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Estimating Process Capability with Truncated Samples

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절단 표본을 이용한 공정능력의 추정

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Process capability has long been viewed as a critical performance measure to indicate how well a process meet the specifications and customer requirements. Several indices, including C_p and C_{pk}, have been proposed and widely implemented to quantify the process capability. However, these indices have been obtained without regard to inspection or screening procedures through which finished products will be truncated at the specifications. Consequently, only a fraction of outgoing products within the specifications will be passed into the customers. From the customer's point of view, it will thus be meaningful to assess the process capability with truncated samples. This article investigates how to estimate the process capability when only incomplete truncated data are available. On the basis of parameter estimation for truncated samples, the proposed methodology may be helpful to evaluate the process capability by examining a sample of items from the lots submitted.

Keywords: process capability, truncated sample, inspection or screening procedures

1. Introduction

As a critical performance measure to indicate how capable a process is in terms of meeting design specifications, process capability is represented by the ratio of a tolerance range to the natural spread of the process. It has been a common practice to set a target level of process capability. For instance, Motorola's Six Sigma program essentially advocates that the capability index is expected to be 2.0. Initiated as a corporate quality objective, this program has become popular even outside the company. Montgomery (1997) and Quensenberry (1997) also presented some recommended guidelines for setting the level of process capability.

Several indices have extensively been implemented to quantify the capability, and the most commonly used capability indices are C_p and C_{pk} (Kane, 1986) which are defined by

$$C_p = \frac{USL - LSL}{6\sigma}$$

and

$$\begin{aligned} C_{pk} &= min\,\{C_{pu}, C_{pl}\} \\ &= min\left\{\frac{USL-\mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right\} \end{aligned} \tag{1}$$

where LSL and USL represent the lower and upper specification limits, respectively, μ is the process mean, and σ is the process standard deviation. Numerous industrial examples of capability indices may be located in the literature including semiconductor manufacturing(Hoskin *et al.*, 1988), jet-turbine engine component(Hubele *et al.*, 1991), flip-chips and

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chip-on-board (Noguera and Nielsen, 1992), speaker drivers (Pearn and Chen, 1997), and memory storage systems (Rado, 1989).

The capability of a manufacturing process is influenced by the materials and components used to a great extent. Thus, the quality of supplied components should be assured to improve the capability of the process. Modern quality management systems stress the need for periodically assessing vendor's capability in meeting the specifications (Asokan and Unnithan, 1999). It has been a common practice to periodically visit the vendor to assess the capability, which can be extremely costly and time-consuming. It will thus be meaningful to investigate how to assess the vendor's capability efficiently and quickly. One such way is to examine the submitted lots to assess the capability since only a fraction of manufactured products passing inspection (i.e., truncated at the specifications) will be supplied. This article investigates how to assessor estimate the process capability by examining truncated samples. The proposed methodology is based on the parameter estimation for truncated samples outlined by Cohen (1957, 1959). The procedures for estimating the process capability with truncated samples will also be demonstrated through numerical examples.

2. Estimation of Process Parameters

In process capability analysis, it has usually been assumed that the process be in statistical control. Additionally, suppose that the quality characteristic of interest follow a Normal distribution even though the proposed methodology may also be applied to other types of process distributions. It is well known that the natural estimators of C_p and C_{pk} can be obtained by replacing μ and σ in equation (1) with their usual estimators $\overline{X}$ and S as follows:

$$\hat{C}_p = \frac{USL - LSL}{6\hat{\sigma}} = \frac{USL - LSL}{6S}$$

and (2)

$$\begin{aligned} \hat{C}_{pk} &= min\{\hat{C}_{pu}, \hat{C}_{pl}\} \\ &= min\left\{\frac{USL-\hat{\mu}}{3\hat{\sigma}}, \frac{\hat{\mu}-LSL}{3\hat{\sigma}}\right\} \\ &= min\left\{\frac{USL-\overline{X}}{3S}, \frac{\overline{X}-LSL}{3S}\right\} \end{aligned}$$

However, equation (2) may not hold when the products are truncated at its specification and only the truncated samples are available for process capability analysis. First of all, the sample standard deviation obtained from truncated samples generally underes-timates the true process dispersion. Furthermore, the sample mean may also be inappropriate to estimate the true process mean especially when the products are truncated in an asymmetric manner about the process mean. For these reasons, this study basically suggests using parameter estimation procedures with truncated samples for the purpose of capability study.

