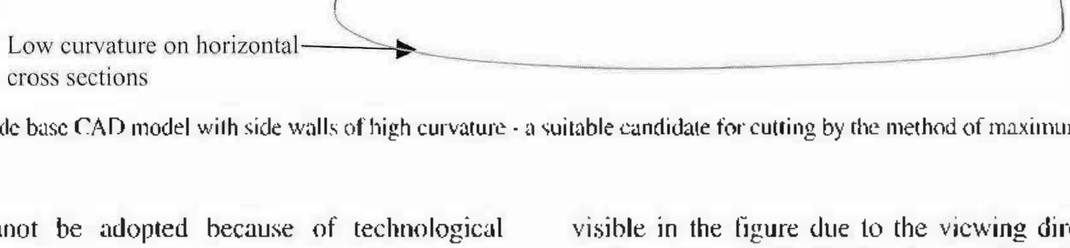


Fig. 15. Wide base CAD model with side walls of high curvature - a suitable candidate for cutting by the method of maximum curvature.

which cannot be adopted because of technological constraints.

The cutting vectors in the NVS (solid arrows) in the projection drawing (side view) appear to be more or less vertical but they do have inclination to the vertical due to the slope of the surface. This is not properly visible in the figure due to the viewing direction. The cutting vectors in NMFS (oval arrows) are having small angle with the vertical but this angle varies with the position. At the uppermost of the three points under consideration in Fig. 14, this angle is quite high, around 35^0 (dashed oval arrow). This is because the uppermost of the three points is a saddle point, where the surface has both concave and convex curvatures with the maximum flatness direction lying in-between. In case of this point, the cutting vector for NMFS is restricted to 30° and is shown with a solid oval arrow.

It is interesting to note that the lower slices of NVS and NMFS are of same thickness. This is because the orientation of cutter trajectory for both methods is almost identical for the lower regions of the part (Fig. 14). However, the slant of the cutter trajectory in the NMCS being too high (>>30°, dashed diamond arrows), the cutter trajectory is oriented instead at ±30° to the vertical (Fig. 14, solid diamond arrows).

The cutting vectors in NMCS at all three points shown in Fig. 14, are inclined at low angles to the horizontal (dashed diamond arrows), which apparently implies a poor applicability of this strategy. This, however, would predominantly occur for bodies with high curvature in the horizontal plane. For bodies wider in the horizontal section with side walls of high curvature, this problem would be less. An example of such a surface is shown in Fig. 15.

4. Volume Comparison

Error estimation in layered Object Manufacturing is mainly carried out in two dimensions, for example : cusp height in NVS (refer section 1.1 Related work). In this respect, a comparison between the respective volumes of the CAD model and the built-up bodies would be more accurate in revealing the respective errors incurred by following different slicing strategies.

The total volume (V) enclosed by the CAD model (2nd Bezier surface) is given by [18]

$$V=\int_0^1\int_0^1 F(u,v)|I|du\,dv \tag{15}$$

where, $I = \begin{vmatrix} \frac{\partial\phi}{\partial u} & \frac{\partial\phi}{\partial v} \\ \frac{\partial\varphi}{\partial u} & \frac{\partial\varphi}{\partial v} \end{vmatrix}$ is the Jacobian and $F(u, v) = Z$,

$\phi \equiv \phi(u,v)=X$

$\varphi \equiv \varphi(u,v)=Y$ (X, Y, Z are the Cartesian co-ordinates of a point)

Now to calculate the volume occupied by the built-up part, the volume of each slice building up the part is evaluated individually and added up. One slice means the portion bounded by two horizontal surfaces on the top and bottom and laterally by four sided ruled patches.

In order to determine the volume of such a body, integration may be performed. However, in that case, the relation between the area of an intermediate section and the height of that section should be known. Since, area of any polygon can be considered to be made of a number of triangles, this relation has been first established for a triangle and then shown to be valid for polygons. The area of the intermediate triangle MNO (Fig. 16) can be expressed vectorially as:

