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원추형 코일스프링의 강성에 대한 연구
A Study on the Stiffness of Frustum-shaped Coil Spring

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Key Words : Frustum-Shaped Coil Spring(원추형 코일스프링), Space Beam Element(빔요소), Step by Step Method(단계법), Local Coordinates System(국부좌표계), Global Coordinates System(전체좌표계), Transformation Matrix(전달행렬), Nodal Displacement(절점변위)

Abstract : 스프링은 가장 널리 이용되어지고 있는 기계요소이다. 본 논문에서는 원추형 코일스프링의 강성을 구하기 위하여, 빔요소를 이용한 유한요소법을 사용하였다. 가상일의 법칙을 이용하였고, 코일스프링의 하중벡터를 압축 분포하중으로 대체하였다. 하중의 증가에 의한 절점에서의 변위는 유한요소법을 이용하여 계산하였다. 단계법으로 절점의 변위를 중첩하여 전체 강성행렬을 구하였다. 유한요소법에 의한 해석치는 실험치와 잘 일치하였다. 본 논문에서 제시한 프로그램을 사용하여, 스프링 강성과 응력을 예측할 수 있을 것으로 사료된다.

Nomenclature

$\{\delta^e\}$: Nodal displacement vector of element in global coordinate system

$\{F^e\}$: Nodal load vector of element in global coordinate system

$[K^e]$, $[k^e]$: Stiffness matrix displacement vector of element in global and local coordinates system

F_{xi}, F_{yi}, F_{zi} : Distributed load to x, y, z-axis directions of beam element

T_{xi}, T_{yi}, T_{zi} : Torsional moment to x, y, z-axis directions of beam element

D_1, D_2 : Minimum and maximum coil diameters of spring

n : Number of active coil

G : Shear modulus

d : Diameter of coil material

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1. Introduction

Frustum-shape coil spring is a machine element with simple structure[1] from that various spring forces could be obtained. It is widely used in deflation washer of an automobile, return spring in clutch assembly and buffer spring of press.

Correction coefficient based on Wahl's theory is mainly applied to spring analysis, but finite element method[2,3] is used to obtain correct stiffness. In addition, previous researchs[4,5] report that cylindrical coil spring received increased tensile load was analyzed.

In this study, finite element method was applied to evaluate stiffness of frustum-shaped coil spring that was increased compressed distributed load and to develop new analysis method by programming using MATLAB.

Spring shape was divided into 1-D beam element. Non-linear part was decoded by applying step by step method in linear simultaneous equation when measuring displacement by load. Therefore analytical solution was compared with experimental results which induced by modeling the shape of sample and then input values of each spring into program.

It is expected that this method can be considered useful when spring was designed because simple simulation can be done by inputting anticipating factor to spring stiffness.

2. Formulation of finite element

2.1 Stiffness matrix

As shown in Fig. 1, the shape of frustum-shaped coil spring was organized using beam element in 3-D space.

In Fig. 2, beam element was pointed according to theory of coil spring divided into beam.

Two nodal points were denoted as 1, 2 in beam element and displacement vector and load vector can be indicated by following equation in global coordinated system.

$$\{\delta^e\} = \{Q_1\ Q_2\ Q_3\ Q_4\ Q_5\ Q_6\ Q_7\ Q_8\ Q_9\ Q_{10}\ Q_{11}\ Q_{12}\}^T \quad (1)$$

$$\{F^e\} = \{F_{x1}\ F_{y1}\ F_{z1}\ T_{x1}\ T_{y1}\ T_{z1}\ F_{x2}\ F_{y2}\ F_{z2}\ T_{x2}\ T_{y2}\ T_{z2}\}^T \quad (2)$$

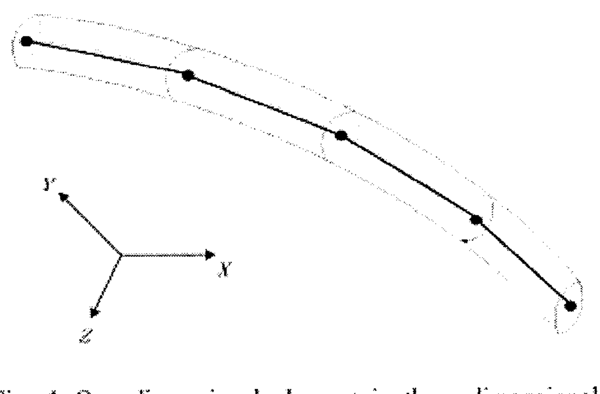


Fig. 1 One dimensional element in three dimensional space

Displacement by load vector is influenced by axis directional force and torsional moment in local coordinate system in Fig. 3. Therefore, x-axis displacement q_1, q_7 is affected by axis directional force and torsional displacement, whereas q_4 and q_{10} are influenced by torsional moment.

