자원·환경경제연구 제 12권 제 1 호
Environmental and Resource Economics Review
Volume 12, Number 1, March 2003 : pp. 157~175

The Application of Discrete Time Optimal Control Theory to Natural Resource Problems

Jin Hyung Kim* · Dug Man Lee**

〈Contents〉

Ⅰ. Introduction

Since Pontryagin *et al.* (1962) developed the maximum principle as a solution technique for the problem of dynamic optimization, optimal control theory has been extensively applied to areas of economics to

---

* Department of International Trade, Dankook University.
** Department of Economics, Sungkyunkwan University. This author acknowledges the support of BK 21 Program.

identify the optimal time paths for economic variables. The features of the maximum principle are essentially characterized by the roles of three distinctive variables. The control variables unambiguously determine the state variables through the state equations describing the laws of motion of the state variables. The costate (adjoint) variables denote the shadow values of the state variables associated with them. In addition, the necessary conditions for the optimization substantially correspond to the sufficient conditions for the optimization (Halkin, 1966). In this respect, optimal control theory has been considered as a more simple but powerful method for solving the problem of dynamic optimization. Previous literature on optimal control theory (Pontraygin *et al.*, 1962; Hestenes, 1966; Dorfman, 1969; Long and Vousden, 1977; Kamien and Schwartz, 1981; Conrad and Clark, 1987; Hanley *et al.*, 1997) have proposed numerous maximum principles to be applied to the various types of optimal control problems. In particular, for the discrete time optimal control problem (DTOCP) the authors stated above provide the maximum principle only for the cases in which the state equation is in the form of the difference equation. However, when we model economic problems using discrete time optimal control theory we have encountered the cases in which the state equation is not in the form of the difference equation but in the form of a general equation. We, thus, assert that the maximum principle proposed by the previous literature does not directly apply to the problems that we have come across. In this context, we want to develop a maximum principle to tackle the DTOCP in which the state equation takes the form of a general equation, not of the difference equation.

For this purpose, we first review the maximum principle proposed by the previous literature for the DTOCP. We, then, present our maximum principle to be used for the DTOCP as a general one. As shown below, our maximum principle, in particular, differs from that proposed by the previous literature in identifying the law of motion of the costate variables. We also show that the maximum principle proposed by the previous literature is a subclass of ours. Finally, we use the problem of natural resource as an example that illustrate the utilization of our maximum principle as a solution technique.

Ⅱ. The Maximum Principles

In this section, we first review the maximum principle proposed by the previous literature for the DTOCP, referring, in particular, to Conrad and Clark (1987). The DTOC problem for this case is described as

$$Max \sum_{t=0}^{T-1} f(u_t\, x_t,\, t) + S(x_T) \tag{1}$$

subject to

$$x_{t+1} - x_t = g(u_t\,,\, x_t)$$
$$x_0 = x^0 \text{ given}$$

where $t = 0,\, 1, 2, \cdots T$ is the set of time periods, and $t = 0$ and $t = T$ denote the intial time period and the terminal time period, respectively. u_t represents a control variable in time period t. x_t represents a state variable describing the system in time period t. $f(\cdot)$ represents the payoff function or the net economic return. $S(\cdot)$ represents the scrap (or terminal) value function at the terminal time period. And $x_{t+1} - x_t = g(u_t, x_t)$ denotes the law of motion of the state variable. The Lagrangian function for this problem can be written as

$$L = \sum_{t=0}^{T-1} f(u_t, x_t, t) + \lambda_{t+1}(x_t + g(u_t, x_t) - x_{t+1}) + S(x_T)$$

where λ_{t+1} denotes the Lagrangian multiplier associated with x_{t+1}. The first order necessary conditions for the optimality are

$$\frac{\partial f(u_t, x_t, t)}{\partial u_t} + \lambda_{t+1}\frac{\partial g(u_t, x_t)}{\partial u_t} = 0 \qquad \text{for } t = 0, 1, 2, \cdots, T-1 \tag{2}$$

$$\lambda_{t+1} - \lambda_t = -\left(\frac{\partial f(u_t, x_t, t)}{\partial x_t} + \lambda_{t+1}\frac{\partial g(u_t, x_t)}{\partial x_t}\right) \qquad \text{for } t = 0, 1, 2, \cdots, T-1 \tag{3}$$

$$x_{t+1} - x_t = g(u_t, x_t) \qquad \text{for } t = 0, 1, 2, \cdots, T-1 \tag{4}$$