Let Y be the quality characteristic of interest, which follows a Normal distribution. Two types of truncation are considered herein: singly and doubly truncated samples. In singly truncated samples, only one of specification limits is specified. When the lower limit is given, products are passed into the customer only if $Y \geqq LSL$, in which case truncation is said to be on the left. On the other hand, products pass the inspection only if $Y \leqq LSL$, and truncation is said to be on the right when only the upper limit is specified. In doubly truncated samples, both upper and lower specification limits are specified. The products will be submitted to the customer only if $LSL \leqq Y \leqq USL$.

When only the lower specification limit is given and thus outgoing products are left truncated, a capability index of C_{pl} is usually used. The process mean and variance need to be estimated to assess the capability. For left truncated samples, the estimators for process mean and variance are given by (Cohen, 1959)

$$\hat{\mu} = \overline{Y} - \theta(\alpha)(\overline{Y} - LSL)$$

and (3)

$$\hat{\sigma}^2 = S^2 + \theta(\alpha)(\overline{Y} - LSL)^2,$$

respectively, where $\theta(.)$ is the auxiliary estimating function and

$$\alpha = \frac{S^2}{(\overline{Y} - LSL)^2}.$$

The auxiliary estimating function $\theta(.)$ is defined as

$$\theta(x) = \frac{\phi(x)}{\phi(x) - x[1 - \Phi(x)]},$$

where $\phi(.)$ and $\Phi(.)$ represent the density and distribution functions of the standard Normal distribution, respectively. Using the estimators given in equation (3), the process capability may easily be assessed by examining supplied products. It can be observed, from equation (3), that the sample variance of truncated samples tends to underestimate the true process variance. For right truncated samples in which a capability index of C_{pu} is usually used, the estimators for process mean and variance can easily be obtained by simply replacing LSL with USL in equation (3). Readers are referred to Cohen (1959) for detailed derivations of the estimators for singly truncated samples.

It is usually the case that both the upper and lower specification limits are specified, and thus the finished products are doubly truncated at both ends. For doubly truncated samples, the process standard deviation and process mean are estimated by (Cohen, 1957)

$$\hat{\sigma} = \frac{ULS - LSL}{\beta_2 - \beta_1}$$

and (4)

$$\hat{\mu} = LSL - \beta_1 \hat{\sigma},$$

respectively, where β_1 and β_2 may be determined by simultaneously solving the following two estimating equations:

$$\frac{\phi(\beta_1) - \phi(\beta_2) - \beta_1[\Phi(\beta_2) - \Phi(\beta_1)]}{(\beta_2 - \beta_1)[\Phi(\beta_2) - \Phi(\beta_1)]} = \frac{\overline{Y} - LSL}{USL - LSL}$$

and

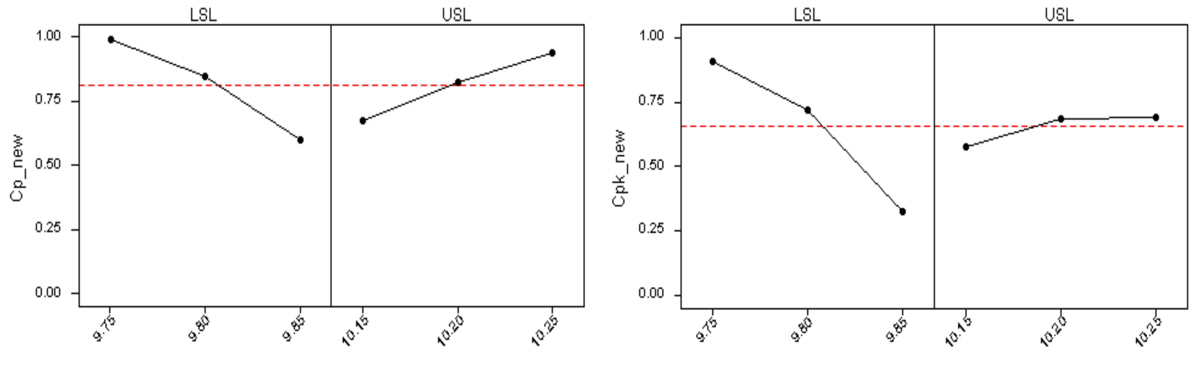
$$\frac{\left\{\begin{array}{l}\Phi(\beta_2) - \Phi(\beta_1) + \beta_1\phi(\beta_1) - \beta_2\phi(\beta_2) \\ + [\phi(\beta_1) - \phi(\beta_2)]^2/[\Phi(\beta_2) - \Phi(\beta_1)]\end{array}\right\}}{(\beta_2 - \beta_1)^2[\Phi(\beta_2) - \Phi(\beta_1)]} = \frac{S^2}{(USL - LSL)^2}$$

More details on the derivation of estimators can be found in Cohen (1957). Using these estimators for process parameters, the process capability can easily be assessed by truncated samples.