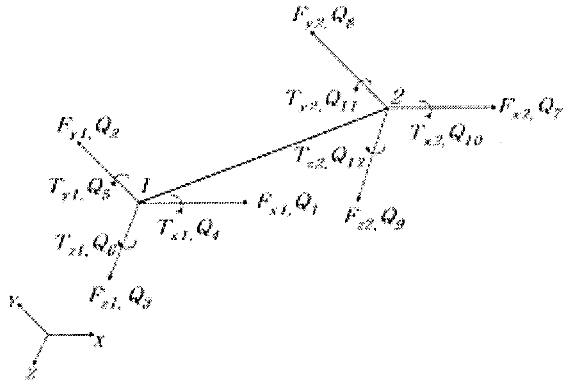


Fig. 2 Displacements and forces on a space beam element in global coordinates

Bending displacement q_2, q_6, q_8, q_{12} in xy-plane were affected by directional shear load in y-axis and by bending moment and load in xz-plane. However, xy-plane and xz-plane agrees with principal axis of section and displacement and bending load in two planes can be considered independent. Therefore, considering each case, 12×12 matrices were obtained in local coordinate system by superposition.

Global stiffness matrix of beam element could be obtained by superposition of each stiffness matrix according to the kinds of load. In addition, global stiffness coordinate $[k^e]$ was found in following local coordinate system. As this equation is defined under local coordinate system, it must be transformed under global coordinate system.

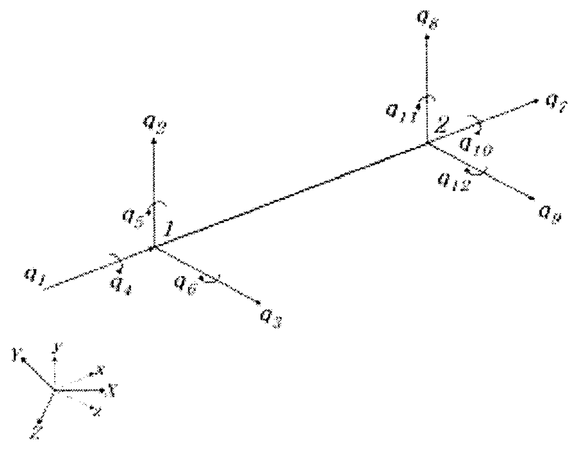


Fig. 3 Displacements of space beam element in local and global coordinates system

2.2 Load vector

Load vector in finite element method was obtained by applying the principle of virtual work.[6] As shown in Fig. 4, frustum-shaped coil spring which was severed in upper parts received distribution load such as tensile or compression of axis direction. Therefore, distribution load is converted to nodal force of beam element. $\{f^e\}$ was denoted load vector of beam element and $[N]$ was denoted shape function of beam element. In node of beam element 1, 2 in Fig. 5, load vector having 12 degree of freedom can be illustrated as following.

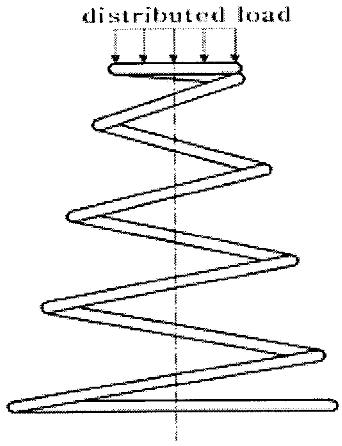


Fig. 4 Distributed load

$$\{f\}^T = \{F_{x1}\ F_{y1}\ F_{z1}\ T_{x1}\ T_{y1}\ T_{z1}\ F_{x2}\ F_{y2}\ F_{z2}\ T_{x2}\ T_{y2}\ T_{z2}\} \tag{4}$$

$$\{f^e\} = \int_0^l [N]^T \begin{Bmatrix} F_x \\ F_y \\ F_w \\ T_x \\ T_y \\ T_w \end{Bmatrix} dx \tag{5}$$