$$\lambda_T = \frac{dS(x_T)}{dx_T} \tag{5}$$

$$x_0 = x^0 \qquad \text{given} \tag{6}$$

If we define the Hamiltonian at time period t as

$$H(u_t, x_t, t, \lambda_{t+1}) = f(u_t, x_t, t) + \lambda_{t+1} g(u_t, x_t)$$

Then, some of the first order necessary conditions are interpreted in terms of the Hamiltonian such as

$$\frac{\partial H}{\partial u_t} = 0 \qquad \text{for } t = 0, 1, 2, \cdots, T-1 \tag{2a}$$

$$\lambda_{t+1} - \lambda_t = -\frac{\partial H}{\partial x_t} \qquad \text{for } t = 0, 1, 2, \cdots, T-1 \tag{3a}$$

$$x_{t+1} - x_t = \frac{\partial H}{\partial \lambda_{t+1}} \qquad \text{for } t = 0, 1, 2, \cdots, T-1 \tag{4a}$$

These last three equations plus equation (5) and (6) denote the maximum principle proposed by the previous literature for the DTOCP.

The Maximum Principle for the General Case

Let us consider the DTOCP in which the state equation is not in the form of the difference equation but takes the form of a general equation. The DTOCP for this case can be described as

$$Max \sum_{t=0}^{T-1} f(u_t\, x_t,\, t) + S(x_T) \tag{7}$$

subject to

$$x_{t+1} = h(u_t, x_t)$$
$$x_0 = x^0 \quad \text{given}$$

where $x_{t+1} = h(u_t, x_t)$ represents the law of motion of the state variable. We build up the Lagrange function for this problem as

$$L = \sum_{t=0}^{T-1} f(u_t, x_t, t) + \lambda_{t+1}(h(u_t, x_t) - x_{t+1}) + S(x_T)$$

Then, the first order necessary conditions for the optimality are

$$\frac{\partial f(u_t, x_t, t)}{\partial u_t} + \lambda_{t+1} \frac{\partial h(u_t, x_t)}{\partial u_t} = 0 \qquad \text{for } t = 0, 1, 2, \cdots, T-1 \tag{8}$$

$$\lambda_t = \frac{\partial f(u_t, x_t, t)}{\partial x_t} + \lambda_{t+1} \frac{\partial h(u_t, x_t)}{\partial x_t} \qquad \text{for } t = 0, 1, 2, \cdots, T-1 \tag{9}$$

$$x_{t+1} = h(u_t, x_t) \qquad \text{for } t = 0, 1, 2, \cdots, T-1 \tag{10}$$

$$\lambda_T = \frac{dS(x_T)}{dx_T} \tag{11}$$

$$x_0 = x^0 \quad \text{given} \tag{12}$$

If we establish the Hamiltonian at time period t as

$$H(u_t, x_t, t, \lambda_{t+1}) = f(u_t, x_t, t) + \lambda_{t+1} h(u_t, x_t)$$

Then, some of the first order necessary conditions are transformed into

$$\frac{\partial H}{\partial u_t} = 0 \qquad \text{for } t = 0, 1, 2, \cdots, T-1 \tag{8a}$$

$$\lambda_t = \frac{\partial H}{\partial x_t} \qquad \text{for } t = 0, 1, 2, \cdots, T-1 \tag{9a}$$

$$x_{t+1} = \frac{\partial H}{\partial \lambda_{t+1}} \qquad \text{for } t = 0, 1, 2, \cdots, T-1 \tag{10a}$$

Thus, our maximum principle can be gathered from these last three results and equation (11) and (12). These differ from the maximum principle obtained above only in the law of motion of the state and costate variable. In addition, we can view $S(x_T)$ as the solution function for this same maximization problem over the time horizon between T and ∞, and $S(x_{T-1})$ can be viewed as the solution function over time horizon from $T-1$ to T. This is an application of the Bellman's (1957) optimality principle and backward recursion. With this we can state that equation (11) holds for $t=0,1,2,\cdots,T-1$. For time $T-1$, we can write

$$S^*(x_{T-1}) = f^*(u_{T-1}, x_{T-1}, T-1) + S^*(h(u_{T-1}, x_{T-1}))$$

where the superscript (*) denotes the optimum. Applying equation (11) for $T-1$ we get

$$\lambda_{T-1} = \frac{dS(x_{T-1})}{dx_T} = \frac{\partial f^*(u_{T-1}, x_{T-1}, T-1)}{\partial x_{T-1}} + S^{*\prime}(h(u_{T-1}, x_{T-1}))\frac{\partial h(u_{T-1}, x_{T-1})}{\partial x_{T-1}} \tag{13}$$

and applying equation (11) to equation (13), we obtain

$$\lambda_{T-1} = \frac{dS(x_{T-1})}{dx_{T-1}} = \frac{\partial f^*(u_{T-1}, x_{T-1}, T-1)}{\partial x_{T-1}} + \lambda_T\frac{\partial h(u_{T-1}, x_{T-1})}{\partial x_{T-1}} \tag{11a}$$