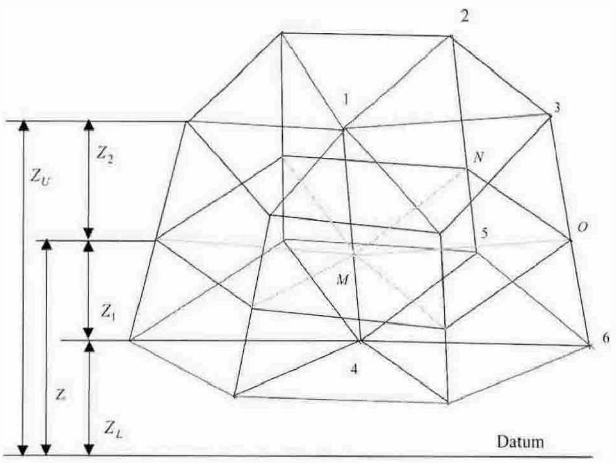


Fig. 16. Division of sliced planes into triangles for volume calculation.

$$A=\frac{1}{2}\left|(\vec{P}_m-\vec{P}_n)\times(\vec{P}_n-\vec{P}_o)\right| \tag{16}$$
$$=\frac{1}{2}\left|\vec{P}_m\times\vec{P}_n-\vec{P}_m\times\vec{P}_o+\vec{P}_n\times\vec{P}_o\right|$$

Where P_i refers to the position vector of a point i.

$$\therefore A=\frac{1}{2}\left|\vec{P}_m\times\vec{P}_n+\vec{P}_n\times\vec{P}_o+\vec{P}_o\times\vec{P}_m\right|$$

$$=\frac{1}{2h^2}\left|\begin{matrix}(\vec{P}_4Z_2+\vec{P}_1Z_1)\times(\vec{P}_6Z_2+\vec{P}_2Z_1)+(\vec{P}_6Z_2+\vec{P}_2Z_1)\\ \times(\vec{P}_5Z_2+\vec{P}_3Z_1)+(\vec{P}_5Z_2+\vec{P}_3Z_1)\times(\vec{P}_4Z_2+\vec{P}_1Z_1)\end{matrix}\right|$$

$$=\frac{1}{2h^2}\left|\begin{matrix}(\vec{P}_4\times\vec{P}_6)Z_2^2+(\vec{P}_1\times\vec{P}_2)Z_1^2+(\vec{P}_4\times\vec{P}_2+\vec{P}_1\times\vec{P}_6)Z_2Z_1\\ +(\vec{P}_6\times\vec{P}_5)Z_2^2+(\vec{P}_2\times\vec{P}_3)Z_1^2+(\vec{P}_6\times\vec{P}_3+\vec{P}_2\times\vec{P}_5)Z_2Z_1\\ +(\vec{P}_5\times\vec{P}_4)Z_1^2+(\vec{P}_3\times\vec{P}_1)Z_2^2+(\vec{P}_5\times\vec{P}_1+\vec{P}_3\times\vec{P}_4)Z_2Z_1\end{matrix}\right|$$

$$=\left|\begin{matrix}(\vec{P}_4\times\vec{P}_6+\vec{P}_6\times\vec{P}_5+\vec{P}_5\times\vec{P}_4)Z_2^2+(\vec{P}_1\times\vec{P}_2+\vec{P}_2\times\vec{P}_3+\vec{P}_3\times\vec{P}_1)Z_1^2\\ +(\vec{P}_4\times\vec{P}_2+\vec{P}_1\times\vec{P}_6+\vec{P}_6\times\vec{P}_3+\vec{P}_2\times\vec{P}_5+\vec{P}_5\times\vec{P}_1+\vec{P}_3\times\vec{P}_4)Z_1Z_2\end{matrix}\right| \tag{17}$$

But here, $Z_1=Z-Z_L$ and $Z_2=Z_U-Z$

Hence, $A=K_1(Z_U-Z)^2+K_2(Z-Z_L)^2+K_3(Z-Z_L)(Z_U-Z)$ (18)

where $K_1=\vec{P}_4\times\vec{P}_6+\vec{P}_6\times\vec{P}_5+\vec{P}_5\times\vec{P}_4$,

$K_2=\vec{P}_1\times\vec{P}_2+\vec{P}_2\times\vec{P}_3+\vec{P}_3\times\vec{P}_1$,

Table 2. Volume comparison between CAD Model and built-up parts

Sl. No.	CAD Model Volume (mm^3)	Strategy	Total Volume (mm^3)	Volume Difference (abs in mm^3)	Volume Difference, %
1		NVS	19.2	0.7	3.8
2	18.5	NMCS	17.9	0.6	3.3
3		NMFS	17.5	1.0	5.4

and

$$K_3 = \vec{P_4}\times\vec{P_2}+\vec{P_1}\times\vec{P_6}+\vec{P_6}\times\vec{P_3}+\vec{P_2}\times\vec{P_5}+\vec{P_5}\times\vec{P_1}+\vec{P_3}\times\vec{P_4}$$

$$\therefore A = C_1 + C_2 Z + C_3 Z^2 \text{ (a polynomial function of } Z\text{)} \quad (19)$$

where C_1, C_2, C_3 are three constants obtained from eqn. (18) by expanding the terms (since are Z_U, Z_L are also constants).