$[N]^T$ can be obtained from previous investigation as follows.

$$[N]^T = \begin{bmatrix} N_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & N_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & N_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & N_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & N_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & N_6 \\ N_7 & 0 & 0 & 0 & 0 & 0 \\ 0 & N_8 & 0 & 0 & 0 & 0 \\ 0 & 0 & N_9 & 0 & 0 & 0 \\ 0 & 0 & 0 & N_{10} & 0 & 0 \\ 0 & 0 & 0 & 0 & N_{11} & 0 \\ 0 & 0 & 0 & 0 & 0 & N_{12} \end{bmatrix} \tag{6}$$

$N_1 = 1 - \frac{x}{l},\ N_2 = (2x^3 - 3lx^2 + l^3)/l^3,\ N_3 = N_2$

$N_4 = N_1,\quad N_5 = (x^3 - 2lx^2 + l^2x)/l^2,\ N_6 = N_5$

$N_7 = \frac{x}{l},\quad N_8 = -(2x^3 - 3lx^2)/l^3,\quad N_9 = N_8$

$N_{10} = N_7,\quad N_{11} = (x^3 - lx^2)/l^2,\qquad N_{12} = N_{11}$

$$[k^e] = \begin{bmatrix} \frac{EA}{l} & & & & & & & & & & & \\ 0 & \frac{12EI_{zz}}{l^3} & & & & & & & & & & \\ 0 & 0 & \frac{12EI_{yy}}{l^3} & & & & & & & \text{sym.} & & \\ 0 & 0 & 0 & \frac{GJ}{l} & & & & & & & & \\ 0 & 0 & \frac{6EI_{yy}}{l^2} & 0 & \frac{4EI_{yy}}{l} & & & & & & & \\ 0 & \frac{6EI_{zz}}{l^2} & 0 & 0 & 0 & \frac{4EI_{zz}}{l} & & & & & & \\ \frac{-EA}{l} & 0 & 0 & 0 & 0 & 000 & \frac{EA}{l} & & & & & \\ 0 & \frac{-12EI_{zz}}{l^3} & 0 & 0 & 0 & \frac{-6EI_{zz}}{l^2} & 0 & \frac{12EI_{zz}}{l^3} & & & & \\ 0 & 0 & \frac{-12EI_{yy}}{l^3} & 0 & \frac{-6EI_{yy}}{l^2} & 0 & 0 & 0 & \frac{12EI_{yy}}{l^3} & & & \\ 0 & 0 & 0 & \frac{-GJ}{l} & 0 & 0 & 0 & 0 & 0 & \frac{GJ}{l} & & \\ 0 & 0 & \frac{6EI_{yy}}{l^2} & 0 & \frac{2EI_{yy}}{l} & 0 & 0 & 0 & \frac{-6EI_{yy}}{l^2} & 0 & \frac{4EI_{yy}}{l} & \\ 0 & \frac{6EI_{zz}}{l^2} & 0 & 0 & 0 & \frac{2EI_{zz}}{l} & 0 & \frac{-6EI_{zz}}{l^2} & 0 & 0 & 0 & \frac{4EI_{zz}}{l} \end{bmatrix} \tag{3}$$

Substituting equation (6) in Eq. (5), the load vector could be solved in nodal points 1, 2. Global load vector could be obtained by combining load vector in each element.

$$F_{x1}=\frac{l}{2}F_{xw},\ F_{y1}=\frac{l}{2}F_{yw},\ F_{z1}=\frac{l}{2}F_{zw}$$

$$T_{x1}=\frac{l}{2}T_{xw},\ T_{y1}=\frac{l^2}{12}T_{xw},\ T_{z1}=\frac{l^2}{12}T_{yw}$$

$$F_{x2}=\frac{l}{2}F_{xw},\ F_{y2}=\frac{l}{2}F_{yw},\ F_{z2}=\frac{l}{2}F_{zw}$$

$$T_{x2}=\frac{l}{2}T_{xw},\ T_{y2}=-\frac{l^2}{12}T_{xw},\ T_{z2}=-\frac{l^2}{12}T_{yw}$$