Based on this result, we can state that the law of motion for the costate variable can be expressed as follows:

$$\lambda_t = \frac{dS(x_t)}{dx_t} \qquad \text{for} \quad t = 0, 1, 2, \cdots, T$$

or

$$\lambda_T = \frac{\partial f^*(u_t, x_t, t)}{\partial x_t} + \lambda_{t+1} \frac{\partial h(u_t, x_t)}{\partial x_t}$$
$$\text{for} \quad t = 0, 1, 2, \cdots, T-1$$

Now, let us apply our maximum principle to the cases in which the state equation is in the form of the difference equation. To elicit the maximum principle stated above using the Hamiltonian that we used, let us build up the following maximization problem by changing the difference equation of state variable, $x_{t+1} - x_t = g(u_t, x_t)$, into $x_{t+1} = x_t + g(u_t, x_t)$. The maximization problem denoted as equation (7) is, then, changed into

$$Max \sum_{t=0}^{T-1} f(u_t\, x_t,\, t) + S(x_t + g(u_t, x_t))$$

Thus, we get

$$\frac{\partial f(u_t, x_t, t)}{\partial u_t} + \frac{dS(x_{t+1})}{dx_{t+1}} \frac{\partial g(u_t, x_t)}{\partial u_t} = 0$$
$$\text{for} \quad t = 0, 1, 2, \cdots T-1$$

If we establish the Hamiltonian function at time period t as

$$H(u_t, x_t, t, \lambda_{t+1}) = f(u_t, x_t, t) + \lambda_{t+1}(x_t + g(u_t, x_t))$$

We obtain

$$\begin{aligned}\frac{\partial H(u_t, x_t, t, \lambda_{t+1})}{\partial u_t} &= \frac{\partial f(u_t, x_t, t)}{\partial u_t} + \lambda_{t+1}\frac{\partial g(u_t, x_t)}{\partial u_t} \\ &= 0 \qquad \text{for} \quad t = 0, 1, 2, \cdots, T-1\end{aligned}$$

Thus, we finally have that

$$\lambda_{t+1} = \frac{dS(x_{t+1})}{dx_{t+1}} \qquad \text{for} \quad t = 0, 1, 2, \cdots, T-1 \qquad (14)$$

With this result, we illustrate the procedure of deriving the law of motion of costate variable. At time t, the optimum value of S will be

$$S^*(x_t) = f^*(u_t, x_t, t) + S^*(x_t + g(u_t, x_t))$$

Then,

$$\begin{aligned}\frac{\partial S^*(x_t)}{\partial x_t} &= \frac{\partial f^*(u_t, x_t, t)}{\partial x_t} \\ &\quad + \frac{dS^*(x_{t+1})}{dx_{t+1}}\left(1 + \frac{\partial g(u_t, x_t)}{\partial x_t}\right) \qquad (15) \\ &\qquad \text{for} \quad t = 0, 1, 2, \cdots, T-1\end{aligned}$$

Using equation (14), equation (15) can be changed into

$$\lambda_t = \frac{\partial f^*(u_t, x_t, t)}{\partial x_t} + \lambda_{t+1}\left(1 + \frac{\partial g(u_t, x_t)}{\partial u_t}\right) \quad (16)$$

$$\text{for} \quad t = 0, 1, 2, \cdots T-1$$

Thus,

$$\lambda_{t+1} - \lambda_t = -\left(\frac{\partial f^*(u_t, x_t, t)}{\partial x_t} + \lambda_{t+1}\frac{\partial g(u_t, x_t)}{\partial u_t}\right)$$

$$\text{for} \quad t = 0, 1, 2, \cdots, T-1$$

In this respect, we derive the first order necessary conditions and transversality conditions stated above using our Hamiltonian. As a result, we can state that our maximum principle is a general one that contains the maximum principle proposed by the previous literature.