Hence, the total area of the arbitrarily chosen horizontal plane (obtained by summing up the individual areas) is also a polynomial function in Z and, thus, the required volume (V) is obtained as:

$$\therefore V = \int_{Z_L}^{Z_U} A_z dz \text{ where } A_Z \text{ is the overall area of the horizontal plane.} \quad (20)$$

The results obtained after carrying out all the above-mentioned operations are tabulated below in Table 2 :

The calculation of volume of the CAD Model and the built up parts (by the three methods mentioned) reveals that:

a) The difference in the volume of the CAD model and the part built up by the method of maximum curvature is the least. Hence, orienting the cutter trajectory in the direction of maximum curvature of the surface would result in the closest approximation of the total volume of the Model. In addition to this, the cusp height calculations would yield exact replication of the form of the CAD model. Thus, the maximum curvature method yields closest approximation of volume of the model for the same degree of accuracy among the three methods discussed here.

b) As expected, the difference in volume between CAD model and built-up part is maximum for the method of maximum flatness, minimum for the method of maximum curvature and intermediate when the cutting vectors lie in NVS. However, an interesting point to be noted here is that the total volume of the built-up part exceeds that of the CAD model when cutting vectors are contained in NVS. This is simply because a major portion of the second CAD model has concave curvature in the vertical direction (Fig. 14) and thus cutting vectors contained in NVS leave huge overcuts. This results in the excess volume.

5. Conclusions

NVS-based calculation of slice height has the advantage of having the calculations done in a plane normal to the CAD model as well as normal to the slicing plane. However, it does not necessarily minimize any parameter like number of slices etc.

Slice thickness should be calculated by maximum curvature principle if most accurate prototypes are required. The reason, as already explained in section 2, is that the deviation between CAD Model and built up part is maximum in the NMCS. Hence, if the NMCS is considered for cusp height (error) calculation and subsequent slice height determination, the worst cases of deviation will have been checked. In the other methods (NVS and NMFS), the deviation which is considered for slice height calculation is not necessarily the maximum between the CAD Model and the built-up part. Thus, maximum deviation, in that way, is unchecked by these methods and this leads to ultimate loss in accuracy of the built up part. This observation is supported by the results of the volume calculations and subsequent discussions presented in section 4.

If fast production is required and if the maximum flatness direction is close to the vertical in most parts of the surface, maximum flatness principle may be used. It will minimize the number of slices as the heights of the respective slices are maximized. Again, if number of slices is minimized, the part building time is generally reduced. Then it is better to go for the minimum curvature principle instead of the NVS principle as has been already discussed in detail in section 2.3.

Acknowledgement

The authors sincerely thank the Department of Science and Technology (DST), India for kindly funding this work.

References

[1] Choi, B. K. (1991), *Surface Modeling for CAD-CAM*, Elsevier.

[2] de Jager, P. J., Brock, J. J. and Vergeest, J. S. M. (1997), "Comparison between zero and first order approximation algorithms for layered manufacturing," *Rapid Prototyping Journal*, **3**, Issue 3, 144-149.

[3] de Jager, P. J. (1996), "Using Slanted and Ruled Layers for Rapid Prototyping," Proceedings of the 5th European conference on Rapid Prototyping and Manufacturing, Finland, June 4-6, 15-30.

[4] de Jager, P. J., Broek, J. J. and Vergeest, J. S. M. (1996), "Rapid prototyping: extending the layer concept."

Proceedings of the 2nd International Conference on Rapid Product Development, Stuttgart, 325-334.

[5] Faux, I. D. and Pratt, M. J. (1979), *Computational Geometry for Design and Manufacture*, Ellis Horwood/ Simon & Schuster.