3. Numerical Example

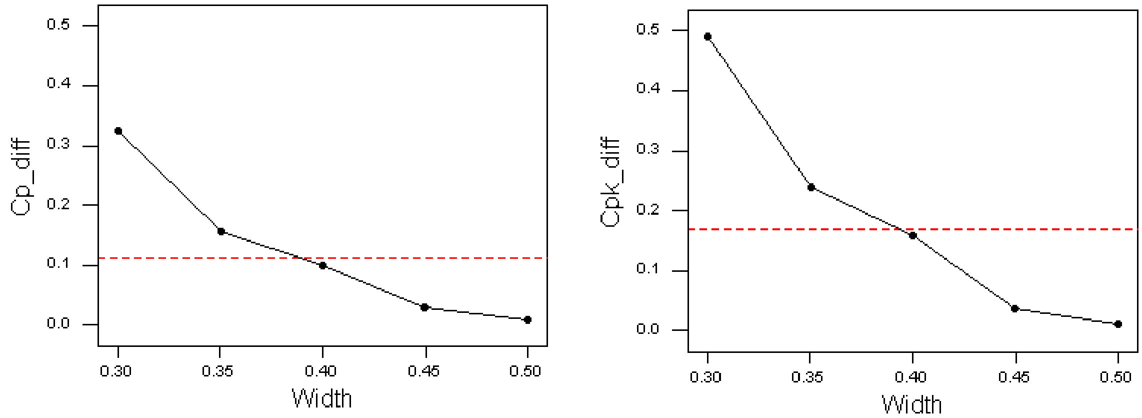
The procedures for estimating the process capability with truncated samples are briefly outlined in the previous section. To demonstrate the proposed procedures, suppose that the width of a component is the quality characteristic of interest with the specifications of 10 ± 0.2 millimeters and components are supplied after inspection procedures so that they are truncated at the specifications. Random samples are drawn from the supplied components, of which mean and standard deviation are calculated as 9.9728 and 0.07397, respectively. Using the sample mean and standard deviation as it stands, the process capability indices are estimated as $\hat{C}_p = 0.901$ and $\hat{C}_{pk} = 0.779$. On the other hand, the proposed procedure yields the capability indices of 0.856 and 0.729 for C_p and C_{pk}, respectively. The values of β_1 and β_2 are calculated as -2.18745 and 2.94976, respectively, from which the process mean and standard deviation are estimated as 9.9703 and 0.07786, respectively. The calculations are performed using the Solver option in Microsoft Excel.

Sensitivity analysis has been conducted to examine the characteristics of the proposed procedures with respect to specifications. Figure 1 depicts the capability indices with truncated samples for different values of the lower and upper specifications. It is intuitive that



(a) C_p Index (b) C_{pk} Index

Figure 1. Capability indices with truncated samples.



(a) C_p Index (b) C_{pk} Index

Figure 2. Comparison of capability indices.

the process capability indices decrease as product specifications become narrower, and vice versa.

Capability indices with truncated samples are also compared with capability indices obtained without considering truncations in Figure 2. It can be seen that the differences become larger for narrower specifications. This implies that the process capability tends to be overestimated when directly calculated with sample mean and standard deviation for truncated samples. It may be attributed to the fact that the standard deviation for truncated samples tends to underestimate the true process variability. Therefore, the proposed procedures may be efficiently applied to estimate the true process capability especially when it is evident that the supplied lots are inspected and thus truncated at the specifications.

4. Conclusion

Process capability indices have widely been recognized as a critical measure based on which how well a manufacturing process meets the specifications is evaluated. From the customer's standpoint, it will be meaningful to assure the vendor's process capability to improve one's own product quality and productivity. Along this line, this article investigates how to quickly and efficiently assess the vendor's process capability and one such way is to estimate the process capability only by examining the supplied products. It has been discussed that parameter estimation techniques for truncated samples can easily applied to estimate the process capability. The overall procedures are briefly outlined, and the effects of truncation on the process capability have been examined through a numerical example. Although simple, the procedures presented in this article may be helpful to facilitate the assessment of process capability.

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