Ⅲ. An Economic Example

In this section, we take a look at the Timber Supply Model 2000 (TSM 2000) developed by Lee and Lyon (2001) as an example of applying our maximum principle to economic problems. The TSM 2000 is developed to analyze the dynamic behavior of the global timber market by incorporating important additional components of the global timber market that has been occurred in recent years.[1] To see how our maximum principle is applied to the TSM 2000, we first summarize the

1) Refer to Lee and Lyon (2001) for more detailed information about important additional components of the global timber market.

formulation of the TSM 2000 and then examine the procedure of deriving the equations that we solve to find the optimal time paths for economic variables.

Equations

The objective of the TSM 2000 is to maximize the total benefit of the society as a whole, not the private profit of an individual landowner. Also, the TSM 2000 decomposes the total industrial wood harvests into solidwood and pulpwood production. Thus, net surplus of year j is defined as

$$s_j = \int_0^{Q_j} D_j^s(n)dn + \int_0^{\hat{Q}_j} D_j^p(n)dn - C_j$$

where Q_j is the quantity of timber for solidwood harvested in year j; $D_j^s(n)$ is the inverse demand function of industrial solidwood in year j; $\hat{Q}_j$ is the quantity of timber for pulpwood harvested in year j; $D_j^p(n)$ is the demand of industrial pulpwood in inverse form; and C_j is the total cost in year j. The total cost are the summation of harvest, access, transportation costs (CH_j), and the regeneration cost (CR_j). Harvesting and transportation costs in year j depend on the total volume harvested by land class, and regeneration costs depend on hectares harvested (regenerated) and the level of input used.

For the formulation, define x_{hj} to be a state vector of hectares of trees in each age group for land class h in year j with element x_{hij}. The subscript h, i, and j correspond to land class, age group, and the year, respectively. Let z_{hj} be the state vector for the regeneration input with element z_{hij}, which is the level of regeneration input associated with age group i in year j for land class h. Next, u_{hj} is the control vector of hectares harvested. The elements of u_{hij} denote for land class h the portion of the hectares of trees in age group i harvested in year j. Let w_{hj} be the level of regeneration input per hectare for those hectares regenerated in year j, and p_{wh} be the price of regeneration input for land class h.

The merchantable volume of timber per hectare for land class h in year j for a stand regenerated i time periods ago depends on i and on the magnitude of the regeneration input used on this stand (z_{hij}). We denote this merchantable volume as

$$q_{hij} = f_h(i, z_{hij})$$

This volume is divided between solidwood and pulpwood using variable proportions which vary by land class, with ϕ_h the portion going to solid wood and $(1-\phi_h)$ the portion going to pulpwood.2) With these definitions, the volume of commercial timber harvested for solidwood and pulpwood from land class h in year j is given by

2) The proportion ϕ_h is a constant elasticity function of the price of solidwood relative to pulpwood (p_j^s/p_j^p). It is given by $\phi_h = A_h(p_j^s/p_j^p)^{\varepsilon}$ where p^s and p^p are solidwood and pulpwood price, relatively; ε is the elasticity of ϕ with respect to rlative price, which is the same for all land classes, and A_h is a scaling factor that varies by land class.

$$Q_{hj} = \phi_{hj} u_{hj}' X_{hj} q_{hj}$$

$$\hat{Q}_{hj} = (1-\phi_{hj}) u_{hj}' X_{hj} q_{hj}$$

and

$$Q_h = \sum_h Q_{hj}, \ \hat{Q}_h = \sum_h \hat{Q}_{hj}$$

where X_{hj} is a diagonal matrix using the elements of x_{hj}, and the total volume harvested in the responsive regions is the summation of these over all land classes. Costs including harvest, access, and transportation cost for land class h is a function of the volume harvested in that land class;

$$CH_{hj} = c_h(Q_{hj} + \hat{Q}_{hj})$$

and regeneration cost for land class h in year j is given by

$$CR_{hj} = (u_{hj}' x_{hj} + v_{hj}) p_{wh} w_{hj}$$

where the inner product in parenthesis gives the hectares harvested in land class h, v_{hj} is the exogenously determined number of hectares of new forest land in land class h, and the product of the last two terms gives expenditure per hectare. This yields total cost of

$$C_j = \sum_h (CH_{hj} + CR_{hj})$$

With these definitions, the objective function of TSM 2000 will be the discounted present value of the net surplus stream as follows;

$$S_0(x_0, z_0, u, w) = \sum_{k=0}^{J-1} \rho^k s_k(x_k, z_k) + \rho^J S_J^*(x_J, z_J) \qquad (17)$$