[6] Hope, R. L., Jacobs, P. A. and Roth, R. N. (1997), "Rapid Prototyping with sloping surfaces," *Rapid Prototyping Journal*, **3**(1), 12-19.

[7] Hope, R. L., Jacobs, P. A. and Roth, R. N. (1997), "Adaptive Slicing with sloping layer surfaces," *Rapid Prototyping Journal*, **3**(3), 89-98.

[8] Hope, R. L., Rick, A. T. and Roth, R. N. (1996), "Layer Building with Sloping Edges for Rapid Prototyping of large objects," Proceedings of the 5th European conference on Rapid Prototyping and Manufacturing, Finland, June 4-6, 47-57.

[9] Im, Y. and Walczyk, D. F. (2002), "Development of a Computer-Aided Manufacturing System for Profiled edge lamination Tooling," *Transactions of the ASME*, Journal of Manufacturing Science and Engineering, **124**, Aug, 754-761.

[10] Jamieson, R. (July, 1994), "CAD methods in Rapid Prototyping," Proceedings of the third European Conference on Rapid Prototyping and Manufacturing, The University of Nottingham, Nottingham, Dickens, P.M. (Ed), July 1994.

[11] Jamieson, R. and Hacker, Herbert (1995), "Direct slicing of CAD model for rapid prototyping," *Rapid Prototyping Journal*, **1**(2), 4-12.

[12] Kulkarni, P. and Dutta, D. (1996), "An accurate slicing procedure for layered manufacturing," *Computeraided Design*, **28**(9), 683-697.

[13] Kumar, A., Banerjee, A. G., Paul, S. and Roy Choudhury, A. (2003), "Maximisation of slice height with uniformity in Surface Roughness in Direct Slicing of free form surfaces," *Journal of Engineering Manufacture*, IMECHE, Proceedings of the Part B, Issue B6, **217**, 765-777.

[14] Kumar, M. and Roy Choudhury, A. (2002), "Adaptive Slicing with Cubic Patch Approximation," *Rapid Prototyping Journal*, **8**, Issue 4, 224-232.

[15] Luo Ren, C. and Ma, Y. (1995), "A Slicing Algorithm for Rapid Prototyping and Manufacturing," IEEE International Conference on Robotics & Automation, 2441-2846.

[16] Mortenson, M. E. (1985), *Geometric Modeling*, John Wiley and Sons, New York.

[17] Novac, A. S., Lee, C. H. and Thomas, C. L. (1997), "Automated technique for adaptive slicing in layered manufacturing," Proceedings of the 7th International Conference on Rapid Prototyping, 31 March-3 April, San Francisco, C.A, 85-93.

[18] Piskunov, N. (1979), "Differential and Integral calculus," Part II, MIR Publishers, Moscow.

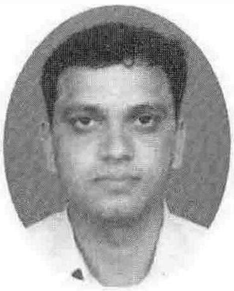
Ashis Gopal Banerjee is currently a final year undergraduate student in Manufacturing Science & Engineering (Mechanical Engineering Department) at Indian Institute of Technology, Kharagpur, India. His research interests include CAD\CAM, rapid prototyping and manufacturing systems design.

Aloke Kumar is currently pursuing a B.Tech. course in Manufacturing Science & Engineering from the Department of Mechanical Engineering, Indian Institute of Technology, Kharagpur. His research interests include CAD\CAM, rapid prototyping, computational fluid dynamics, soft computing.

Shankar Tejavath is a mechanical engineer with a Master's degree in Manufactuirng Systems Engineering from the Indian Institute of Technology, Kharagpur. He is currently working with the Oil and natural Gas Commission of India.

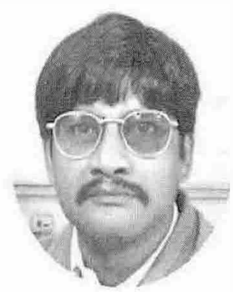
Asimava Roy Choudhury is an Assistant Professor in the Mechanical Engineering Department of the Indian Institute of Technology, Kharagpur. His interests include Rapid Prototyping, CNC, Laser processing of Materials, Non-conventional Manufacturing processes and conventional metal cutting.

Ashis Gopal Banerjee



Shankar Tejavath

Aloke Kumar



Asimava Roy Choudhury