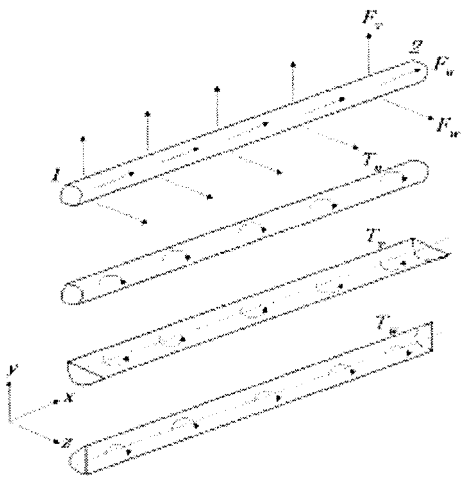


Fig. 5 Load vector in beam element

3. Analysis of frustum-shaped coil spring

Formulized equations by finite element method was programmed. Shape of frustum-shaped coil spring was discrete as 133 nodal points and was divided into 132 element in Fig. 6. Analysis was implemented by applying boundary condition in Tables 1

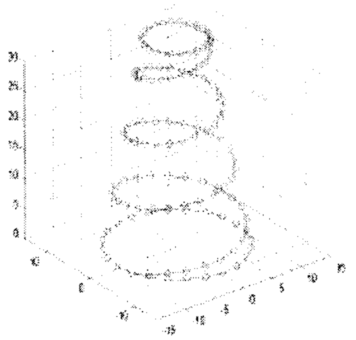


Fig. 6 Modeling of frustum-shaped coil spring

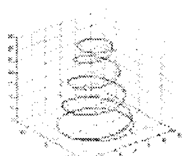
Fig. 7 illustrated deformation aspect of frustum-shaped coil spring as various 6 types of load conditions.

Table 1 Specification of frustum-shaped spring

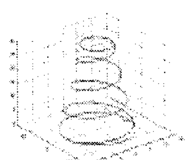
Items	Value
Compression force (N)	10
Upper diameter D_1 (mm)	9.42
Lower diameter D_2 (mm)	20.75
Height H (mm)	35.4
Coil material (mm)	1
Number of active coils	3.5
Young's modulus (GPa)	205.8
Shear modulus (GPa)	80
Number of element	132

Table 2 Boundary condition of model

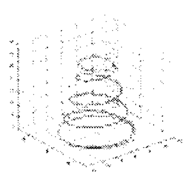
Nodal point	Displacement			Distortion		
	x	y	z	x	y	z
109~132	fixed	free	fixed	free	free	free
133	fixed	fixed	fixed	fixed	fixed	free



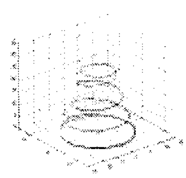
(a) F=1.76 (N)



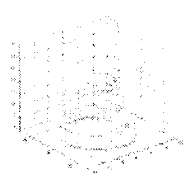
(b) F=2.744 (N)



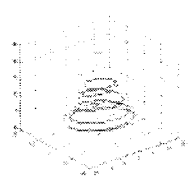
(c) F=4.41 (N)



(d) F=6.174 (N)



(e) F=8.036 (N)



(f) F=9.996 (N)

Fig. 7 Deformation of frustum-shaped coil spring by compression load

4. Experiments

Photo 1 shows the specimen of this experiment and material specification. Dimensions are presented in Table 1 and Fig. 8 respectively. Spring tester (FMC-A 200D), which was used in this experiment, was presented in Photo 2. Preparing three samples, distributed compression load was measured after setting displacement of each six steps.