where ρ is the discount factor; J is the last time period of the model time horizon; u is any admissible set of control vectors; w is any set of admissible control scalars; and $S_J^*(\cdot)$ is the optimal terminal value function. Equation (18) is to be maximized over the control variables subject to the state equations and the constraints. The constraints for control variables and the state equations for the given system are given by

$$0 \leq u_{hj} \leq 1 \qquad \text{for all } h, i, j \qquad (18a)$$

$$0 \leq w_{hj} \qquad \text{for all } h, j \qquad (18b)$$

$$x_{h,j+1} = (A + BU_{hj})x_{hj} + v_{hj}e \qquad \text{for all } h, j \qquad (19a)$$

$$z_{h,j+1} = Az_{hj} + w_{hj}e \qquad \text{for all } h, j \qquad (19b)$$

where

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ 1 & 0 & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 1 & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & 1 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & 0 & 1 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \qquad e = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix}$$

$$
B = \begin{bmatrix}
1 & 1 & 1 & 1 & \cdot & \cdot & \cdot & 1 \\
-1 & 0 & 0 & 0 & \cdot & \cdot & \cdot & 0 \\
0 & -1 & 0 & 0 & \cdot & \cdot & \cdot & 0 \\
0 & 0 & -1 & 0 & \cdot & \cdot & \cdot & 0 \\
0 & 0 & 0 & -1 & \cdot & \cdot & \cdot & 0 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0
\end{bmatrix}
$$

$A, B,$ and U are M-square matrices; U_{hj} is a diagonal matrix using the elements of u_{hj}; and e is an M-vector where M is equal to or greater than the index number of the oldest age group in the problem.

Maximum Principle

The problem of maximizing objective function (17) subject to the constraint equations (18a) through (19b) is the DTOCP that can be solved by the discrete time maximum principle. As stated above, the maximum principle is a theorem that the constrained maximization of equation (17) can be decomposed into a series of subproblems. In each time period, the following Hamiltonian is maximized with respect to u_{hj} and w_{hj} subject to constraints. The Hamiltonian for year j can be written as

$$
\begin{aligned}
H_j = & \int_0^{Q_j} D_j^s(n)dn + \int_0^{\tilde{Q}_j} D_j^p(n)dn - C_j \\
& + \sum_h \lambda'_{h,j+1}[(A + BU_{hj})x_{hj} + v_{hj}e] \\
& + \sum_h \phi'_{h,j+1}(Az_{hj} + w_{hj}e)
\end{aligned}
$$

where

$$\lambda_{hj} = \rho[d S_j^*(x_j, z_j)/dx_{hj}] \tag{20a}$$

$$\lambda_{hj} = \rho[(d s_j^*(x_j, z_j)/dx_{hj}) + (A+BU_{hj}^*)'\lambda_{h,j+1}]$$

and

$$\psi_{hj} = \rho[dS_j^*(x_j, z_j)/dz_{hj}] \tag{20b}$$

$$\psi_{hj} = \rho[(ds_j^*(x_j, z_j)/dz_{hj}) + A'\psi_{h,j+1}]$$

The derivatives with respect to vectors are a gradient vector, and $S_{j+1}^*(\cdot)$ is the solution function in year $j+1$. The λ_{hj} and ψ_{hj} is the costate variables associated with x_{hj} and z_{hj}, respectively, and identify the shadow values of the hectares of forest and regeneration input. For this problem, the Lagrangian function can be written as

$$L_j^H = H_j + \sum_h \xi_{hj}'(1-u_{hj})$$

and the Kuhn-Tucker necessary conditions are

$$\partial L_j^H/\partial u_{hj} = [\phi_{hj}D_j^s(Q_j)+(1-\phi_{hj})D_j^p(\hat{Q}_j)-c_h'(Q_{hj}+\hat{Q}_{hj})]$$

$$X_{hj}q_{hj} - x_{hj}p_{wh}w_{hj} + X_{hj}'B'\lambda_{h,j+1} - \xi_{hj}' \le 0 \quad \text{for all } h$$

$$(\partial L_j^H/\partial u_{hj})u_{hj} = 0 \quad \text{for all } h \text{ and } i$$

$$\partial L_j^H/\partial w_{hj} = -u_{hj}x_{hj}p_{wh} + \psi_{h,j+1} \le 0 \quad \text{for all } h$$

$$(\partial L_j^H / \partial w_{hj}) w_{hj} = 0 \qquad \text{for all } h \text{ and } i$$

$$\partial L_j^H / \partial \xi_{hj} = (1 - u_{hj}) \geq 0 \qquad \text{for all } h$$

$$(\partial L_j^H / \partial \xi_{hj}) \xi_{hj} = 0 \qquad \text{for all } h \text{ and } i$$

These Kuhn-Tucker conditions, the state equations (equation 19a and 19b), and the laws of motion for the costate variables (equation 20a and 20b) identify a two-point boundary problem that can be used to solve both theoretical and numerical problems. These are the equations that we solve to find the optimal time paths for economic variables.

Ⅳ. Conclusive Remarks

Because the maximum principle proposed by the previous literature can be applied only to the problem in which the state equation is in the form of the difference equation, we developed our maximum principle to be used for the DTOCP in which the state equation takes the form of general equation. When we compare our maximum principle with that proposed by the previous literature, we identify that the difference between them lies in the laws of motion for the costate variables. In particular, by applying the Bellman's optimality principle and backward recursion we found that the maximum principle proposed by the previous literature is a subclass of our maximum principle. We, therefore, can state that our maximum principle is a general solution technique for the DTOCP in economics. Finally, we took the TSM 2000 as an illustrative example to apply our maximum principle as a solution technique, and derived the equations that we use to identify the optimal time paths for the variables. We furthermore feel that our maximum principle can be elaborated for the stochastic DTOCP in economics.

◎ References ◎

1. Bellman, R., *Dynamic Programming, Princeton*, New Jergy; Princeton University Press, 1957.
2. Conrad, J. M. and C. W. Clark, *Natural Resource Economics: Notes and Prolems,* Cambridge; Cambridge University Press, 1987.
3. Dorfman, R., "An Economic Interpretation of Optimal Control Theory," *American Economic Review*, Vol. 59(4), 1969, pp. 817～831.
4. Halkin, H., "A Maximum Principle of the Pontryagin Type for Systems Described by Nonlinear Difference Equation," *Journal of SIAM Control*, 4(1), 1966, pp. 90～111.
5. Hanley, N., Shogren, J. F. and B. White, *Environmental Economics in Theory and Practice*, Oxford; Oxford University Press, 1997.
6. Hestenes, M. R., *Calculus of Variations and Optimal Control Theory*, New York; Wiley, 1966.
7. Kamien, M. I. and N. L. Schwartz, *Dynamic Optimization: The Calculus of Variations and Optimal Control in Economics and Management*, New York; North Holland, 1981.
8. Lee, D. M. and K. S. Lyon, "A Dynamic Analysis of the Global Timber Market under Global Warming: An Integrated Modeling Approach," Working Paper, Department of Economics, Utah State University, 2001, *Forthcoming at Southern Economic Journal.*
9. Long, N. V. and N. Vousden, Optimal Control Theorems, in J. D. Pitchford and S. J. Turnovsky (eds.), *Application of Control Theory to Economic Analysis,* North Holland,

Amsterdam; Netherland, 1977, pp. 11 ~ 34.

10. Pontryagin, L. S., Boltyanskii, V. G., Gamkrelidze, R. V. and E. F. Mishchenko, *The Mathematical Theory of Optimal Processes*, Translated by Trigonoff, K. N., Interscience, New York, 1962.

이산시간 적정제어이론의 응용사례에 관한 연구

김진형 · 이덕만

본 연구는 이산시간 적정제어이론을 사용하여 동태적 경제 문제의 최적해를 구하기 위해 사용되는 두 가지 형태의 극대원리를 분석하고 있다. 이산시간 적정제어이론에 관한 선행연구들은 상태방정식이 차분방정식의 형태를 띠는 경우에 적용할 수 있는 극대원리를 제안하고 있다. 그러나 본 연구는 상태방정식이 차분방정식이 아닌 일반방정식의 형태를 띠는 경우에 적용할 수 있는 극대원리를 개발하였다. 그리고 본 연구를 통해 개발된 극대원리와 선행연구들이 제안한 극대원리와의 차이는 공동상태변수의 운동법칙에 있음을 보여주고 있다. 특히 본 연구는 Bellman의 최적원리를 이용하여 본 연구에서 개발된 극대원리는 선행연구들이 제안한 극대원리를 포함하는 일반적인 극대원리임을 설명하고 있다. 마지막으로 본 연구는 목재공급모형인 TSM 2000에 포함된 경제변수의 동태적 최적화를 구하는 과정을 통해 본 연구에서 개발된 극대원리가 경제 문제에 적용되는 사례를 보여 주고 있다.