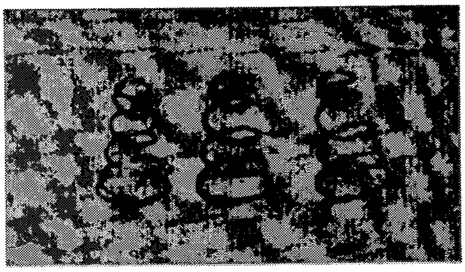


Photo 1 Samples for frustum-shaped coil spring

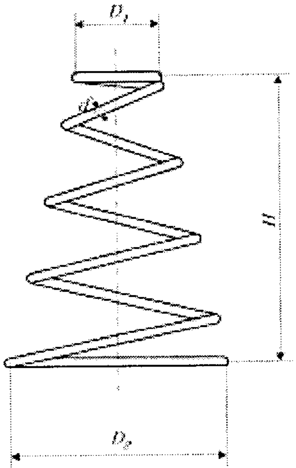


Fig. 8 Dimensions of specimen

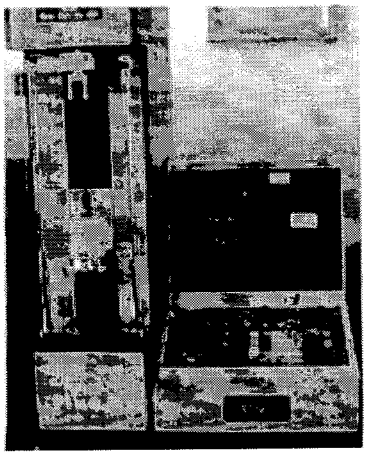


Photo 2 General view of spring tester

5. Analysis and considerations

Equation resolving displacement of frustum-shaped coil spring is as following:

$$\delta = \frac{16Pn}{Gd^4}\,\frac{R_2^4 - R_1^4}{R_2 - R_1} \tag{7}$$

Table 3, Fig. 9 and 10 presented the comparisons among the value of analytical solution, experiment and general theoretical equation.

Fig. 9 illustrated the coincidence between experiment results and analytical values. Less than 4% in discrepancy between solid mechanics and the results of finite element method, it is satisfactory that first grade for spring constant was permitted by KS B 2405[8], in accordance with the rule which is within less than 5%.

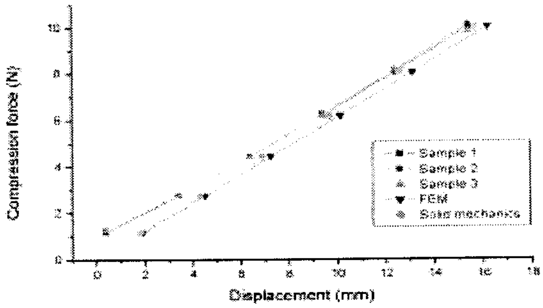


Fig. 9 Relationships between compression force and displacement for frustum-shaped coil spring

In Fig. 10, overall spring constant from finite element method showed consistency, but early period of experimental results showed different patterns from those of finite element method and solid mechanics. This discrepancy was mainly due to parallelism, perpendicularity and form error. However, experimental results and values of finite element method showed good agreement as load increased.

Table 3 Comparison of deformation

Item	Solid mechanics	Test	FEM
Displacement (mm)	15.6437	15.4	16.2025
Error(%)	0	1.56	3.57

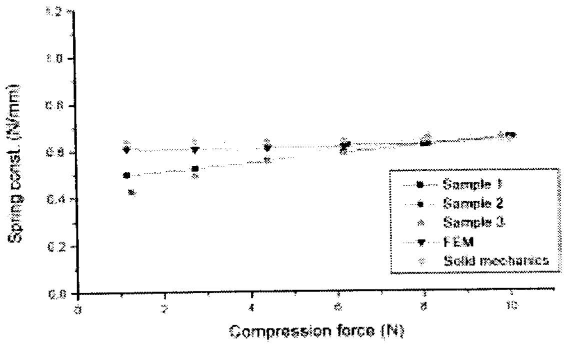


Fig. 10 Spring constant of frustum-shaped coil spring

6. Conclusions

The following conclusions were reached;

1. Finite element method using beam element of twelve degree of freedom when distributed compression load was worked could be used to analyze the stiffness of frustum-shaped coil spring.

2. The results induced from finite element method by beam element showed good agreement with those of experiment when applying to frustum-shaped coil spring which was received distributed compression load.

3. Generally, spring is made from slender coil and its external diameter is larger than that of material. The softness of spring is mainly attributed to these properties. Therefore, geometrical non-linearity must be considered analyzing the results from finite element method.

4. Using developed program, it is expected that interrelation of load and displacement were estimated applying to other springs, such as torsion coil spring, torsion bar, beam which is not straight and stabilizer of automobile.

Acknowledgement

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